Linear State Changes

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The Process Dilation Equation

Let v and c define rates of change for an arbitrary process such that v is independent of c so that $(v,c) \equiv v \perp c$. Let c be a constant and require that a relation between v and c be consistent for all possible values of v for $v \leq c$.

1. The relation is trivial for $v = c$.
2. For $v < c$, the scaling factors t_0 and $t\,'$ must be introduced such that $ct\,' \geq ct_0$ and the condition $v \perp c$ requires that the processes must be related by the Pythagorean Theorem:

$$\left(ct\,'\right)^2 = \left(ct_0\right)^2 + \left(vt\,'\right)^2$$
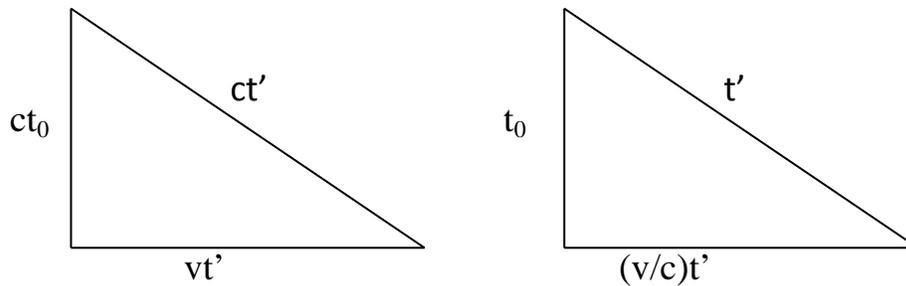
Where the relation $\beta = {v}/{c}$, $v \leq c$ defines the process rate of change as a fraction of c .

This gives us the rate change equation as a function of the scaling factors $t\,'$ and $t\,'$, since

$$t\,' = \frac{t_0}{\sqrt{1-\beta^2}} = \Gamma t_0, \quad \text{and } \Gamma = \frac{t\,'}{t_0} = \frac{1}{\sqrt{1-\beta^2}}$$

Note that this relation is independent of any physical interpretation of the parameters; in particular, any "space-time" coordinate system. This relationship is invariant with respect to a global scaling factor T on the rates of change, since:

$$t' = \frac{t_0}{\sqrt{1 - \left(\dfrac{v}{c}\right)^2}} = \Gamma t_0 = \frac{t_0}{\sqrt{1 - \left(\dfrac{vT}{cT}\right)^2}} \, .$$

If T is said to be a "gauge", then the relation is said to be "gauge invariant".

Covariance

Consider the relation $t' = \Gamma t_0$ for some (constant) final state t', where t_0 defines a (variable) initial state and Γ is a (variable) linear transformation to the final state.

If the relation is scaled by some constant (e.g. c, which can be equal to 1), so that $ct' = \Gamma(ct_0) = (c\Gamma)t_0$, the transformation c that transforms the r.h.s. into the l.h.s. (i.e. $c : t_0\Gamma \to t'$) is said to be co-variant in the parameters t_0 and Γ, since they transfer in the same way (in direct proportion) with respect to c to maintain the final state ct'.

Contra-variance

Consider the relation $\dfrac{t'}{\Gamma} = \Gamma^{-1}t' = t_0$, for some (constant) initial state t_0, where t' defines a (variable) final state and Γ^{-1} is a (variable) linear transformation back to the initial state t_0 (i.e., an inverse transformation between initial and final states w.e.t. covariance). If this relation is again scaled by c so that $c\left(\dfrac{t'}{\Gamma}\right) = (ct')\Gamma^{-1} = (c\Gamma^{-1})t' = ct_0$, we say that the transformation c is contra-variant in the parameters Γ^{-1} and t', since they must change in inverse proportion with respect to c to maintain constant initial condition t_0.

State Change

Let $S_0 = ct_0$ define an initial state S_0, $S_\Delta = vt_0$ define an intermediate state relative to this initial state, and let $S_f = ct'$ define the final state, based on the scaling factors t' and t'.

Then $t' = \dfrac{t_0}{\sqrt{1-(\beta)^2}}$ and $ct' = \dfrac{ct_0}{\sqrt{1-(\beta)^2}}$, so that $\left(\dfrac{S_f}{S_0}\right)^2 \left(1 - \left(\dfrac{S_\Delta}{S_0}\right)^2\right) = 1$ so that

$\left(\dfrac{S_f}{S_0}\right)^2$ and $\left(1 - \left(\dfrac{S_\Delta}{S_0}\right)^2\right)$ are contravariant w.r.t. unity. Note that if $S_\Delta = S_f$ there is a contradiction, which is resolved by the fact that in the original equation this implies that $vt' = ct$ so that a second "initial state" is created: $ct' = \sqrt{2}$.

Theory of Special Relativity

To fully understand the Special Theory of Relativity as applied to the Space-time and Mass-Energy domains, one has to deconstruct v/c (interpreted as "velocity") into its space-time components; one then finds that in the space domain, "length contracts" and in the time domain "time expands", and that Γ is interpreted as a "mass density", covariant in time, and contra-variant in space.

Stay tuned…..