The Rest Mass of Light

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(My take on Relativity)

One of the most important results of Maxwell's analysis was his conclusion about the "speed of light", which is given by: $c = 1/\sqrt{\varepsilon_0 \upsilon_0}$ (along with his equations) , which is imprinted on the sweatshirts of many undergraduate students in physics and electrical engineering every generation. The constants ε_0 and υ_0 are the permittivity and permeability constants, which refer to Gauss's definition of the static E field and Faraday's concept of the static B field in the space domain, as well as Coulomb's and Ampere's laws in the charge and time domains. By analyzing the consequences of these laws, and using the ideas of the integral forms Stokes' and Green's theorems (dot and cross products), as well as "infinite charge sheets", etc., he was able to arrive at a result that modelled the propagation of light in the vacuum (which could be adjusted for non-vacuums by adding the D and H fields for transparent mediums). The term above comes from the "displacement" current, which Maxwell had to add to model the transport of electromagnetic energy across space between the plates of a capacitor as alternating current (and provides for the concept of the propagation of light).

This resulted in a huge step in technology, but theoretically the concept of the medium raised questions, since the motion of an emitter through the medium should result in a detectable difference in direction for experiments such as the Michaelson-Morley apparatus.

When no such motion was detected, Lorentz analyzed the results and produced a mathematical formula that was consistent with the results (called the "Lorentz Transform". When analyzed in the domain of space-time, the difficulty could be explained by "shrinking" the apparatus in the direction of motion, retaining Newton's laws and the Galilean Cartesian coordinate system to define velocity.

However, this was unsatisfying, since Maxwell's original equations of electromagnetism were now inconsistent with the model.

Einstein's solution was to ignore the concepts of space and time as developed from the permittivity and permeability constants; and developed the Special Theory of Relativity which rests on the following two assumptions:

1. The speed of light is homogeneous and isotropic.
2. The speed of light is constant and independent of the velocity for all possible values of v: and "inertial frame" in spacetime was assigned to each value of velocity, with orthogonal coordinates assigned to the specific value of c determined by measurement experimentally.

By simply declaring the speed of light constant, Einstein avoids the difficulties of the Lorentz transform, since the medium is now a constant no matter what velocity is modelled.

The difficulty now was that the field is modelled as a coordinate "net" but no particle is located; that is, there is no origin, since the equation $E_0 = m_0 c^2$ involves negative values only if one is prepared to accept a negative rest mass (and Energy) as possibilities. This implies that the energy and mass must be either both positive or both negative.

However, from Maxwell's equations, we have: $1^2 = (\upsilon_0 \varepsilon_0) c^2$; that is, $m_0 c^2 = 1^2$ if we identify $m_0 = (\upsilon_0 \varepsilon_0)$ with rest mass, which is reasonable if one equates charge with mass, since Ampere's and Coulomb's laws involve forces and Maxwell's equations model transport of energy across space in the displacement current term as mentioned above. However, if this is applied, the rest mass is now positive definite only, which is emphasized by the relativistic energy equation for the relation between v and c:

$$P = m'\left(\frac{v}{c}\right)$$

$$(E')^2 = (Pc)^2 + (E_0)^2$$

This means that there cannot be a negative velocity either for v or c in the underlying coordinate system. The issue is resolved by defining the rest mass by a mass creation time T and a mass creation rate C, resulting by the rest mass $M_0 = \rho C T^0, \quad \rho = 1$ and the relation between a "velocity defined" mass $M_v = VT'$ with the total mass defined by the "Mass Creation" equation:

$T' = \dfrac{T_0}{\sqrt{1 - V^2/C^2}} = T_0 \Gamma$, where Γ is interpreted as a density. This equation is often referred to as a "Time Dilation" equation, but notice that there is no explicit reference to space.

This approach is explained more fully in the document:

Mass Energy Derivation

Minkowski developed a matrix to describe the fundamental concepts behind the Theory of Special Relativity as related to Cartesian coordinate system consisting to "three dimensions of space and one of time", but this can be shown to be misleading in terms of the fundamental assumptions of STR (homogeneity and Isotropy). The imaginary coordinate $i = \sqrt{-1}$ is shown to be a "destruction" operator which "destroys" the coordinate system, leaving the masses intact in two dimensions.

The reason this happens is because mass in the system above can be interpreted as a spatial "ruler" common to both v and c in the spatial coordinates, so that v/c is only defined by time.

$v/c = \frac{x_v}{t_v}\frac{t_c}{x_c} = \frac{t_c}{t_v}$, $x_v = x_c$; the resulting analysis gives the same result for $x_v = x_c = c_t = CT = X_c$ with no corresponding "space" parameters for M_v.

However, one can show that a density $\Gamma_x = 1/\Gamma_t$, $\Gamma_t = \Gamma^{C,V}$ can also be related by assuming $t_v = t_c$ that is the inverse of the "time" density of the coordinate domain, but has no independent existence in the Mass-Energy Domain (since one is only considering rest mass M_0). This result is the common concept of space "contracting" with increasing velocity, as opposed to "time dilation".

That is, the name "Rest Mass" is misleading, since one is really modeling an initial condition defined by length, not velocity.

The Mass of light from Coulomb's Law and the Vacuum Permeability

The equivalent of rest mass for light can be derived directly from Coulomb's Law and the Vacuum Permeability.

Coulomb's Force Law

$$F_{Coulomb} = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{r^2}$$

Vacuum Permeability

The force per unit length for two wires is given by:

$$F = \frac{\mu_0 I^2}{2\pi r} = \frac{\mu_0 dq^2}{2\pi dt^2}$$

For a specific value of q and t, and considering only the effect of a single wire:

$$F = \left(\frac{\mu_0 I^2}{4\pi r}\right) = \left(\frac{\mu_0 q^2}{4\pi r t^2}\right)$$

For $r = ct$, we have:

$$F_{Permeability} = \frac{\mu_0 c^2}{4\pi r^2} q^2$$

Equating the two, we have:

$$F = \frac{1}{4\varepsilon_0} = \frac{\mu_0 c^2}{4}, \text{ so that } c^2 = \frac{1}{(\varepsilon_0 \mu_0)}$$

$$1 = (\varepsilon_0 \upsilon_0) c^2 = m_0 c^2$$

I discuss these topics more fully in the documents:

The Creation of the Universe

The Destruction of the Universe

Note: Although the space-time "destruction" operator $i = \sqrt{-1}$ eliminates the coordinate system from the Minkowski matrix, it is applied in the Pauli matrices (non-relativistic), modeling the "destruction" of the B field only with no "E" initial condition) and the Dirac Equation (which models the rest mass as an initial condition).