The Universe and STR *(by Chuck Keyser)*

(06/08/2012 at 2:25 PM pm, PST) I am currently working on the section in blue text.
(06/10/2012) Added a section in Lorentz transform discussion..
06/11/2012) Added section close to pg. 36 (possibly redundant)
(06/17/2012) Added section in Lorentz transform discussion.
(07/10/2012) Added section describing spacetime circles as foundation of QED

Note (07/26/2012): I have also started on a paper which analyzes the Lorentz Transform. It can be found via my Web Page at http://www.flamencochuck.com and clicking on the "Theory of Relativity" link.

I'm still working on this, but think I'm on the right track. But the blind men's elephant has turned out to be a large centipede……

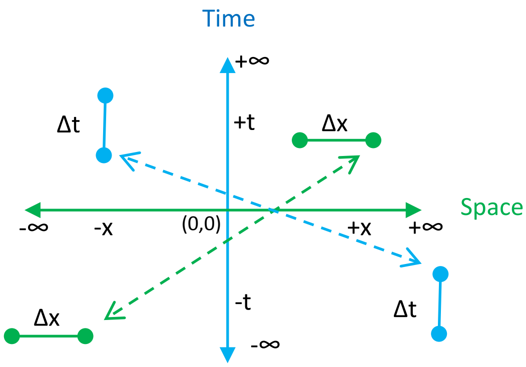
Before we start: I am sure that someone has provided this analysis somewhere; I just haven't seen it myself. I am attempting to clarify the physical relationships between Newton's equations, STR, and Maxwell's equations, with obvious extrapolations to both QED and Gravitation. This is very, very much a work in progress.

Light has two roles to play in this discussion of the Special Theory of Relativity (STR). The first is its role as a velocity in the space-time; that is, as the characterization of motion between coordinate points as a mathematical abstraction, and the second as in defining mass in the energy-momentum of a physical particle. Newton's laws treat these concepts as independent by specifying mass as independent of velocity in space and time. Einstein modifies the laws to show that mass defines space and time for an individual particle specified by its "rest" mass, which is then modified by its velocity (and thus its momentum and kinetic energy).

We begin with an analysis of Newton's ("Galilean") velocity in terms of space and time, a brief discussion of Newton's laws of motion, and a discussion of Special Relativity.

"Galilean" Space and Time

The concept of Galilean Space-Time (as clarified by Minkowski) is expressed by independent (orthogonal) space-time axes, extending to infinity in three dimensions of space and one of time, with the fundamental assumption that space and time are homogeneous and isotropic. This means that all physical laws are translationally invariant in space and time (i.e., true "anyplace" and "anytime"). Since space is isotropic, any point in space can be defined in these (3+1=4) dimensions with reference to an origin (0,0,0,t) in terms of a radius $r^2 = x^2 + y^2 + z^2$ (the equation of a sphere, so that a point with respect to the origin in isotropic spatial dimensions can be specified by two dimensions (r,t) = (x,t).



All physical objects actually exist at the point t=0; i.e., the distance from the four dimensional origin (0,0,0,0). An individual point is located at a space-time point (x,y,z,0), or (r,0). Physical laws must remain true for any value of t, so this "distance" is translationally invariant in time; (r,0) = (r,t) = r, independent of time.

Since space is isotropic, we can identify a specific radius with a discrete increment $\Delta r = \Delta x$, which defines a circle that is then translationally invariant in two dimensions of space and one of time (i.e., a "ruler" or "stick" can be moved over a spatial area and time without changing it). In three dimensions, the object would be a spherical volume, which again could be moved "anywhere) in four dimensional space-time. The radius in each of these cases would assume the role of a geometric "metric" for the space under analysis.

Similarly, a discrete interval in time Δt (a "clock") can be moved anywhere in space-time, so a spatial geometric object can exist "any-when" in time. (r,t) – that is, space and time are independent of each other, and therefore can be represented by orthogonal axes.

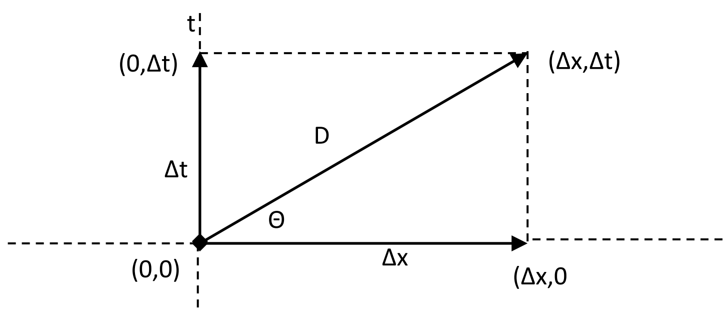
For the present, we will restrict our analysis to a two dimensional "space-time" consisting of one dimension of space and one dimension of time (x,t).

The space-time diagram can be viewed in two ways:

1. "Static" space-time: As an infinite mesh in space and time, where all objects are located at points (x_i,t_i)
2. "Dynamic" space-time: As a "time line" moving upward, with all objects moving back and forth in space as the timeline progresses.

A **velocity** "v" is specified by two points in space-time; if both points are on the space axis, the velocity is infinity ("instantaneous" movement, $v = \infty$). If both points are on the time axis, the velocity is 0 ($v = 0$)

In Galilean coordinates, the velocity is specified by the ratio between space and time, $v = \frac{\Delta x}{\Delta t}$



Velocity can be specified in terms of other parameters (the "diagonal" D, the angle Θ, etc.) Any combination of two of these parameters (i.e., Δx, Δt, D, or Θ) is sufficient to define velocity using trigonometric relationships, e.g. $v = \frac{\Delta x}{\Delta t} = \frac{\mathrm{D}\cos\Theta}{\mathrm{D}\sin\Theta} = D\cot\Theta$.

The velocity then defines the rate of change of position of a coordinate "particle" with respect to time along the space axis since$\Delta x = v\Delta t$, where Δx and Δt are distances from the origin (0,0). Multiple particle cooridnates can have different origins (which can be co-located), since the velocity relationship is translationally invariant; different values of Δx and Δt define different "meshes", or "frames", which define particle coordinates of different velocities – specifying an origin determines to position of the particle in space-time, or its movement along the space axis relative to other particles.

The line represented by D is called the "world line" of the particle. In "Static" space time, it can be thought of as a "line of existence" in space time. In "Dynamic" space-time, the particle moves along the x-axis as the time axis moves upward, with the position of the particle at the juncture of the moving space-time origin (space-time axis).

Newton's Three Laws of Motion

Newton adds the idea of the mass of a particle to Galilean coordinates, which is defined by Newton's laws of motion:

1. **Every object in a state of uniform motion tends to remain in that state of motion unless an external force is applied to it.**

This relation is characterized by the momentum "P", which is a constant defined by a constant mass multiplied by a constant (by definition) velocity; that is, mass and velocity (and thus momentum) for each object ("particle") are translationally invariant in space-time, since space and time coordinates are not defined explicitly (only implicitly in the definition of velocity).

That is, $\mathbf{P_i = m_i v_i}$ for each individual particle i.

2. **The relation between an objects mass m, its acceleration a, and the applied force is F= ma.**

Acceleration "a" is defined by a change in velocity (and thus a change in momentum) for an individual particle so that $a = \Delta v$.

$F_i = m_i a_i = m_i(\Delta v_i) = \Delta P_i$ Again, this relationship is translationally invariant – space and time coordinates are not involved explicitly.

3. **For every action there is an equal but opposite reaction**. This is the law of inertia; that is, if a force is applied to initiate a momentum $\Delta P = m(\Delta v)$ from a state P=0 (i.e., $v = 0$), then the resistance to this force is equal and opposite, so the final state (P = constant) has no further forces operating on it. This is expressed by the concept of kinetic energy, which is constant and positive (independent of the sense of velocity).

This can be expressed by the total kinetic energy $T_2 = m_2(\pm\Delta v_2)^2$. (This is a model of two particles traveling in equal and opposite directions (i.e., with positive and negative velocities).

The kinetic energy for a single particle in one direction is then given by $T_1 = \frac{1}{2} m_2 \Delta {v_2}^2$.

Newton's laws imply that the dynamics of a single particle can then be expressed as a polynomial expansion to second order of velocity:

$1 = m \pm (m\Delta v) + \frac{1}{2} m(\Delta v)^2$ for a single particle with positive OR negative velocity
$2 = 2m + (mv) + (-mv) + (mv^2)$ - For two particles, in a system satisfying all three laws (including conservation of kinetic energy).

More particles can be added to the system; if there are no interactions, the kinetic energy only depends on the velocities of the individual particles, and the description of the system is again invariant in space and time.

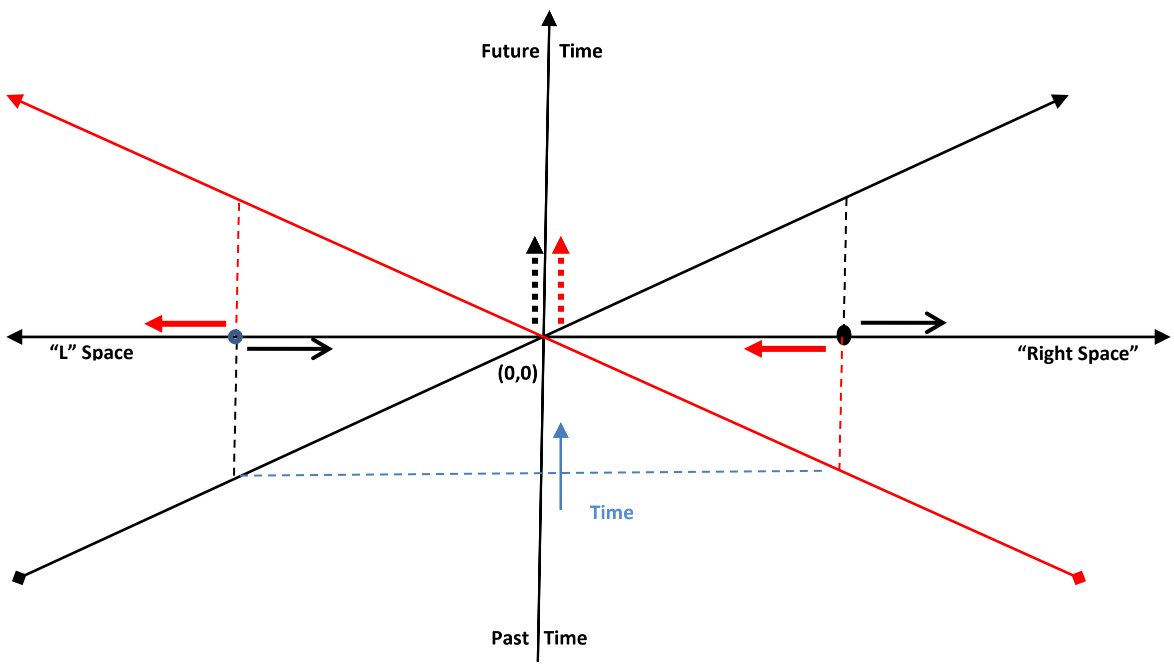
(Interactions between particles involve the characterization of "potential" energy, which depends on space and/or time, and beyond the scope of this context).

Degeneracy

Consider the case where two particles start toward each other from the same distance, meeting at the origin (which is the "center of mass" in this example). Since we have specified that they don't interact, they will pass through the origin and continue in opposite directions afterwards (by Newton's second law, the creation and destruction of the particles lie in the infinite past and infinite future, respectively).

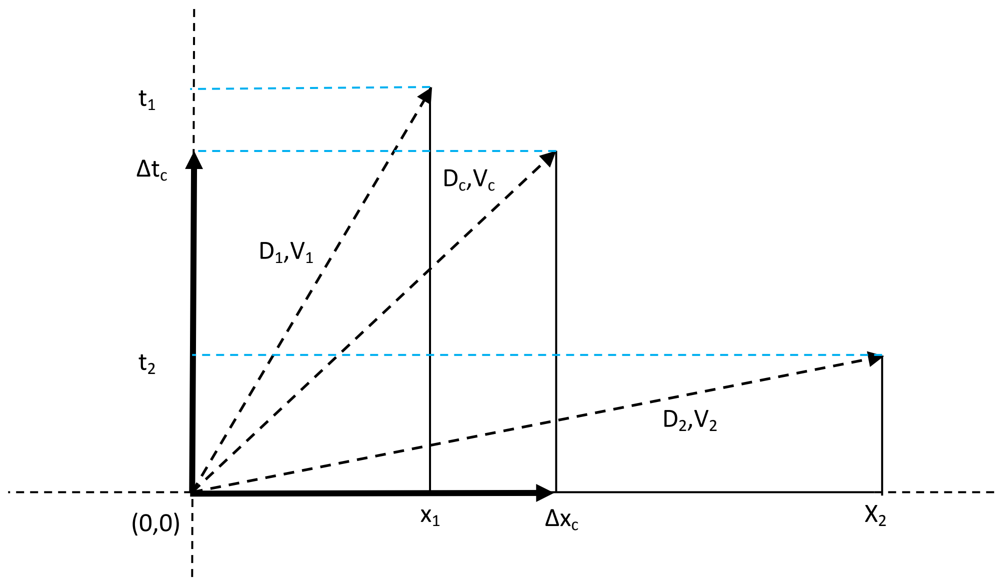
Nevertheless, we have not specified location in Newton's laws. We can imagine that the two particles can be represented by a single particle with both positive and negative momentum at the origin. In addition, we can say the system is represented by that particle for all past and future times – that is, the system as a whole is **degenerate**.

That is, we could imagine the particles stopping at the origin, merging, and continuing on a vertical world line at the origin with total mass = 2m, with $v = 0$; in this case, the kinetic energy expended in the distant past in starting the particles on their way (at an "emitter) is returned at the origin when the particles merge (at an "absorber"). In this case we would say the degeneracy was "lifted" at the emitter and "returned" at the absorber, which in this case is located at the origin.



Comparison of velocities in Space-Time

Newton's laws characterize each particle as having its own unique velocity; that is, the momentum and energy of each particle (i) are defined individually by m_i and v_i which "travel with" the particle. As a prerequisite to STR, it is important to establish one velocity in terms of which all other velocities can be defined. Consider a number of velocities defined at a common origin:



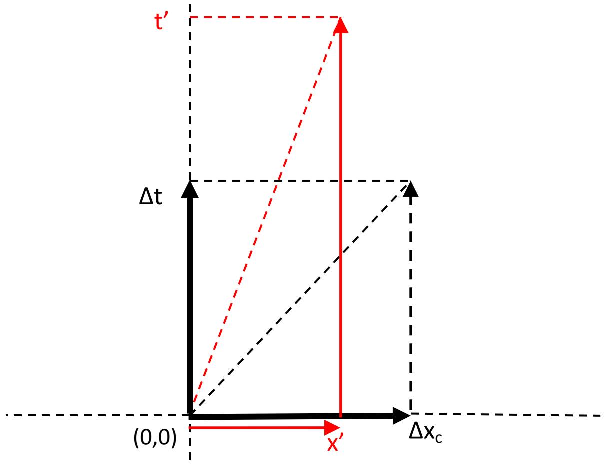
We define a single velocity "c" where $c = V_c = \frac{\Delta x_c}{\Delta t_c}$, in terms of which all others can be defined by the factor $\beta_i = \frac{V_i}{V_c}$, since $V_i = \beta_i V_c$. In the diagram above, $V_1 < V_c < V_2$; that is, x_2 moves faster than x_1 along the x axis, while t_2 moves slower than t_1 up the time axis. That is, it takes longer for x_1 to travel a given length than x_2 , and it takes longer to get there. If t = 0 (no time dimension), x can get anywhere instantaneously, but if x = 0 (no space dimension) it takes x an infinite amount of time to go an infinitesimal distance – (actually 0).

From now on, we will refer to the velocity $c = V_c = {}^{x_c}/_{t_c}$, the velocity of light.

(Important: note that no assumptions about limits on velocity with respect to the speed of light have been made at this point.)

Comparison of velocities in terms of c

We choose a velocity v= x'/t' **<** c = x_c/t_c = 1; that is v < c, and note that the space and time values are shorter and longer, respectively, than those of c.

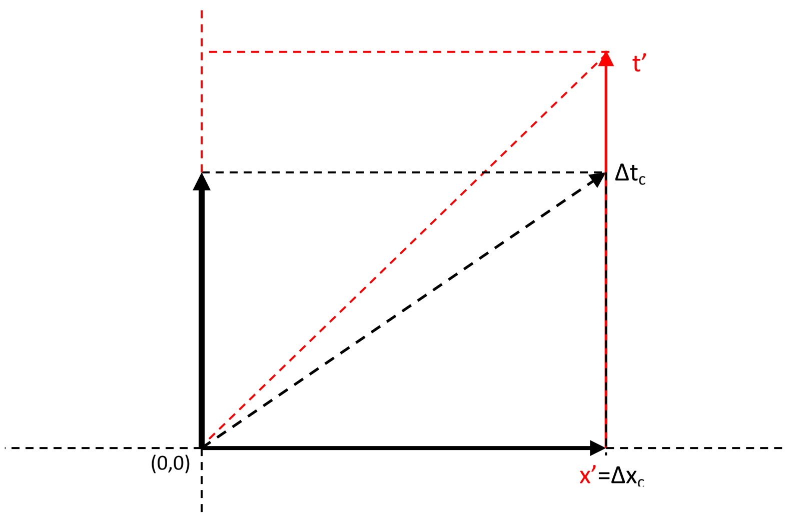


$c = v_c = \frac{\Delta x_c}{\Delta t_c}$ and $v = \frac{x'}{t'}$

We can express the arbitrary velocity v in terms of the basis v_c by setting one of the space-time intervals (Δx or Δt) to be common to both velocities.

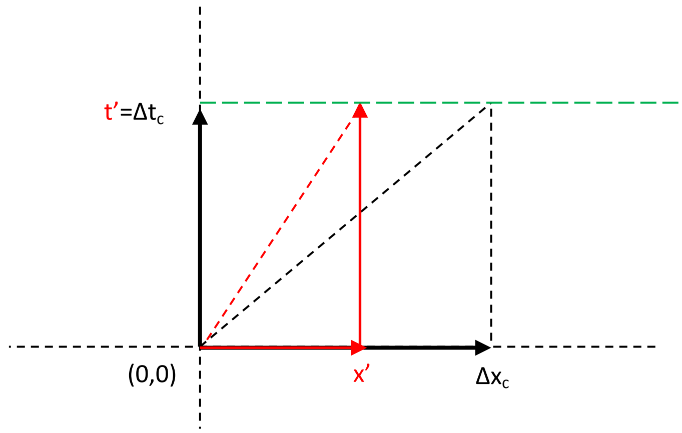
Common Space Length ($x' = \Delta x_c$)

For example, if the space lengths of the velocities are set equal so that $t' > t_c$ in the example below, the velocities can expressed in terms of time alone:



Then β = $\frac{v}{c} = \frac{\Delta t_c}{t'}$ where $t' > \Delta t_c$ (in this example) , so that $v = \frac{\Delta t_c}{t'} c$. Since $t' > \Delta t_c$ time is greater ("clocks slow down" in the definition of this (slower) velocity in comparison to c. That is, it takes longer for a coordinate point traveling at velocity v to travel a length $x' = x_c$ than it does for a photon (point of light).

Common Time Length ($t' = \Delta t_c$)



Similarly, a point with velocity v travels less far than a photon with velocity c (x' > x_c) in the same time interval $t' = \Delta t_c$.   Then $= \frac{v}{c} = \frac{x'}{\Delta x_c}$, with $x' < \Delta x_c$, so $v = \frac{x'}{\Delta x_c} c$. That is "space contracts" since $x' < \Delta x_c$ (in the above example).

The relation $\beta = \frac{v}{c}$ is then defined for any velocity expressed as a fraction of the speed of light by specifying one parameter of the velocity in either space or time provided we set the second parameter (time or space, respectively) to be equal in our definition. Since the origin is arbitrary in space and time, β is translationally invariant as well.

This means that both photons and coordinates "move" along the same axis – space if time is the common coordinate, and time if space is the common coordinate, with v expressed as a fraction of c.

Therefore, by this definition, **all velocities are dependent on the speed of light**, since $v = \beta c = \frac{v}{c} c$.
This analysis defines velocity in terms a "Galilean" coordinate system as defined above, which is the basis of Galileo's coordinate system. Also, v and c have the same "direction" along the x axis.

(Important - note that calculus – the introduction of infinitesimals in the sense of $v = \frac{dx}{dt}$ – is not required for this description (and even wrong in this context, since it implies ambiguity in definition of velocity relative to c. According to my knowledge, Newton actually invented calculus to explain gravity).

Summary

We refer to the parameter of c selected as common as "proper".

Proper Space - If we select a common space value for comparing velocities (i.e., $x_v = \Delta x_c$), we have $\beta = \Delta t_c / t_v$; that is, $t' = t_v = \frac{\Delta t_c}{\beta}$ Again, as v decreases relative to c, β decreases, and t' increases (i.e., time "slows down" in its role in defining velocity relative to c) . That is, velocity is "inversely proportional" to time relative to c.

Proper Time - If we select a common time value for comparing velocities (i.e., $t_v = t_c$), we have $\beta = x_v / x_c$; that is, $x' = x_v = \Delta x_c \beta$ (as v decreases, β decreases, and x' decreases (i.e., space "contracts" in its role in defining velocity relative to c). That is, velocity is "directly proportional" to space traveled relative to c.

(Note: The **"Twin" paradox** is a clever play on conceptual misdirection. See the addendum.)

The Special Theory of Relativity

Newton's laws define mass and velocity independently of each other (m,v), with the momentum given by a mass particle defined by $P = mv$ and its kinetic energy defined by $T = \frac{1}{2}mv^2$. The velocity is defined in terms of independent space and time (x,t) by the relation $v = \frac{x}{t}$, and the mass m is a constant for all values of v, with different masses defining different particles (and thus different momenta and kinetic energies.

The Special Theory of Relativity (STR) is a revision of Newton's Laws by defining velocity in two incompatible ways, one in Newton's Space-Time in "Galilean" coordinates (x,t), and the second in a "Momentum – Energy" (K,E) space. Both of these spaces are characterized by an unchanging parameter called an "invariant", and it is necessary to understand both in the analysis of STR.

Space-Time ("Galilean") Coordinate System

Einstein introduces the velocity of "c" (the speed of light) as an **invariant** to Galilean Space-Time The velocity c is then an reference in terms of which all other velocities can be defined by the relation $v = \frac{v}{c}c = \beta c,\ \beta = \frac{v}{c}$. Since $c = \frac{\Delta x_c}{\Delta t_c}$ and $v = \frac{\Delta x_v}{\Delta t_v}$ for discrete values of space and time, the ratio β can be defined in space-time by either assuming a common space increment OR a common time increment.

In the case of a common space increment, $\Delta x_v = \Delta x_c$, in which case $\beta = \frac{\Delta t_c}{\Delta t_v}$, and $\Delta x = \Delta x_v = \Delta x_c$ is called "proper length".

In the case of a common time increment, $\Delta t_v = \Delta t_c$, in which case $\beta = \frac{\Delta x_v}{\Delta x_c}$, and $\Delta t = \Delta t_v = \Delta t_c$ is called "proper time".

Again, it must be emphasized that either proper length or proper time must be selected for a consistent definition of β in defining the ratio of v to c. The proper length or time selected is then the **invariant** in the definition of the velocity in terms of c.

Note: Einstein proposes his "train" example to illustrate the idea of proper length (see the addendum).

If the coordinate system (Δx_c,Δt_c) is thought of as a "mesh" (frame), then it defines an absolute coordinate frame for SRT; namely, the frame in which c is defined, with all other velocities defined as frames with differing meshes in EITHER space or time, according to whether proper time or space is selected for the analysis. In particular, infinitesimals (i.e., calculus: $v = \frac{dy}{dx}$) cannot be used to define the relations between the meshes uniquely, since infinitesimals are ambiguous in defining the "proper" length or time interval.

In our discussion, depending on which is chosen, the parameter indicating velocity other than c will be indicated by a prime, so that $\beta = \frac{v}{c} = \frac{x'}{x} = \frac{t}{t'}$.

Note that no assumption regarding the limits of v relative to c has been made at this point, and different velocities can still be added, e.g. $v = \beta c = \left(\frac{v_1}{c} + \frac{v_2}{c}\right)c = \frac{(v_1+v_2)}{c}c = \frac{v_{12}}{c}c = \beta_{12}c$

In Galilean Space-Time the velocity v is dependent on the velocity c via the parameter β; in particular, they are aligned on the space axis (the "reality" axis) in space-time, on which particles actually exist (i.e., move).

A bit of Background

The Special Theory of Relativity was initially highly controversial, since the physics community had been convinced that Maxwell's theory of electromagnetism required waves to propagate signals. Because waves supposedly require a medium (the "aether") to propagate, the derivation of the speed of light using space and time in Maxwell's equations was considered a fundamental proof that the equations were universally correct.

However, the failure of the Michaelson-Morley experiments to detect such an aether was explained by Lorentz in terms of a modification to space and time to compensate for this failure. There is much to discuss regarding these issues, but Einstein's solution was to model light as a particle ("photon") as above (interference can be explained by a further modification of light as a "wavicle", i.e., a "radar" pulse, with the pulse width the "coherence length" of the particle (interferometer), and the frequency described by the DeBroglie relation.

It is important to note that according to Maxwell's equations, light has no mass. This means that the electromagnetic field equations are not unique with respect to the derivative of a scalar field. However, a "vector potential" can be defined which adds and subtracts this field when working with the electromagnetic parameters. This effectively allows the expression of mass as a "charge to mass" ratio in the Lorentz force equation – which effectively changes the permeability constant if mass is added, so the speed of light can remain constant in the field equations.

This question can be avoided in SRT by observing that the speed of light can be derived directly from Coulomb's and Ampere's laws, so that (as in the Maxwell derivation) the speed of light is constant to the extent that the permittivity and permeability are constant.

(Note: this means that c is not a universal constant even in empty space, and is only constant at an "effective LaGrange point", where there is no acceleration – e.g., the center of the earth, or you or I on its surface – that is, where all forces are balanced so there is no acceleration. However, further discussion is beyond our present scope).

The "Speed" of Light from Coulomb's and Ampere's laws.

Consider two charges far apart, which are brought together to a distance r. As they approach each other, they will feel a force which can be measured in terms of the permittivity constant ε_0.

Coulomb's Law is then defined as $F = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{r^2}$

Now consider two wires far apart, brought together to the same distance r. We **define** the current in terms of the charge in the wires per unit time to have an equal force at the same distance in terms of the permeability constant μ_0.

Ampere's Law is then defined as $F = \frac{\mu_0}{4\pi}\frac{i^2}{r^2}$

Setting these two forces equal, so there is no acceleration, we have $\frac{i^2}{q^2} = \frac{1}{\mu_0\varepsilon_0}$

If current i is defined as a given charge q traveling through a given length x in a given time t, we can define current as $i = \frac{qx}{t} = qv$, where v is the velocity of the charge in space-time. Then $\frac{i^2}{q^2} = \frac{q^2v^2}{q^2} = \frac{1}{\sqrt{\varepsilon_0\mu_0}} = c$, where c has the numerical value of the "speed of light", but is actually defined by the permittivity and permeability constants, in which mass is subsumed as a result of equating the electric and magnetic forces.

Since Maxwell's equations derive the speed of light by implicitly including these constants in when defining the E and B fields, the mass can be set to 0 (up to the derivative of a scalar field, which will again introduce the vector potential so as to negate these in the field equations.

1. Note that that the above results are independent of the orientation of charge or the relative position of the wires, which implies that the "speed" of light is isotropic (independent of direction), and therefore so is v. The distance r can then be considered a radius r=ct in two dimensions (x, y), which are independent of direction.

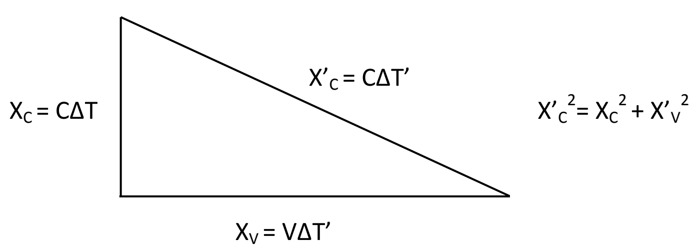
2. Note that the "speed" of light is constant to the extent that ε_0 and μ_0 are constant in this context.

The Energy-Momentum Coordinate System

Contrary to the Space-Time axis, the quantities V and C are defined as independent (V,C), and so form orthogonal (E-K) axes, where C is an invariant for all values of V. This is often paraphrased as "the speed of light is independent of the motion of the source". (We will distinguish parameters in the E-K coordinate system by using capital letters).

Since any relation between V and C must hold for all values of V, we can relate them through a distinguishing parameter ΔT and ΔT', so that we form a new "LENGTH" space, (CΔT,VΔT').

If we then require that the relation between CΔT and VΔT' be linear, then we form CΔT' as the diagonal of a corresponding right triangle (since V≠C and T≠T'), so that $(C\Delta T')^2 = (C\Delta T)^2 + (V\Delta T')^2$. Solving this equation gives us the relation $V = C\sqrt{1 - \frac{\Delta T^2}{\Delta T'^2}}$, where ΔT' ≥ ΔT by the requirements of linearity (i.e., the right triangle).



Note that the "Lengths" are squared in the above diagram, and that the same relationship holds for both positive and negative lengths (corresponding to +/- lengths). This suggests that mass (and thus momentum and energy) can result as second order quantities, implying conservation of energy and momentum for a single particle, as in Newton's theory.

If the dimensions of C and V are "velocities" and those of T and T' are "times" then this describes a relation between "lengths" defined in terms of "velocity" multiplied by "time"; multiplying the lengths by a "density" ρ allows us to define a relation between masses, with an invariant ("rest") mass defined by $m_0 = \rho C\Delta T = \rho X_C$, a "kinetic" mass defined by $m_v = \rho V\Delta T' = \rho X_V$, and a "total" mass defined by $m_t = \rho C\Delta T' = \rho X'_C$

It should be emphasized that this density is a density per unit "light stick", that defines mass in terms of the "mass" component of light, NOT a "density" of mass per unit length in space-time.

We then have $m_t{}^2 = m_0{}^2 + m_v{}^2$ as the relation between total mass, rest mass, and kinetic mass.

Multiplying the relation by C^4, we have:

$$(m_t C^2)^2 = (m_0 C^2)^2 + (m_v C^2)^2$$

If we identify relativistic momentum as $P = m_0\beta = m_0\frac{V}{C}$, total energy $E_t = m_t C^2$ and rest energy $E_0 = m_0 C^2$, we have the relativistic relation between energy and momentum:

$$E_t{}^2 = E_0{}^2 + P^2C^2$$

Then $\frac{V}{C} = \sqrt{1 - \frac{m_0{}^2}{m_t{}^2}}$, is defined as a ratio between the rest mass and the total mass (and not space and time, since t' is unrestricted in space time, but T' ≥ T in Energy Momentum; i.e., X=CT is an invariant, where different proper "Lengths" define different "rest masses" $m_{0_i} = \rho_i CT$

The "Time Dilation" equation

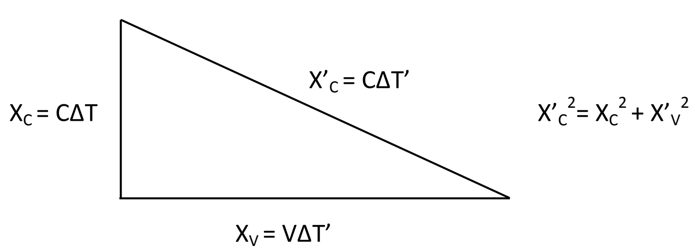
If we assign ρ=1, then we have $T' = \frac{T}{\sqrt{1-\frac{V^2}{C^2}}} = T\gamma, where\ \gamma = \frac{1}{\sqrt{1-\frac{V^2}{C^2}}}$

The equation $T' = \frac{T}{\sqrt{1-\frac{V^2}{C^2}}}$ is often called the "Time Dilation" equation, even though it is derived from independent velocities (v and c), which contradicts the Galilean space-time model. To see this, we compare the two models; in both coordinate systems, we will use the common space models.

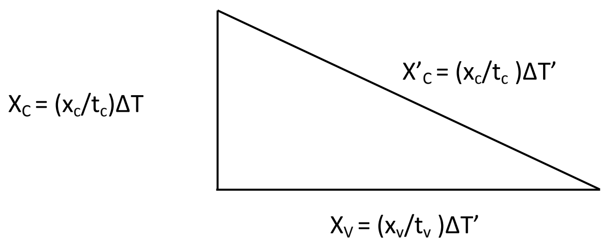
Space-Time and E-K relationship

The E-K relation (V,C) and the space-time relationship (v,c) are related by Coulomb's and Ampere's Laws (which determine $C = \frac{1}{\sqrt{\varepsilon_0\mu_0}}$ and the Maxwell derivation of C in space-time, show that a point where C is constant (to the gradient of a scalar field) is related to c by the definitions of electro-magnetic fields propagating through space-time).

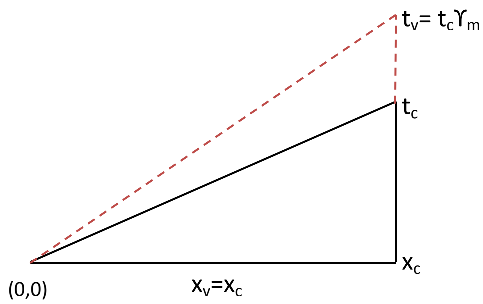
This means that the E-K relationship changes the definition of velocity in space time, which relates how "far" a massive particle traveling at velocity v will move in space-time compared to a photon traveling at c in terms of an increase in mass due to an increase in kinetic energy by a scaling factor that arises because the rest mass is declared independent of its kinetic energy. The rest mass is defined by $m_0 = \rho CT$, where ρ is a density (which is set equal to 1 in our example), which is applied to all values of mass, energy, and momentum, and C and T have the dimensions of space and time, respectively. The relationship can then be defined by "mass equivalent lengths", as illustrated (again) in the following diagram



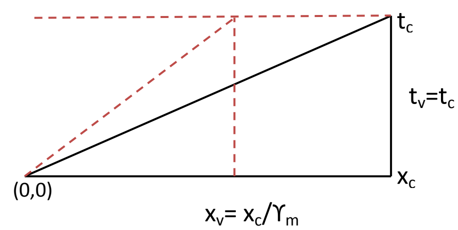
In order to connect the EK relationships to space-time, we assume that v = V and c = C (as in Maxwell's equations). Then $C = \frac{x_c}{t_c}$ and $V = \frac{x_v}{t_v}$, where the subscripted parameters are assumed to be discrete lengths (e.g., $x_c = \Delta x_c$)



The above relation yields: $(\frac{x_c}{t_c})^2(\frac{t_v}{x_v})^2 = \frac{1}{1-\left(\frac{T}{T'}\right)^2} = \frac{1}{1-\left(\frac{\rho c T}{\rho c T'}\right)^2} = \frac{1}{1-\left(\frac{m_0}{m'}\right)^2} = \gamma_m$, where m' is the total relativistic mass, and $\gamma_m = \frac{1}{1-\left(\frac{m_0}{m'}\right)^2} = \frac{m'^2}{m'^2 - m_0{}^2} = \frac{m'^2}{m_v{}^2}$, so that γ_m is the relation of total relativistic mass to kinetic mass (referred to the "speed" of light, c). In space-time, the velocity "mesh" is now defined by these relationships, so that space time for a given rest mass, and with velocity referred to c through the EK relationships, and defining the relationship via "proper space (x_v=x_c)" OR "proper time (t_v=t_c)", respectively, gives:



OR



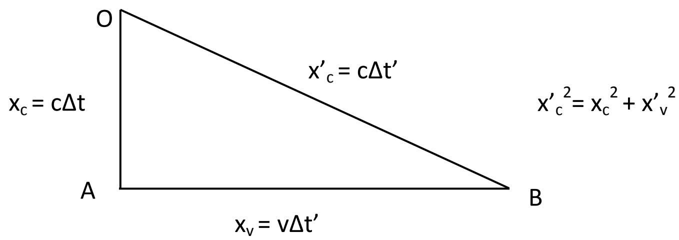
Notice that if x_v=x_c AND t_v = t_c, then T'=T so that V=0 in EK space. IF neither "proper time" or "proper space" is assumed, then $(\frac{t_v}{t_c})^2(\frac{x_c}{x_v})^2 = \gamma_m$; that is, the space and time ratios (squared) are inversely proportional to each other (time "increases", OR space "contracts") according to the kinetic mass relationships, so that if neither equality holds then. Since $(\frac{x_c}{t_c})^2(\frac{t_v}{x_v})^2 = (\frac{c^2}{v^2}) = \gamma_m$, and $v = \frac{c}{\sqrt{\gamma_m}} = \frac{m_v}{m'}$, then, $\frac{v}{c} = \frac{m_v}{m'} = \frac{m_v c^2}{m' c^2}$ so that when v is referred to c, it is defined by the ratio of the kinetic mass (energy) to the total relativistic mass (energy).

In summary, the requirement that c be an invariant constant for all velocities ensures that v is defined by an invariant rest mass for each particle, which varies according to density ρ and is scaled by a factor T/T' (required by independence).

(Note: The very specification of v means that a force has been applied; i.e., there has been an acceleration, so that $F = m_0 a$. In space-time, this is an "impulse" which changes the vertical world line of the rest mass to a world line defined by the velocity, which is in turn defined by the change in mass. If Einstein's equivalence principle holds, this impulse is gravitational in nature, if no other particle has been specified – this implies a change in permittivity/permeability, in which case the scalar field has changed to restore c to its original value (c is only constant at LaGrange points(?)). In the case a particle has been absorbed to produce the change in velocity, then the density must have changed if c is to remain constant.)

Critique of "Observer Example"

The analysis of the time dilation equation is often "explained" by the following diagram, with all variables in the space-time domain:



An "observer" at O sends a beam of light to a second observer traveling at velocity v on a track perpendicular to OA. A third observer remains motionless at A. During the time of travel Δt to point A, the second observer will have moved to point B in a time Δt', at which point he will see the light from O. The observers at A and B will disagree on the time light takes to travel to their positions if the speed of light is assumed to remain the same for all observers. The "time dilation" equation can then be worked out in the space-time domain to obtain the time dilation equation:

$$\Delta t' = \frac{\Delta t}{\sqrt{1 - \frac{v^2}{c^2}}}$$

This diagram is misleading in a number of ways. First of all, note that the possibility that v = c is inconsistent with the diagram unless Δt=0. This means that v and c cannot be collinear on the space-axis (which is where all real particles exist). That is, it is a diagram comparing three lengths in space and therefore involves two spatial dimensions instead of (x,t), where OA is the y axis and AB is the x axis, with the time axis perpendicular to the plane of the diagram.

Finally it is inconsistent with $v = \beta c = \frac{\Delta t_c}{\Delta t_v} c$ since $v = c\sqrt{1 - \frac{t^2}{t'^2}}$ with t = Δt_c and Δt' = t_v

However, there is a sense in which it is true; if the diagram is defined as an area, then the (constant area) is invariant in time (the two-dimensional plane moves upwards (or out towards the reader) as time progresses, provided it is considered a "snapshot" in time, but that the Pythagorean relationship holds for such snapshots, and all values of velocity.

In fact if the diagram is rotated around observer O, the diagram is rotationally invariant as well, so OA and OB can be considered as the radii of circles in the x-y plane, with OB defined as a linear relationship which defines the relationship between their areas; i.e.

$$\Delta r' = \frac{\Delta r}{\sqrt{1 - \frac{v^2}{c^2}}}$$

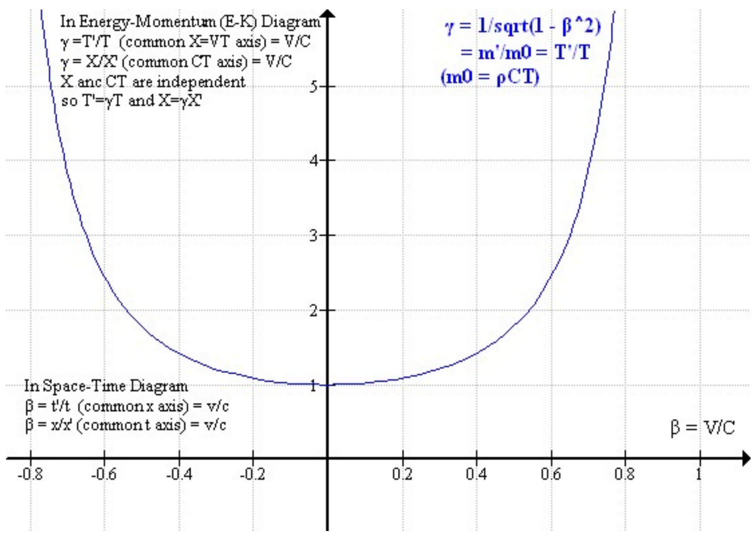
However, v and c are now rotationally invariant as well, which means they no longer have the relationship $v = \beta c = \frac{\Delta t_c}{\Delta t_v} c$ since now the velocity depends on the areas of circles: $v_r = c_r\sqrt{1 - \frac{\Delta r^2}{\Delta r'^2}}$ where $\Delta r = r_c$ and $\Delta r = r_c$.

This means that the interpretation of r as a linear speed in one dimension of space is again inconsistent, but must be re-interpreted as a radius.   (We will see a physical interpretation when we discuss the Michelson-Morley experiment and the derivation of the Lorentz transforms.

Energy-Momentum

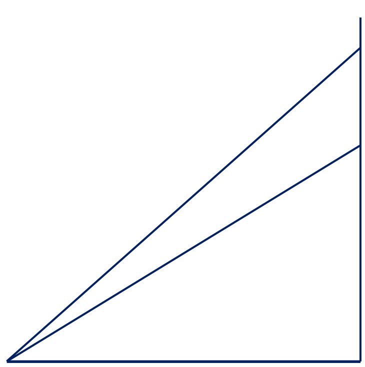
In Energy-Momentum Space, we have $T' = T\gamma = \frac{T}{\sqrt{1-\frac{V^2}{C^2}}}$. i.e., $V = C\sqrt{1-\frac{T^2}{T'^2}}$, with T' > T.

Note that $T' = T \ \rightarrow V = 0$ and that $T' = \infty \ \rightarrow V = C$



We can plot the effect of energy-momentum on a space-time diagram if we assume the common "space" parameter $\Delta x_c = \Delta X_c$ =CΔT. The velocity in space-time coordinates can then be re-defined so that

$t' = t\gamma = \frac{\Delta t_c}{\sqrt{1-\frac{V^2}{C^2}}}$; that is, $v = V = c\sqrt{1-\frac{t^2}{t'^2}}$ instead of v = βc = $\frac{t_c}{t'}c$



$t'_t = T' = t_\gamma = \frac{t_c}{\sqrt{1-\frac{{m_0}^2}{{m_t}^2}}} = \frac{T_c}{\sqrt{1-\frac{{m_0}^2}{{m_t}^2}}}$, v_γ = c$\sqrt{1-\frac{{m_0}^2}{{m_t}^2}}$

$\Delta t_c = \Delta T_c$, v=c, V=0, $m_t = m_0$, T'=T

$\Delta x_c = c\Delta t_c$, $\Delta X_c = C\Delta T_c$

v < c   t'= $\Delta t_v > \Delta t_c$

The "time" t=T is now a function of x_c,t and m(v)=m_t, where the mass depends on velocity, as opposed to Newton's laws, which separate mass from space-time coordinates(x,t). This means that NONE of the space-time coordinates refer to distance between two points, but rather are definitions of velocity in the space-time frames of c and C, which are related by $\gamma(\frac{m_0}{m_t})$. In particular, the relativistic relationship above only defines the properties of a single **mass** point in space-time, which depends on its mass. Its world line is then defined by v(γ), instead of v(β).

The diagram above emphasizes that although the graph mesh is determined by (Δx_c,Δt_c), the value of elapsed time for an object covering the common distance is now determined by the relation of the rest mass to the total relativistic mass.

That is, (to the extent that c is independent of position in space and time (i.e., a "flat" space-time) the frame that defines c is "absolute", i.e., the same for all "observers" (particles), even though the velocity for a given particle is now defined by its mass. This means that "coordinate frames" in space-time are now dependent on **each particle mass** as related to the frame defined by c and differ by $t'_i = t_i\gamma_i$ rather than $t'_i = t_i\beta_i$ or, as with Galileo/Newton, one frame that defines velocity in a single space-time (x,t) frame for all particles.

Note that time has a "first order" relation to space in Galilean coordinates (indicating that both are independent of Newtonian mass), but that the "space-time" relation in E-K coordinates is second order, which implies the mass (and thus the energy-momentum) relationship.

This also implies that this definition of velocity implies an increase of mass from a "rest" mass to a "moving mass" and thus a transformation in which the change in mass can be characterized by a density (=1 for photons), the speed of light c, the velocity v and a scaling factor T/T' to (called a Lorentz Transform to distinguish it from a Galilean coordinate transformation).

Since this transformation is now independent of the Galilean velocity (v = βc) for a given rest mass, any change in velocity (i.e., an acceleration) is also independent of Galilean coordinates in which $x_c = ct_c = X_c = CT_c$. However, a change in rest mass (which will change (ΔX_c ΔT_c) = (Δx_c,Δt_c) can be considered a change in C (since T' is just a scaling factor). This means that the product of the permeability and/or the permittivity constants must change, since $c = \frac{1}{\sqrt{\varepsilon_0\mu_0}}$. The relation defining charge and current can change to compensate, since (at least) permeability is adjusted to define current, and is arbitrary. However, Maxwell's equations will now be inconsistent, since the derivation of the speed of light from them depends on the equations being translationally invariant in space-time.

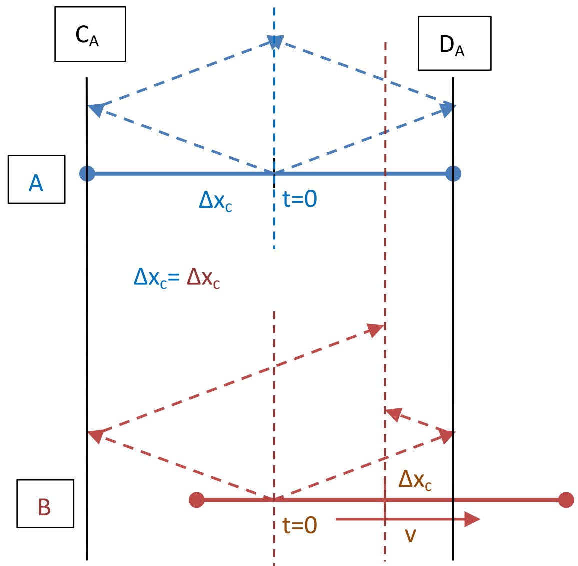
This is solved by declaring Maxwell's equations consistent with the addition of the gradient of a "scalar field" at the origin (position of the particle), which compensates for the change in mass, and a "vector potential" which the scalar field is subtracted out when the matrices of the E and B fields are multiplied to recover the rest mass via the Poynting vector. Maxwell's equations are then valid for any mass, which is again neutral.

Simultaneity and Einstein's trains

Consider two trains A and B of equal lengths of $\Delta x_c = \Delta x_c$ on parallel tracks with observers at the center of each train with mirrors on the embankment at positions C_A and D_A.

At time t=0 both trains sent a light signal toward the mirrors at the ends of train A, C_A and D_A; Train A remains stationary, while train B begins moving. Since both signals start from the center of the trains (and the speed of light is independent of the velocity of train B by assumption, the two signals from each train will reach C_A and D_A simultaneously. On the return trip, however, the observer at the center of train A will observe the signals to be simultaneous, but the observer at the center of train B will see the signal returned from mirror D_A sooner than that returned from mirror C_A, and both signals will be arrive at different times than those observed on train A.

Then if the "A" reference "frame" is defined by the length of train A (equal to that of train B)and the positions of mirrors C_A and D_A at the ends of train A, the observers at the centers of those trains will see the returns of the signals differently in that frame.



These roles can be conceptually reversed, since if B runs to the rear of his train at velocity –v, he will observe the signals to be simultaneous on his arrival, since the rear of his train now corresponds to the midpoint of A's (this is equivalent to re-aligning B's time coordinate with A's).

So far, so good; the above analysis is consistent with Galilean coordinate transformations (e.g. radar), and illustrates Einstein's contention that light is a particle (a pulse) rather than a (modulated) wave.

According to Einstein, there is therefore no "absolute" reference frame, since these each observer will have a different definition of simultaneity.

However, it is not the whole story. It ignores the fact that the absolute reference frame in which c is defined is also applicable to the rest masses of the trains and observers. In particular, the B train (and the observer) will have increased their masses relative to the system consisting of the A train and its observer. Their velocities are now defined by:

$v_\gamma = c\sqrt{1 - \frac{\Delta t_c{}^2}{t'^2}}$, that is: $t' = \Delta t_c \gamma = \frac{\Delta t_c}{\sqrt{1-\frac{v_\gamma{}^2}{c^2}}} \neq \Delta t_c \beta = \frac{\Delta t_c}{\sqrt{1-\frac{v_\beta{}^2}{c^2}}}, \beta = \frac{v}{c}$

Although the signals at the end of the B train will appear to be simultaneous, the will have taken longer to arrive than the simultaneous signals that A observes. If A and B are unaware of their masses, they will assume that the signals travel slower in B's frame of reference.

If both trains are at rest, they will have equal "rest" masses/energies. However, if one of them begins moving because of an impulse of energy, it will have more energy than the train at "rest" (according to its observer). If the stationary observer merely switches spatial coordinates to that on the moving train, his train will now seem stationary, and it is the other train that will seem to have received the impulse.

According to Newton's third law, this can be resolved if one defines velocity from the center of mass of the system; each train will have an equal but opposite half-velocity in relation to the motionless space-time "center of mass" (each train will have received a half-impulse). At the center of mass, the mass of the system as a whole will have increased according to the impulse that produced both motions.

In the example above, observer B running to the back of his train changes his position to the center of mass of both systems, but the mass contribution of B at that center will be greater than the contribution of A.

Photons

A distinction can thus be made beween particles that travel the "proper" distance x_c in time t_c (i.e., "photons") and those that travel the proper distance in a longer length of time t_v ("matter"). Photons do not interact with each other, but with matter as described by a Lorentz transform. Note that a Lorentz transform can be considered an elastic "acceleration" from a rest mass to a kinetic mass. Such interactions can be described by the light cone to be discussed below, and in QED by Green's function.

However, energy and momentum conservation for photon/matter collisions (Compton effect) requires quantum mechanics for the energetic description of the photon.

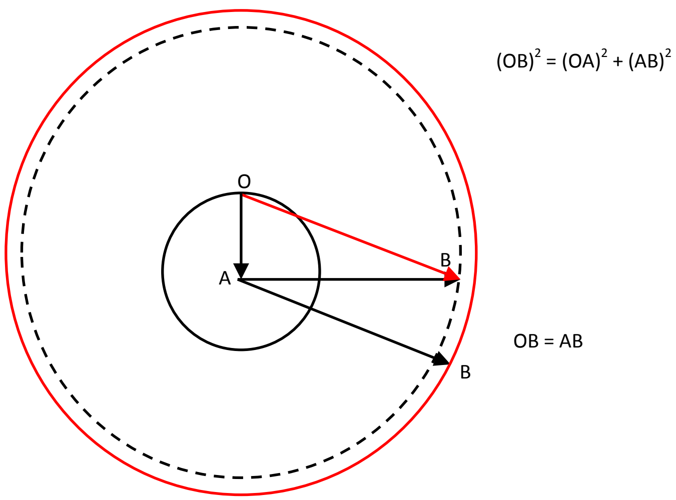
The distinction between photons and matter can be described as "no structure" vs. "structure", where the "structure" can be characterized by the "rest" mass (ρ)cT.

Further Discussion

In E-K space, the fact that the quantities are squared, means that the dynamic "lengths" CT,CT' and VT' are independent of "sense", corresponding to negative values of mass and/or velocity; for example, negative values of rest mass can be interpreted as anti-particles, with the total rest energy defined as twice either of the negative or positive values $\pm m_0=\pm(\rho CT)$, with $\rho=1$. Note that the sense can be applied to any of the properties ρ, C, T, or in the case of dynamics, V and T' as well.

Also, we have specified that space and time be rotationally invariant, which means that these "lengths" can also be interpreted as the radii of circles (or even spheres) – again, these circles are interpreted as masses, since (e.g.) in the case of the circle, $m_0(R)=\pi(\rho CT)^2=)=\pi(\rho R)^2$, where the sense is now degenerate.

For example, in this interpretation, OB = Total relativistic mass (energy), OA = rest mass (energy), and AB = Kinetic mass (energy) for a single particle (ρ=1).



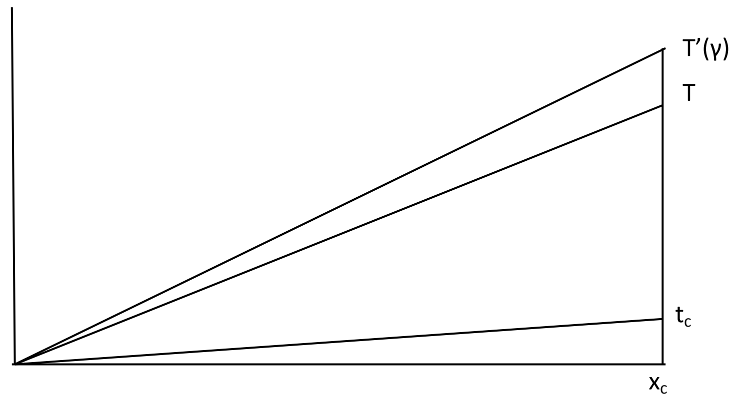
(This perspective is consistent with the Michelson-Morley experiment and Peter Bergmann's derivation of the Lorentz transforms, and Barut's characterization of the Lorentz transform in terms of rotation matrices).

Interpretation in Space-Time

In Space-Time, the quantity $m_c = \rho(ct_c) = c(t_c) = x_c$ (ρ=1) can then be interpreted as the "inertial mass" of a photon, where x_c is interpreted as the distance a photon of mass m_c travels in a time t_c (at a velocity c.

(Note: Planck's constant and spectra are irrelevant in this context, although it can be added for relativistic QED definition of a field in terms of Green's function, beginning with the Klein-Gordon equation. However, even in this context, "t" must be interpreted in terms of mass and its relation to dynamics, rather then time alone).

For a given particle, then, with rest mass defined as m_0 = cT, and total energy defined in terms of T' = T(γ(v,c)) where $\gamma(\mathrm{v},\mathrm{c}) = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$ in the space-time domain, interpreted with a given "proper space" (for all particles) x_c, we have the following:



Note that "time" is now interpreted in terms of mass; i.e., as velocity increases, "time" increases. T' is then interpreted as the time it will take for a mass T' to travel a distance x_c (if hit with a photon of mass $m_{ph} = \rho c t_c$), compared with the distance a free photon of mass $m_{ph} = \rho c t_c$ will travel in the "proper time" t_c.

If multiple particles are involved then the relation is described by re-interpreting the density; thus if all the triangles are multiplied by a single density, the relationships remain the same. Multiplying both time and space by a constant (density) changes the total "gauge" of the space time diagram (i.e., the "x,t" grid spacing), but not the space-time or mass-energy relationships, so the relationships can be said to be invariant under gauge transformations, or "gauge invariant" in this context.

The relationships are similarly invariant under changes in velocity (space-time diagram) and or rest mass (E-K diagram) under the conditions of "proper space" as above, where c is defined for all densities and particle masses by $c = \frac{x_c}{t_c}$.

Space-time length as radii; the foundation of relativistic QED)

(added 6/10/2012)

Recall that we have now defined STR as a relation between three concentric circles in Energy-Momentum space, with axes (radii) CT and VT', and hypotenuse CT'. We can identify each of these circles as a mass by multiplying by a "density" p=1 so that (e.g.) M0 = pCT=CT. From there, we can tweak the triangle (the "Time Dilation" equation, which is actually a "Mass Increase with Velocity" equation) by multiplying by C^2 to get the Relativistic Momentum Energy Relation, which describes the whole system.

It is important to re-emphasize that we still are talking about ONE particle (galaxy, black hole, universe); if a particle is created, it was created initially with its velocity v at the same time as c in the infinite past), not through an interaction…. The "origin" can be anywhere, anywhen in spacetime.

HOWEVER, one can now re-identify space and time with the scaling factors T and T' (remember that x=X=CT and x have already been spedified as identical), so that instead of time being defined linearly by t'=(1/Beta)t, we now have t'=Gamma(M)*t, where M is given in terms of p, C, and T. This means that in space-time, a particle with velocity v(M) will still take longer to travel the distance x than a photon will, but the time will now be defined in terms of mass (possibly modified by a velocity) instead of the space traveled and a velocity defined by the coordinate system alone.

Now draw the three concentric circles around the origin and imagine the time axis oscillating between t=-ct to t=+ct. The "particle" will have two components, traveling in cw and ccw directions along the circumference (now a "world line" as the axis progresses upwards. (These components are degenerate in E,P, because of the squaring; now consider the area covered in the circle(s) as time progresses to describe E,P). In this interpretation, the circles (the particle) is interpreted as an "event" – it is created once in space-time (possibly in the infinite past and/or by "god") with energy-momentum in one cycle, and then the description is invariant in space-time, so the origin (center of mass) travels along its global world line.)

That is, the description of the particle(s) around the circumference is periodic, and thus can be described as a "wave" equation. This "wave" equation forms the basis of that described by the Klein-Gordon equation in QED (particle physics) This wave physically repeats only for a limited number of cycles, forming a group of "pulses"; the length of the group is called its "lifetime", or "coherence length".

The following section was revised 6/4/2012 at 6:30 pm PST.
(I'm on the right track, but might have some of these relations wrong.. I'm working on it…..)

Mass as a "vector"

Let $\beta = \frac{V}{C}$, $\Gamma_v = \frac{1}{\sqrt{1-\beta^2}}$ in (E,P) space, so that V , C , M_0 , and M' apply to energy and momentum. Then V increasing implies that β increases. But $\beta \le 1$, so β^2 decreases. Then $1-\beta^2$ increases. But $1-\beta^2 \le 1$ so that $\sqrt{1-\beta^2}$ decreases and Γ_v increases. Since $M' = \Gamma_v M_0$, we can think of Γ_v as a linear transformation on the basis "vector" M_0. Then the "vector" M' is directly proportional to Γ_v which we can think of as the coefficient of the transformation, and we say that the "vectors" transform in the same way, that is, they are "**covariant**"; (increasing Γ_v (or V) has the same effect as increasing M_0).

Space-Time Coordinate transformations

Consider the coefficient of linear transformation Φ, as yet undefined. Then we can define covariant "vectors" v and c such that $v = \Phi c$.

Then $\frac{x_v}{t_v} = \Phi \frac{x_c}{t_c}$.

Space Invariance ($x_v = x_c$)

Then $\frac{1}{t_v} = \Phi \frac{1}{t_c}$ and $t_c = \Phi t_v$. If we set $\Gamma_v = \frac{1}{\Phi}$, we have $t_v = \Gamma_v t_c = \frac{t_c}{\sqrt{1 - \frac{v^2}{c^2}}}$, which is the "time dilation" equation. As above, Γ_v increases as v increases, so the time "vectors" transform in the same way as the coefficient, with t_c the basis "vector", and are therefore covariant like the mass vector and the velocity vector

If the velocities are the same in both (x,t) and (E,P), then this means that an point particle with a greater mass due to an increase in velocity will take longer to travel a fixed length than a particle at light speed.

Time Invariance ($t_v = t_c$)

Then $x_c = \Phi x_v = \frac{1}{\Gamma_v} x_v$. Then Γ_v increases as v increases, but now Φ decreases, space "vectors" transform in the opposite way as the coefficient, so the space transform is said to be "**contravariant**". This means that a point particle with a greater mass due to an increase in velocity will not travel as far in a given amount of time as a point particle at light speed.

The fundamental relationship of STR

We now assume that $v = V$ and $c = C$, so that the velocities are the same in both (x,t) and (E,P) space

Starting with $M' = \frac{M_o}{\sqrt{1-\frac{v^2}{c^2}}} = pCT' = \frac{pCT_0}{\sqrt{1-\frac{v^2}{c^2}}}$ $(p=1,\ C=c)$ in Energy-Momentum space (invariant length $X_0 = X'$ in (E,P) space), we have $T' = \frac{T_0}{\sqrt{1-\frac{v^2}{c^2}}}$, with T_0 and T' scaling factors in rest and kinetic masses.

Invariant Length "Space"

In space-time, we have $\frac{v}{c}=\frac{x'/x_c}{t'/t_c}$. If we postulate that $x'=x_c=ct_c$ (invariant length in coordinate space), we have that $\frac{v^2}{c^2}=\frac{t_c^{\ 2}}{t'^2}$, and length is irrelevant to the discussion. Then $T'=\frac{T_0}{\sqrt{1-\frac{v^2}{c^2}}}=\frac{T_0}{\sqrt{1-\frac{t_c^{\ 2}}{t'^2}}}$, so that

$\left(\frac{t_c}{t'}\right)^2+\left(\frac{T_0}{T'}\right)^2=1$ for $x_c=CT_0$

Note that $v<c,\ t_c\le t', and\, T_0\le T'$. This equation relates the scaling factors in flat space-time $\left(\frac{t_c}{t'}\right)$ to those in Energy-Momentum space $\left(\frac{T_0}{T'}\right)$, with the squared values indicating symmetry in invariant time and mass, since $\left(\frac{t_c}{t'}\right)^2+\left(\frac{T_0}{T'}\right)^2=1$ The ratio $\frac{t_c^{\ 2}}{t'^2}$ relates the time it takes a coordinate point to travel the invariant length at a velocity c to the time it takes a coordinate point to travel the invariant length at a velocity v. The ratio $\frac{T_0^{\ 2}}{T'^2}$ is a scaling factor relating the invariant length of a stationary "light stick" to the increased length of a moving "light stick" in (E,P).

We can write this equation as $\left(\frac{\rho ct_c}{\rho ct'}\right)^2+\left(\frac{\rho CT_0}{\rho CT'}\right)^2=\left(\frac{m_0}{m'}\right)^2+\left(\frac{M_0}{M'}\right)^2=1$

which suggests that if mass exists at any coordinate point in (x,t) , the ratio between the rest mass of the point to its moving mass can be expressed by the ratio m_0/m'. Similarly the ratio of a the mass of a stationary mass in (E,P) to that of a moving mass can be expressed by M_0/M'. Note that an increase in t_c or m_0 implies a decrease in M_0 and M', respectively, (e.g. $t_c = 1/T_0$). Since t_c and T_0 (m_0 and M_0) vary inversely with respect to each other, we say that such a change is "contravariant". Similarly, since t_c and t' (and M_0 and M') are directly proportional, a change that relates them is called "covariant".

Invariant Time

We begin $x_c = x'$

If we consider invariant time, with $t_c=T_0$ instead, we have that $\left(\frac{x'}{x_c}\right)^2+\left(\frac{VT'}{VT_0}\right)^2=1$.

The ratio x'/x_c relates the distance a coordinate particle moving at velocity v will travel to the distance a coordinate particle moving at velocity c will travel. Similarly, the ratio X'/X_0 similarly relates rest mass to moving mass as represented by "light sticks" $X_0=\rho CT_0, X'=\rho CT'$. Note that $x'\le x_c,$,

(the following section is a bit redundant with the above; I'll revise it when I can….)

The Scalar Field

So far we have assumed a density of $\rho = 1$ as the coefficient of CT in (E,P) space. However, the same can be done in spacetime (x,t), so that space can be interpreted as a scalar field instead of a distance. The length $m_0 = ct_c = \rho ct_c$ can then be interpreted as a scalar "mass field", with this invariant (coordinate) mass at every coordinate point. For invariant space, we have that

$$\left(\frac{\rho t_c}{t'}\right)^2 + \left(\frac{\rho T_0}{T'}\right)^2 = 1,\ \rho = 1$$

Now indicate the density associated with the (x,t) as ρ_c and that associated with (E,P) as ρ^c so that

$$\left(\frac{\rho_c t_c}{t'}\right)^2 + \left(\frac{\rho^c T_0}{T'}\right)^2 = 1,$$

Consider a linear transformation that changes the densities; for this equation to be valid, the densities must be inversely proportional; that is for constant coefficient k, the densities must change inversely in magnitude (thus the transformation is said to be "contravariant"):

$$k = p_c / p^c$$

Here ρ_c can be thought of a "scalar" potential at each point in the coordinate system, and ρ^c a scalar density defining mass. Note that setting $\rho_c = \rho^c = 1$ removes the distinction between covariance and contravariance.

Setting k = 1 makes the densities inverses of each other, so that adding a "vector" mass to (E,P) is compensating by subtracting an equivalent "scalar" mass at its point in (x,t), so that physical laws involving mass are unchanged (e.g., electromagnetism). In particular, this process changes the description of Maxwell's field (first order equations) to the EM field tensor (second order equations) as the foundation of particle field theory.

To recap, since Maxwell's Equations don't involve an explicit coordinate system, the scalar field is introduced so that any mass added to a point particle is subtracted from the EM field so that the local mass is irrelevant to the geometric (integral) and differential expressions of the equations. This allows Maxwell's equations to be expressed as the second order EM field tensor, conserving momentum-energy with respect to its equations, and permitting Maxwell's equations to be expressed as tensor).

Since the coordinate transformations are no longer relevant, the EM field equations are said to be "gauge invariant" (the coordinate meshes no longer apply, and the analysis is done exclusively in Energy-Momentum Space). In two and three dimensions (at invariant time), the space "length" is interpreted as a radius r_c, so the geometry applies to circles and spheres instead of lengths. The same is true in (E,P) of the rest mass "length" R^c. For invariant length, time can be considered as a circle, which suggests periodic functions ("waves"); in one dimension, a restoring "spring constant", in two dimensions a force from the origin of the point, for three dimensions, from the sphere, etc.….

ρ can then be parameterized by time $\rho = \rho(t)$ (for invariant space) ; or space (for invariant time) or both $\rho = \rho(x,t)$ (which changes rest mass with time in (E,P), and suggest a restoring force (or centrifugal/centripetal acceleration) and/or gravity.

QED then adds the concept of an invariant "action" ħ relating (x,t) and (E,P) to the area of this space-time circle.

06/11/2012 --

In (E,P) space, the scaling factor defining mass is defined as:

$T' = \dfrac{T_0}{\sqrt{1-\dfrac{V^2}{C^2}}}$ Then $CT' = \dfrac{CT_0}{\sqrt{1-\dfrac{V^2}{C^2}}}$, $X' = \dfrac{X_0}{\sqrt{1-\dfrac{V^2}{C^2}}} = X_0\Gamma$, so that $X_0 = \dfrac{1}{\Gamma}X'$

Here, X_0, X', V and C represent "light sticks" and "velocities" characterizing the energy content of these parameters, instead of coordinate lengths in (x,t). Since X_0 is a constant characterizing rest mass under "velocity" changes (transforms) characterized by Γ , we see that increasing 1/Γ (a "velocity" transform inverse to Γ) requires decreasing X', and vice versa, so we say that the "vectors" are characterized a "contra-variant" in (E,P) space.

In (E,P) space, we have that $M' = \dfrac{M_0}{\sqrt{1-\dfrac{V^2}{C^2}}} = M_0\Gamma$, so that velocity transforms as:

$\dfrac{V}{C} = \sqrt{1-\dfrac{M_0{}^2}{M'^2}}$ If we stipulate that the physical quantity represented by mass is numerically the same in both (x,t) and (E,P), then $\dfrac{v}{c} = \sqrt{1-\dfrac{M_0{}^2}{M'^2}}$

(TBD)

Analyze covariance/contravariance of $\Gamma_{(x,t)}$, $\Gamma^{(P,E)}$, $\Gamma_{(x,t)}{}^{(P,E)}$
The role of +/- in symmetry (x,t) vs (E,P)
Positive/negative energy – chg vs current.
Photon "field" vs structure
Spin and QED

Gravitation

Note that STR describes non-interacting particles if the context of invarant length, with no origin (rest mass and velocities are independent of location – they can be defined "anywhere, anywhen". The physical system is invariant with change in position (with constant c and the vector potential applied), since rest mass is an invariant with p = 1.

Einstein's theory adds new ideas; it applies analysis, so that the densities in (x,t) and (E,P) are functions of spacetime $\rho_c = \rho_c(x,t)$ and mass $\rho^c = \rho^c(E,P)$, and coordinate transforms (e.g, $x = f(x), E = E(m)$ with all variables continuous and differentiable. (Thus applying differential geometry where the velocity of light is now scaled (not constant) in either (x,t) or (E,P) and everything gets WAY beyond the scope of this effort.

Addendum(s) *(by Chuck Keyser)*

(And notes to myself)

The Twins "Paradox"

Einstein imagined a set of twins (call them Bob and Mary), each with a clock and a ruler. At time t=0, they synchronize their clocks and rulers, and define a velocity for Mary, and a distance she must travel to and return from while Bob remains motionless. Since Mary is moving, her clock runs slower, so when she meets Bob, she finds that she is younger in comparison.

Discussion

In a space-time diagram, two points are necessary to define a velocity. If we ignore the kinetic energy velocity in the space-time diagram (Bob and Mary have rest masses = 0), then the first point is given when Mary passes Bob at the origin (the vertical time axis), assuming they agree on the distance Mary is to travel. The second point (at which they synchronize their watches) is given when the meet on Mary's return to the time axis, at which Bob and Mary agree on Bob's velocity (=0) and Mary's velocity (=v).

Even if one assumes that their masses are not 0, and that Mary has kinetic energy governed by gamma (they agree on the relation), they still agree on the velocity (and the difference in rest vs. kinetic mass) at the second click.

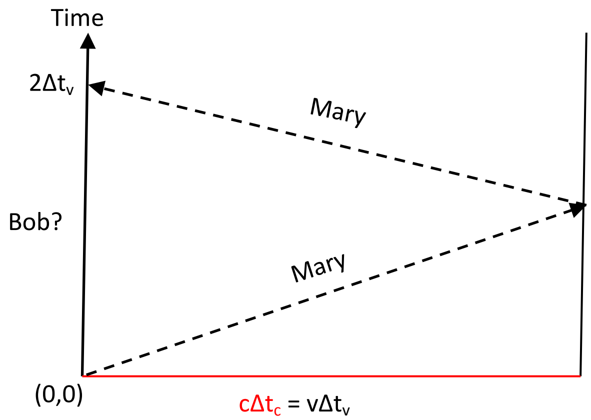
The "paradox" only arises if one agrees to the kinematic relationship (gamma), but associates t' with T' so that ct = CT.

Let's set the scene in a space-time diagram. (We can eliminate the factor of mass increasing by setting Bob's stationary mass equal to Mary's relativistic mass $m_0(Bob) = m_v(Mary)$, since we haven't included any inertial effects in the problem (this means that we have ignored Mary's initial acceleration from v=0 to v, which by Einstein's equivalence principle would be gravity, and thus the domain of the General Theory).

We select proper length as our basis for comparison, and note that Mary's velocity is defined by two points in space-time. Bob remains at the origin, and Mary starts her journey at t = 0. She travels a proper length (on which she and Bob agree, since both agree as well that Mary's velocity is defined only with respect to c) and arrives back at the origin after a time interval of $t = 2\Delta t_v$ for her round trip.

But what of Bob? We know that he synchronize his watch at the origin, but until he meets Mary at time $t = 2\Delta t_v$ he hasn't defined his second velocity point so he can say he has velocity $v = 0$. That is, Bob's velocity is defined by $v_B = \frac{v_B}{c}c = \beta_B$ and in particular, $v_B = \frac{t_c}{t_B}c$ so that $t_B = 0$ means that v_B is undefined (time could be anything).

In order to define a velocity $v = 0$ so that he moves no distance in a given time, he must compare his watch with Mary again after her journey of twice the proper length (upon which both she and Bob agree) so that $\Delta t_v(Bob) = \Delta t_v(Mary)$ where $\Delta t_c = \frac{\Delta t_c}{\Delta t_v}\,\Delta t_v$ for both (which hasn't been mentioned in the presentation of the problem..:-)



Of course, one can define the physical effect on Mary by her getting fat (due to gravity), but I don't think it would be a wise idea in her presence….:-)

In summary, then, Bob must define period travel as Mary for their watches to actually be synchronized (an infinitesimally short period doesn't count, since then the proper length is infinitesimal, and the velocity is ambiguous.

The Michaelson-Morley Experiment

(Blackbody source and sensor, in addition to QM uncertainty of phase responsible for null results in interference experiments. Relation of coherence length (pulse width) of photon to cosmological considerations.)

(Peter Bergmann's derivation of the Lorentz transforms)

(TBD)

The Lorentz Transform

As observed earlier, the fact that quantities are squared in E-K coordinates suggest energy-momentum conservation, when positive and negative lengths or masses are involved. This means that both positive and negative energies are included in the "Time Dilation" equation, meaning the equations are independent of direction in the space-time domain, and thus correspond to the radius of a circle.

This is actually suggested by a modification of the Lorentz transformation, which sets the time parameter = 0 in the space transform $x' = \frac{x-vt}{\sqrt{1-\frac{v^2}{c^2}}}$ so that $x' = \frac{x}{\sqrt{1-\frac{v^2}{c^2}}}$. The modification also sets the space parameter = 0 in the time transform $t' = \frac{t-\frac{vx}{c^2}}{\sqrt{1-\frac{v^2}{c^2}}}$, so that $t' = \frac{t}{\sqrt{1-\frac{v^2}{c^2}}}$ (the time dilation equation). Setting x=ct and x'=ct' in the modified space transform shows that the two equations are then equivalent.

This is, of course, the procedure involved in choosing one of the common parameters in the space-time diagram to compare velocities.

(06/10/2012) To recap, I would argue that the Lorentz transforms (derived in Peter Bergmann's "Introduction to the Theory of Relativity) include the idea that observers are separated by a "distance" in a "frame of reference", a concept that Einstein developed in his explanation (I assume to be able to apply the integral representation of Maxwell's equations, which involve geometric objects). However, if one assumes that any interaction (observation of an event) requires that observers be at the same place at the same time, then all definitions of velocity (including c) have the same coordinate location in space-time.

Therefore the concept of frame is eliminated (except to say a coordinate point can exist anywhere in space-time, and definitions of velocity are translation invariant in space-time. Since the observers must exist at the same point to compare clocks and watches to define velocity, the t can be set to 0 in the space transform, and space can be set to 0 in the time frame, so the two equations refer to the same point (a local origin (0,0) – which can be anywhere, any-when in space-time.

(Again, to make Maxwell's Equations consistent with the introduction of (invariant/rest) mass, the vector potential (gradiant of a scalar field) is introduced in (E,P), which implies that the scalar field is introduced in (x,t) with densities = 1, so that the EM field can be expressed in the form of a tensor for QED.

(06/17/2012) The important point about the Lorentz transforms is that the “coordinate systems” to which it is referring are actually the lengths of the perpendicular arms of the Michaelson-Morley experiment, and that the “velocity” is the change in signal space and time that would be observed if there were a medium in which light traveled. Here the initial coordinate system is (x,y,z,t) and the lengths (rods,periods of clocks) along these axes are (X,Y,Z,T)

In the derivation, the y and z axes are eliminated a priori, since the physical experiment showed no difference in direction – thus the analysis rests directly on X as isotropic (think of X as a radius), so that the physical law remains the same no matter which way the arms are pointed. Since the analysis now only refers to space and time, the results must be in terms of the difference in the lengths of the arms as affected by the motion of the “aether” at velocity v, with the length of the arms at v=0 equal to X=cT, so that X’ = X = cT’.

I repeat, these are not “coordinates”, they are the lengths of the arms referred to a coordinate system. One can imagine the arms floating in a circular swimming pool, where the sources and sinks can be located along any orientation, provided they are opposed.

Since no assumption is made about the specifics of the lengths, the terms in (X’,T’) must be taken to mean the **difference** in length and period from that observed in (X,T). Since the analysis must be isotropic, the lengths are squared to remove sense and orientation of the aether from the problem.

That said, I refer the actual calculation of the Lorentz transforms to “Introduction to the Theory of Relativity”, by Peter Gabriel Bergmann, starting on pg. 34. Again, note that both sides of the equation are squared (pg. 35, 4.7) to make each side of the equation a radius (so now we’re actually talking in terms of circles), and setting the coefficients the same eliminates the Y and Z directions (for spatial isotropy), so the analysis is now completely in terms of (X and T). Eliminating the non-linear parts of the equation by solving for the coefficients results in the “coordinate transforms”:

$$X' = (X - vT)\Gamma, \qquad \Gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$T' = (T - \frac{vX}{c^2})\Gamma$$

Note that:

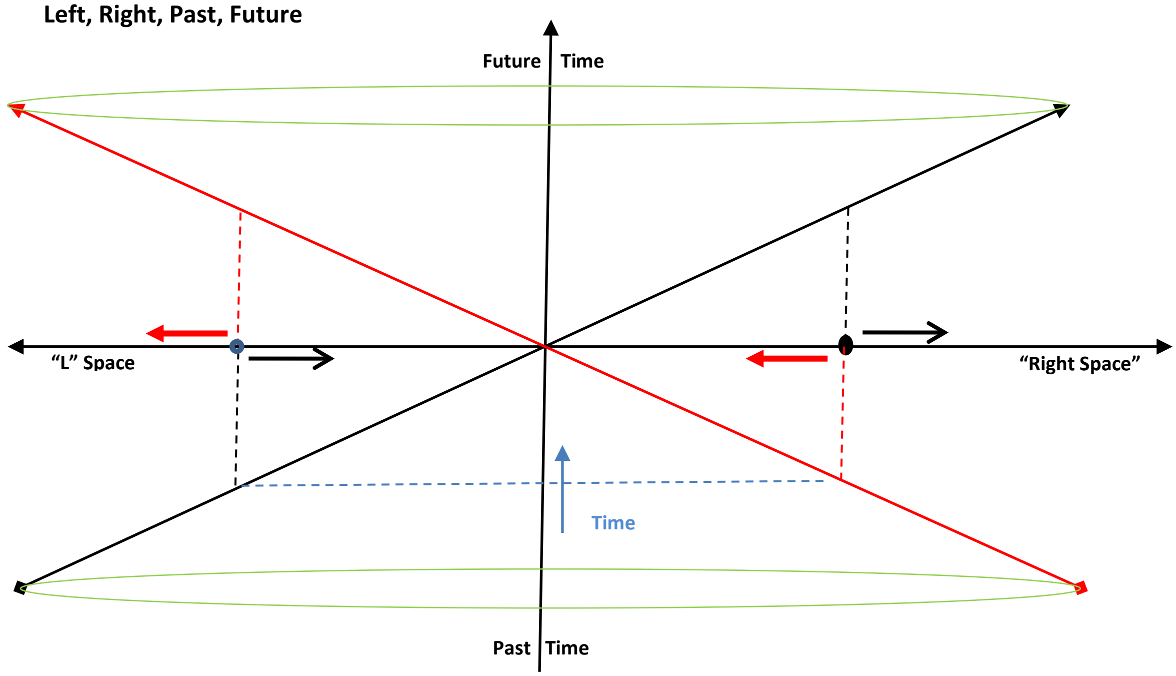
1. For v=0, X’=T’ (the lengths of the arms are the same)
2. For v->c, the difference in lengths (whatever they are) approaches to infinity, (assuming $X \neq 0$).
3. For v=0, the time it takes a signal to travel the T’ arm is the same as the T arm ($T' = T$)
4. For v->c, the time it takes a signal to travel the T’ arm will approach the time it takes to travel the T arm (assuming $T \neq 0$)

Note that in the above analysis, the new “lengths” will depend on the values of v and c, which are not independent of each other, if the above changes are multiplied by a “density” = 1, so that m=px=pcT and m’=pX’=pcT’, the “mass” of the system will change due to its interaction with the light medium.

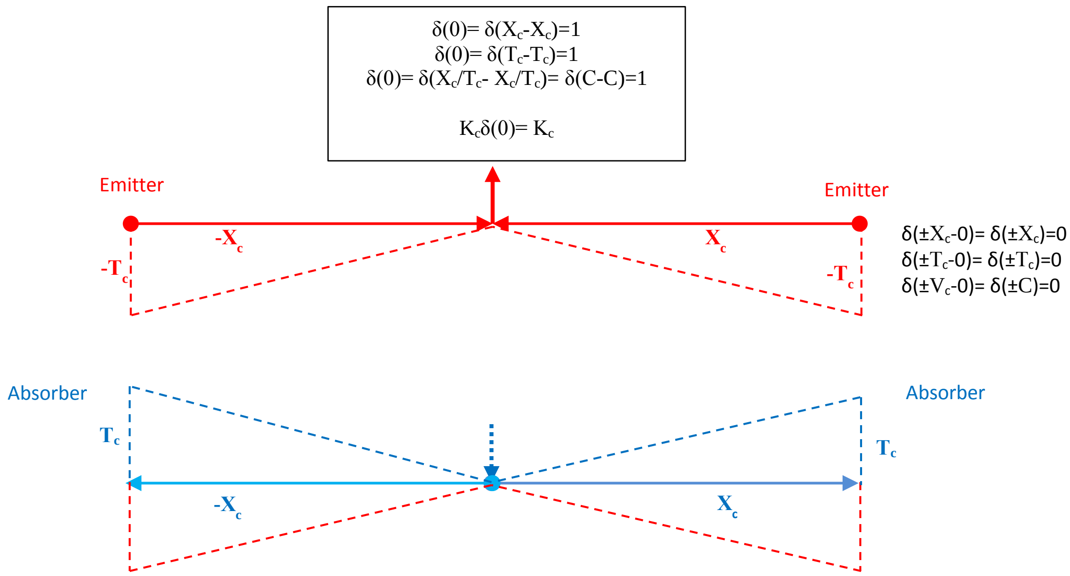
By introducing the concept of an invariant length (arguably a system “rest mass”) that is invariant under velocity changes, Einstein postulates that there is no difference in the lengths of the arms for any orientation, so that the system can be characterized by either (X’ = X,0) or (0,T’=T) for any orientation, which removes the velocity dependence in the numerator for each equation (T=0 for X’=X, or X=0 for T’=T), which gives the “time dilation” equation: $X' = X\Gamma = cT'\Gamma = cT\Gamma$.

Green's function and the light cone

Left, Right, Past, Future



The Dirac Delta Function δ(x)



The "Light Cone"

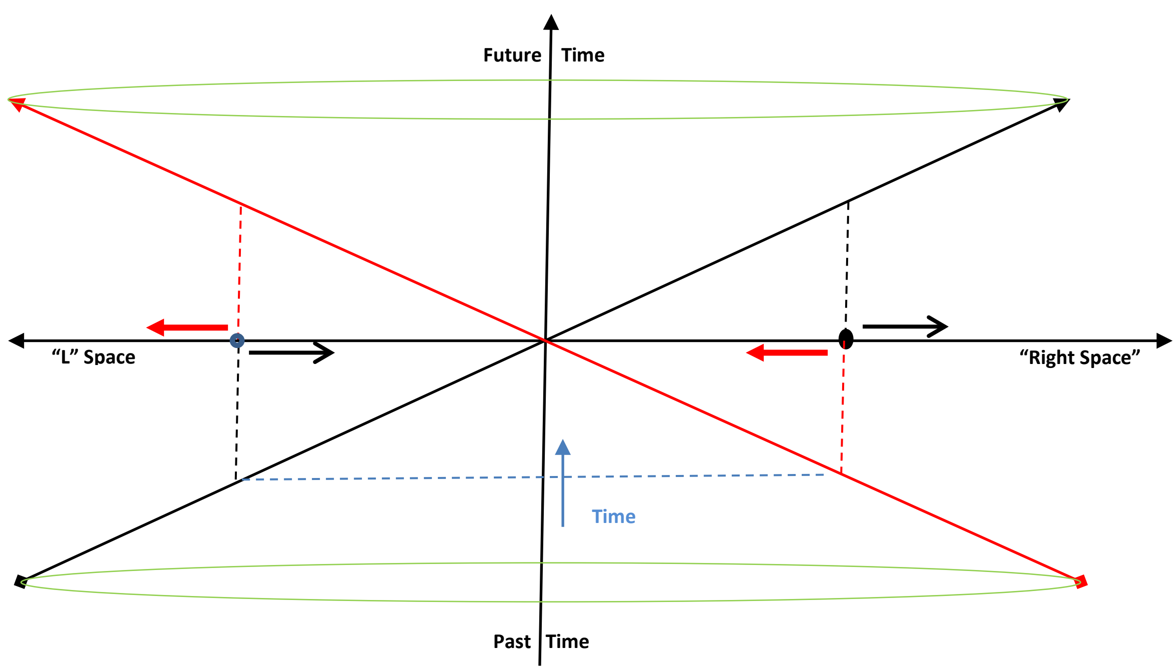
So far we have made no assumptions about the speed of light c with respect to other velocities; in particular, coordinate points are still free to "travel" faster than light – the world line such a coordinate point will be "more horizontal" than that specified for c, and $v/c > 1$.

Consider only the velocity c. The discussion of velocity can be expanded to include past and future in time and "left" and "right" in one dimension of space. Coordinate points in space ("photons") only "exist" on the horizontal space axis and move along it forward and backward in time, expressed by the arrows in the diagram. Their world lines are expressed by the diagonals – these diagonals define the "existence" of the particles in space and time in terms of their velocities.

That is, the ends of the diagonals in the past can be thought of as "sources" (emitters) of the photons, and the ends of the diagonals in the future can be thought of as "sinks" (absorbers). Again, the sources only exist on the space axis, and the photons pass through the origin ("here" and "now") without interfering with each other.

We can then expand our description space-time into two dimensions of space and one of time (x,y,t); a plane extending from the page along the horizontal axis. The sources and absorbers now form circles – one can think of single pulse of photons emitted from the source in the past, arriving at the origin, and continuing on to the absorbers. The world line(s) of these photons would then form a "light cone" – the actual photons would emanate from a circle in the x-y plane, pass through the origin, and be absorbed in a second circle, congruent to the first (for now).

Note that the dimension "x" is now a radius, and the "direction" of light is isotropic (independent of direction in spatial coordinates) since $R^2 = x^2 + y^2$, and the distinction between "L" and "R" disappear.



Note that for $v < c$, a “cone” will be narrower than that of light (radius will be smaller), and emission/absorption of points will occur earlier/later than that of c (photons) in time. Conversely, for $v > c$, the radius will be larger (cone will be flatter) , and emission/absorption will occur later/earlier than that of c in time.

Polarization/Spin

The circumference (disc, pulse train, etc.) can also be “spun” around the time axis with a constant velocity. A single point on any of the radii will then map out a vertical helix (“coil”) around the time axis as time progresses. For photons, this is called polarization; for a coordinate point other than a photon it is referred to as “spin”, and can be either clockwise or counter-clockwise.

The First postulate of the Special Theory of Relativity

The requirement that the velocity of light is spatially isotropic is the first postulate of relativity, and means that STR is described in two dimensions of space and one of time, but space can be interpreted as a radius, so the dimensions really are one of space and time again (r,t) instead of (x,y,t).

(In terms of electromagnetism, this implies that light is a "plane wave" in (x,y) moving through time – with the third spatial dimension z "parallel" to the time dimension. Much more on this is to be discussed in later work, or in other texts.)

Consider an number of coordinate particles (i.e., a pulse) at the origin, with density ρ(0)=1. This can be interpreted as having emanated in a circular source in the past, with circumference C=2πR, so the distribution per unit length (of the circumference) at a point on the circle is ρ(R)=1/C=1/(2πR). In particular, note that this corresponds to a 1/R potential. If the cone is completely filled with emitters, than the density is determined by the integrated circumferences (i.e., the area) so $\rho(R)=1/A=1/(\pi R^2)$. This means that a sold circle passes through the origin, rather than a circumference.

Other configurations can be imagined – for example a periodic sequences of circumferences (corresponding to a pulse train. Note in particular that **R=0 does NOT imply a singularity** in this context, it only means there is no light cone (although a coordinate "density" can exist in the "here and now" referred to as an "event").

(Note to myself - Discuss the meaning or R < 1?)

The second postulate of the Special Theory of Relativity

The second postulate of the theory of relativity is that the speed of light c is independent of the velocity of the source (i.e., any other velocity), which contradicts the Galilean principle that a velocity can be added or subtracted from c to form a third velocity (c' = c ± v), and that time and space coordinates cannot be added or subtracted from those of c to define new velocities.

This means that there is an absolute "frame of reference" in spac-time for STR in space-time (for our "universe", which we will now take to be a single "particle" at the origin; it is that in which the speed of light is defined.

Since v and c are independent of each other, we can express them in an orthogonal relationship analogously to the coordinates of space-time; that is, as right-angle axes (v,c) :

The Michelson-Morley Experiment (Peter Bergmann's derivation of the Lorentz transforms)
The aether
(note; it is not that the aether doesn't exist, it is that one can't detect it, since it would require a detection of change in velocity as a function of direction – i.e. acceleration = a change in gravity).
Lorentz Transforms

The Time Dilation Equation

$$t' = \frac{t}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{t}{\sqrt{1-\frac{T^2}{T'^2}}} \rightarrow \left(\frac{t}{t'}\right)^2 = 1 - \frac{T^2}{T'^2} \rightarrow \frac{(tT')^2+(t'T)^2}{(t'T^2)} = 1$$

For light, $t = T = 1$, so that $t'^2 = T'^2 + t'^2 \rightarrow T'^2 = 0 \rightarrow vT' = 0$ so that the velocity "distance" $vT' = cT' = 0$ has no relevance for in the E-K domain, since v and c are undefined.

The Postulate of STR

1. The speed of light is constant and isotropic. That is, there is an absolute reference frame in which all other velocities can be defined as a fraction of c $\beta_{(v,c)} = \frac{v}{c}$. In this context, v is defined in the same direction as c; that is, the distances $r = ct$ and $r' = vt'$ are each radii of circles, or, equivalently, aligned in the same direction in three dimensional space-time. The analysis will then be carried out in one dimension each of space and time (x,t)).

2. The speed of light is independent of the velocity of a moving coordinate system relative to the frame in which c is defined in space-time. This is true for all velocities.

Black Holes

Event Horizon defined as point (R) at which matters is transformed completely into light. A Black Hole is a spherical shell with inner "core structure" of positive matter (charge) with $k = R^2, R \leq 1$, and an outer shell of negative matter (charge) with $k = \frac{1}{R^2}$, R≥1. The energy of the outer core is the integrated "impulse" out to a distance R=kCT, where k≥1; the energy of the inner core is the integrated "impulse" of positive energy in to a diameter of R=1/kCT. Thus the Hilbert space has an R cutoff depending on the energy of the core $E = M(\pi R^2)$ the imaginary in the exponent is only to declare the whole system invariant in translation (in the sense of an infinite "lattice" of black holes/universes) with no overlap (interaction). That is, one "real", and the rest "virtual", "imaginary", whatever…

However, this description is energy only; there is no extension. CT is the "energy" radius of the Black Hole, which has no kinetic energy, since any impinging matter is immediately converted into ± energy at the event horizon (matter is "crushed") and distributed accordingly into the inner and outer shells at the surface of the event horizon (a black hole "has no hair"); mass increases accordingly. During the conversion, the impinging photon is "virtual", during the time its degeneracy is lifted. In that sense, the Black Hole cores are "virtual" when considered separately, only when considered together is the Black Hole charge neutral and thus "real". That is, motion in Minkowski space time is virtual if c is constant (no charge imbalance, Fermi level = 0).

Since this corresponds to a deceleration, Einstein would equate this to a gravitational "red shift" via the equivalence principle, although strictly speaking, this idea involves a quantum mechanical wave equation, and the question of applying QM to gravity is problematical. (if light slows down, there is either the medium has changed via the permittivity/permeability constants (ie., the system charge/current has changed or the scalar potential has changed – i.e., the mass of light has changed, which is subtracted out in the vector potential).