# Relativistic Energy Derivation

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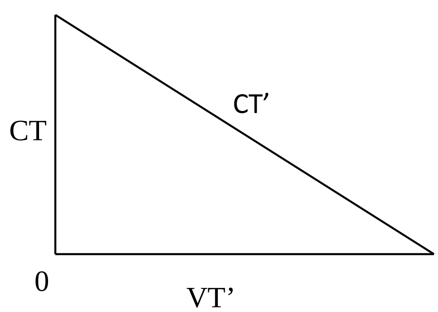
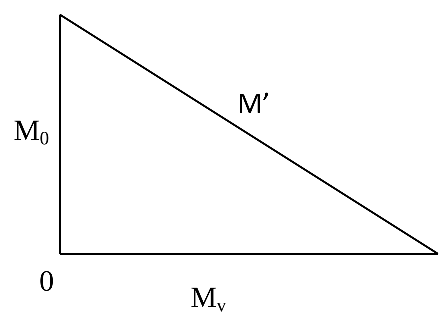
## Mass Derivation (The Mass Creation Equation)

$M_0 = \rho CT \geq 0,\ \rho = 1$ as the initial condition, $C$ the mass creation rate, $T$ the time, $\rho$ a density.

Let $V$ be a second mass creation rate, and $T'$ a second mass creation time, defined at a single mass point (i.e., an affine connection at a single mass "event"). Assume that $V \perp C$ and $VT' \perp CT$ , valid for all values of $CT$ and $VT'$ and scaling factors $T$ and $T'$. We then have the relation:

$$(CT')^2 = (VT')^2 + (CT)^2$$




$$(M')^2 = (M_v)^2 + (M_0)^2$$

Solving gives the relation between the mass creation times in terms of the mass creation rates:

$$T' = \frac{T}{\sqrt{1-\left(\frac{V}{C}\right)^2}} = \frac{T}{\sqrt{1-(\mathrm{B})^2}} = T\,\Gamma, \quad \Gamma = \frac{1}{\sqrt{1-\left(\frac{V}{C}\right)^2}}, \quad \mathrm{B} = \frac{V}{C}$$

Then $M' = CT' = \frac{CT}{\sqrt{1-\left(\frac{V}{C}\right)^2}} = (CT)\Gamma = M_0\Gamma$, and $\Gamma = \rho$ is interpreted as the density. The total mass increases with density (and V) from the rest mass (initial condition). $M'$ and $\Gamma$ are covariant $M_0$ (an increase in one implies an increase in the other) w.r.t the "rest mass" (initial condition)

Note that

$$\frac{V}{C} = \sqrt{1-\left(\frac{T}{T'}\right)^2} = \mathrm{B}$$

**Relativistic Energy Derivation from Relativistic Mass**

(to be consistent with the above, all variables should be in capital letters, but the following is easier to read in my opinion.

$$m' = m_0\Gamma = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$m'\sqrt{1-\frac{v^2}{c^2}} = m_0$$

$$(m')^2\left(1-\frac{v^2}{c^2}\right) = (m_0)^2$$

$$(m')^2\left[\left(\frac{c^2}{c^2}\right)-\frac{v^2}{c^2}\right] = (m_0)^2$$

$$(m')^2\left[\frac{c^2-v^2}{c^2}\right] = (m_0)^2$$

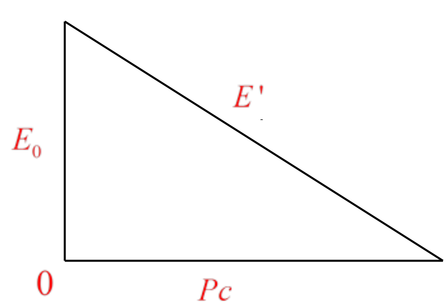
$$(m')^2\left(c^2-v^2\right) = (m_0)^2 c^2$$

$$(m')^2 c^2 = (m')^2 v^2 + (m_0)^2 c^2$$

$$(m')^2 c^4 = \left(m'\frac{v}{c}\right)^2 c^2 + (m_0)^2 c^4$$

$$P = m'\left(\frac{v}{c}\right)$$

$$(E')^2 = (m')^2 c^4 = P^2c^2 + (m_0)^2 c^4 = (Pc)^2 + (E_0)^2$$



Consider the Momentum term $P^2c^2$ . Taking the square root gives

$Pc = [m'\frac{v}{c}]c = M_0\Gamma\frac{v}{c}c = ct(\frac{t'}{t})\frac{v}{c}c = (vt')c$ , so we see that the relativistic momentum is given by:

$P = vt'$ and $P^2c^2 = (vt')^2 c^2$ for conservation of relativistic energy.

## The Minkowski Matrix

Let $f\left(x,y,z\right)$ = r = r(x,y,z) = $\sqrt{x^2+y^2+z^2}$

Define a four dimensional matrix A:

$$A=\begin{vmatrix} ir & 0 & 0 & 0 \\ 0 & r & 0 & 0 \\ 0 & 0 & m_v & 0 \\ 0 & 0 & 0 & m_0 \end{vmatrix}$$

$$A^2=\begin{vmatrix} -r^2 & 0 & 0 & 0 \\ 0 & r^2 & 0 & 0 \\ 0 & 0 & \left(m_v\right)^2 & 0 \\ 0 & 0 & 0 & \left(m_0\right)^2 \end{vmatrix}$$

$$Tr\left(A^2\right)=m'(A)^2=-r^2+r^2+m_v{}^2+m_0{}^2=m_v{}^2+m_0{}^2$$

$\left(m'\right)^2=m_v{}^2+m_0{}^2$ , (Positive Definite). For $r=m_v=m_0=1$ ,

$A=\begin{vmatrix} i & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{vmatrix}$, which gives the "Minkowski Matrix": $A^2=\begin{vmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{vmatrix}$, so that $Tr(A^2)=2$

That is, $\left(m'\right)^2=m_0{}^2+m_0{}^2=2m_0{}^2$, $A^{(M')}=\begin{vmatrix} M_v & 0 \\ 0 & M_0 \end{vmatrix}$, positive definite, as is $\left(A^{(M')}\right)^2$

The signature of this expression of the Minkowski matrix is given by the diagonal $\left(-1,1,1,1\right)$ and is related to the covariant Electromagnetic Tensor in Electromagnetism.

## General Relativity

Of course, the above analysis can be applied to any function $f(x,y,z,t)$

$$A^2 = \begin{vmatrix} -f(x,y,z,t)^2 & 0 & 0 & 0 \\ 0 & f(x,y,z,t)^2 & 0 & 0 \\ 0 & 0 & (m_v)^2 & 0 \\ 0 & 0 & 0 & (m_0)^2 \end{vmatrix}$$

And again,

$$A^{(M')} = \begin{vmatrix} M_v & 0 \\ 0 & M_0 \end{vmatrix}$$

In Quantum Field Theory, another (contravariant) “signature” for the Minkowski matrix given by $(+,-,-,-)$

$$\begin{vmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{vmatrix} = \begin{vmatrix} 1^2 & 0 & 0 & 0 \\ 0 & i^2 & 0 & 0 \\ 0 & 0 & i^2 & 0 \\ 0 & 0 & 0 & i^2 \end{vmatrix},$$

$$\begin{vmatrix} r^2 & 0 & 0 & 0 \\ 0 & (ir)^2 & 0 & 0 \\ 0 & 0 & (im_v)^2 & 0 \\ 0 & 0 & 0 & (im_0)^2 \end{vmatrix} = \begin{vmatrix} r^2 & 0 & 0 & 0 \\ 0 & -(r)^2 & 0 & 0 \\ 0 & 0 & -(m_v)^2 & 0 \\ 0 & 0 & 0 & -(m_0)^2 \end{vmatrix}$$

Eliminating the coordinate (“r”) parameters as before, we have:

$\begin{vmatrix} -(m_v)^2 & 0 \\ 0 & -(m_0)^2 \end{vmatrix}$ so that $Tr\left(\begin{vmatrix} -(m_v)^2 & 0 \\ 0 & -(m_0)^2 \end{vmatrix}\right) = -(m')^2 = -(m_v)^2 + \left[-(m_0)^2\right]$

These can be added to the “covariant” signature Minkowsi matrix as destruction operators, but the total relativistic energy can never be below $E_0 = m_0 c^2 \geq 0 \quad (V \leq C)$ , so the final result is:

$(m')^2 = m_v{}^2 + m_0{}^2 - (m_v)^2 + \left[-(m_0)^2\right] \geq 0$ Note that there are four particles; two “creation” operators and two “destruction” operators.

For the case:

$\begin{vmatrix} 1^2 & 0 & 0 & 0 \\ 0 & 1^2 & 0 & 0 \\ 0 & 0 & 1^2 & 0 \\ 0 & 0 & 0 & 1^2 \end{vmatrix}$ , 4 particles have been created and none destroyed.

(Particle creation can increase the dimension of the matrix arbitrarily, until all the existing particles in the Universe have been taken into account).

## Thoughts and Feelings

One can see that coordinate systems correspond to thoughts and masses correspond to feelings, and that every real thought has its imaginary counterpart. It should be clear that one doesn't have to restrict his coordinate thoughts to circles. Any coordinate function will have its imaginary counterpart in the above analysis. However, if anyone's curved (shall we say kinky?) thoughts should actually produce mass, it can be included in the matrix as either another mass dimension, or within one of the two that already exist.

Of course, production of a kinky thought/feeling-mass may require child support...

Now some may think that kinkiness causes gravity, and in fact there have experiments in the '60's that were performed with this perspective. A number of followers of various gurus in India tried bouncing up and down on their camp cots with their eyes closed trying to levitate. So far there have been no reports of success of which I am aware....

## Displacement Current

Displacement current is the alternating (time varying) electric field Maxwell had to include in his equations so that the emf could propagate through space; the basic idea is to put a capacitor in Ampere’s wire, so that the equations will e consistent.

Here is a basic video explaining Maxwell’s equations:

https://www.youtube.com/watch?v=AWI70HXrbG0

The lecturer introduces displacement current at the very end as the term Maxwell had to add to model the transfer of emf over space (between the plates of the capacitor).

$c = \frac{1}{\sqrt{\varepsilon_0 \upsilon_0}}$: Maxwell’s result, interpreted as “speed”, versus $\varepsilon_0 \upsilon_0 = m_0$, so that $(\varepsilon_0 \upsilon_0) c^2 = m_0 c^2 = 1$,

He kind of waves his hand at Maxwell's interpretation (since his exposition is very basic); to do the derivation you have to imagine the capacitor plates are infinite (so the E field is perpendicular to the plates and the B field is parallel to them, and you have to imagine an alternating current with no "friction" of space (i.e., independent of the dialectric). So it involves more math than just the dot product....

Displacement_current (Wiki article)

The imaginary capacitor plates are conceptually similar to the "branes" general relativists like to imagineer... However, instead of alternating current, one can imagine particles (i.e., photons) being created and destroyed on the plates themselves by somebody, somewhere, a long time ago, far away..

(My own feeling is that the Universe is braneless, but in any case it will be very hard to tell if it has one. Perhaps GTR fans with more branes than me will be able to prove it; on the other hand, it may be a braneless intellectual endeavor ...)

(And somehow I feel uncomfortable being between the plates of an infinite capacitor if it involves alternating current, and the plates are loose or flop around, especially if some want to call them god ...

In any case, that is the physical motivation for identifying the "mass" of light with permittivity and permeability, since the emf between the capacitor plates is a "force" ultimately derived from the laws that form the foundation of Maxwell's analysis.

## The Mass of light from Coulomb’s and the Vacuum Permeability

Coulomb's Force Law

$$F_{Coulomb} = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{r^2}$$

Vacuum Permeability

The force per unit length for two wires is given by:

$$F = \frac{\mu_0}{2\pi}\frac{I^2}{r} = \frac{\mu_0}{2\pi}\left(\frac{dq}{dt}\right)^2$$

For a specific value of $q$ and $t$, and considering only the effect of a single wire:

$$F = \frac{\mu_0}{4\pi}\frac{I^2}{r} = \frac{\mu_0}{4\pi r}\left(\frac{q}{t}\right)^2$$

For $r = ct$, we have:

$$F_{Permeability} = \frac{\mu_0 c^2}{4\pi r^2}q^2$$

**From “Introduction to the Theory of Relativity”, Peter Gabriel Bergmann, Chapter V, pg. 47, ff.**

“We can formally characterize relativistic physics by the invariance of the expression

$$\tau_{12}{}^2 = \left(t_2 - t_1\right)^2 - \frac{1}{c^2}\left[\left(x_2 - x_1\right)^2 + \left(y_2 - y_1\right)^2 + \left(z_2 - z_1\right)^2\right] \quad (5.1)$$

(My Comment) We can re-interpret this in terms of mass by:

$$\tau_{12}{}^2 = \left(t_2 - t_1\right)^2 - \frac{1}{c^2}\left[\left(x_2 - x_1\right)^2 + \left(y_2 - y_1\right)^2 + \left(z_2 - z_1\right)^2\right]$$
$$\left(c\tau_{12}\right)^2 = \left[c\left(t_2 - t_1\right)\right]^2 - \left[\left(x_2 - x_1\right)^2 + \left(y_2 - y_1\right)^2 + \left(z_2 - z_1\right)^2\right]$$
$$\left(c\tau_{12}\right)^2 = \left[c\Delta t_{21}\right]^2 - \left[\Delta x_{21}{}^2 + \Delta y_{21}{}^2 + \Delta z_{21}{}^2\right]$$
$$\left(c\tau_{12}\right)^2 = \left[c\Delta t_{21}\right]^2 - \left[\Delta r_{21}{}^2\right]$$
$$\left[c\Delta t_{21}\right]^2 = \left[\Delta r_{21}\right]^2 + \left(c\tau_{12}\right)^2$$
$$\left(CT'\right)^2 = \left[VT'\right]^2 + \left(CT^0\right)^2$$
$$\left(M'\right)^2 = \left(M_0\right)^2 + \left(M_0\right)^2$$
$$M' = M_0\Gamma$$

Mass Matrix Form (Equivalent of the Minkowski Matrix)

$$\begin{vmatrix} iM_v & 0 & 0 \\ 0 & M_v & 0 \\ 0 & 0 & M_0 \end{vmatrix}$$

$$\begin{vmatrix} iVT' & 0 & 0 \\ 0 & VT' & 0 \\ 0 & 0 & CT_0 \end{vmatrix}$$

$$Tr\begin{vmatrix} iVT' & 0 & 0 \\ 0 & VT' & 0 \\ 0 & 0 & CT_0 \end{vmatrix}^2 = -(VT')^2 + (VT')^2 + (CT_0)^2 = (CT_0)^2$$

$$Tr\begin{vmatrix} i1_v & 0 & 0 \\ 0 & 1_v & 0 \\ 0 & 0 & 1_{CT} \end{vmatrix}^2 = Tr\begin{vmatrix} (i1_v)^2 & 0 & 0 \\ 0 & (1_v)^2 & 0 \\ 0 & 0 & (1_{CT})^2 \end{vmatrix} = -(1_v)^2 + (1_v)^2 + (1_{CT})^2 = (1_{CT})^2$$

The imaginary component "destroys" the real component of the perturbation, leaving the rest mass (i.e. the initial condition) invariant.

## The Final Blow

Consider the Destruction matrix

$D_{Basis} = \begin{vmatrix} i & 0 \\ 0 & 1 \end{vmatrix}$. Then: $D_{Basis}{}^2 = \begin{vmatrix} -1 & 0 \\ 0 & 1 \end{vmatrix} \Rightarrow TR\left(D_{Basis}{}^2\right) = -1 + 1 = 0$

(Nothing to feel here, move right along.)

Now consider the rest mass $M_0$:

$$D_{m_0} = \begin{vmatrix} iM_0 & 0 \\ 0 & M_0 \end{vmatrix}$$

$$D_{m_0}{}^2 = \begin{vmatrix} -(M_0)^2 & 0 \\ 0 & (M_0)^2 \end{vmatrix} \Rightarrow TR(D_{m_0}{}^2) = -(M_0)^2 + (M_0)^2 = 0$$

Didn't you pay attention to what I just said? ☺