

The Relativistic Circle

Chuck (Charles Jr) Keyser

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[Website](#)

Postulates:

1. There are no negative numbers; all countable numbers are positive definite $\sqrt{n^2} = n$. Negative numbers only arise in the context of differences:
 $-c = a - b, b > a \leftrightarrow b - c = a$, with both sides of the equality positive, and $a - a = 0 \leftrightarrow a = a$ or $a = 0$.

2. A prime number n is defined by the relation $1_n = \frac{n}{n}$ ("1 to the base n ") so that

$$n(1_n) = n\left(\frac{n}{n}\right) = n. \text{ In particular, } 1_{f(n)} = \frac{f(n)}{f(n)} \leftrightarrow f(n)(1_{f(n)}) = f(n). \text{ Every number is}$$

therefore prime to its own base. Goldbach's Theorem ("Every even number is the sum of two primes.") immediately follows: $n + n = 2n$

3. Proof of Fermat's Last Theorem:

Hypothesis: $c^n \neq a^n + b^n$ for all a, b, c, n

Thesis: For numbers $(a, b, c, n) > 0$

$$c = a + b$$

$$c^n = (a + b)^n = [a^n + b^n] + [f(a, b, n)] \text{ (Binomial Expansion)}$$

$$c^n = [a^n + b^n] \leftrightarrow [f(a, b, n)] = 0$$

$$[f(a, b, n)] \neq 0$$

$$\therefore c^n \neq [a^n + b^n] \text{ Q.E.D.}$$

Note: of particular interest for physics is the case $n = 2$

4. Complex Numbers

$$\psi = a + ib$$

$$\psi^* = a - ib$$

$$\psi\psi^* = (a + ib)(a - ib) = (a^2 + b^2 - i(ba) + i(ab) - i^2b^2)$$

$$\psi\psi^* = a^2 + b^2 \leftrightarrow i = \sqrt{-1} \leftrightarrow i^2 = -1$$

However, $1 = \frac{1}{1} = \frac{-1}{-1} \neq \frac{i^2}{i^2} = \frac{-1}{-1}$ (i.e., if negative numbers do not exist, then neither do their square roots.

5. Russell's Paradox

"A barber in a village shaves all those and only those that don't shave themselves. Does the barber shave himself?" Ans. The barber cannot both shave and not shave himself, and thus does not exist; the village of those that don't shave themselves is irrelevant. Mathematically,

$$1^2 \neq 1$$

6. Coordinate velocity

$$x := vt$$

$$\frac{x}{t} = v \left(\frac{t}{t} \right) = 1_t$$

$$x = (vt) 1_t = t$$

$$\frac{x}{vt} = 1_v = \frac{vt}{vt} \leftrightarrow x = (vt) (1_v)$$

$$x_n := n(vt)$$

$$\frac{x_n}{(vt)} = n \left(\frac{vt}{vt} \right) = n(1_v)$$

The electromagnetic force of light (single numerical value) is characterized by the expression

$$f_0 := c_0 = \frac{1}{\sqrt{\epsilon_0 \mu_0}} \text{ where } \epsilon_0 \text{ and } \mu_0 \text{ are derive from Coulomb's and Ampere's force laws, and the}$$

expression $c_0 = \frac{1}{\sqrt{\epsilon_0 \mu_0}}$ is the result from Maxwell's equations, which agrees numerically with the

speed of light (at sea level at Greenwich (England) GMT 12AM on any Sunday morning). More generally (e.g. inside a lens) the expression for electromagnetic force in terms of Maxwell's derivation is

$$f := c = \frac{1}{\sqrt{\epsilon \mu}}$$

The electromagnetic force of light is scaled by the variable τ In order to express all possible values

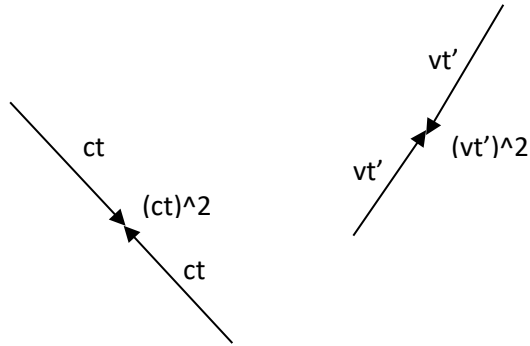
$$f_0 := c_0 \tau, \text{ or more generally } f := c \tau$$

Newton's Third Law is expressed by an equal and opposite force at a coordinate origin:

$$m = f^2 = (c\tau)^2 \text{ and by itself characterizes an initial state (condition).}$$

Non-Interacting Forces

Affine (non-interacting) Forces



Affine forces have no common origin; as such, they do not interact and are represented by the matrices

$$|\#| = \begin{vmatrix} ct & 0 \\ 0 & vt' \end{vmatrix}, \quad Tr(\#) = (ct) + (vt')$$

$$|\#|^2 = \begin{vmatrix} (ct) & 0 \\ 0 & (vt') \end{vmatrix}^2 = \begin{vmatrix} (ct)^2 & 0 \\ 0 & (vt')^2 \end{vmatrix}, \quad Tr(\#) = (ct)^2 + (vt')^2$$

Interacting forces have a common origin (force center) and are characterized by the relations

$$\# = (ct) + (vt')$$

$$\#^2 = (ct)^2 = (ct)^2 + (vt')^2 + 2(ct)(vt')$$

For any Pythagorean Triple (e.g., 5,4,3)

$$7 = 4 + 3$$

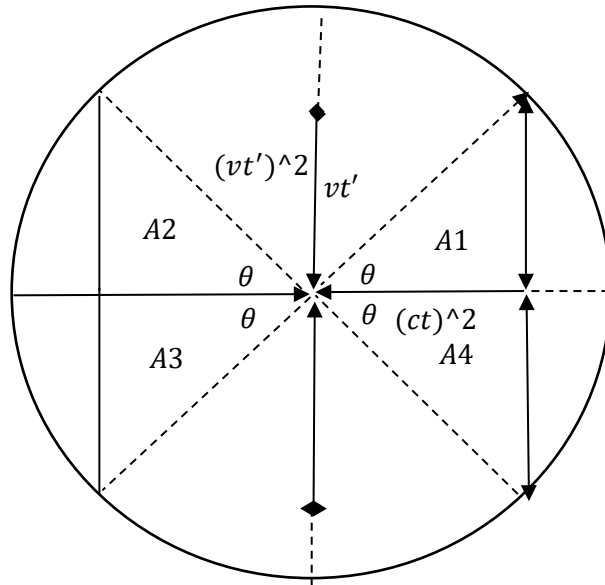
$$49 = 7^2 = (4 + 3)^2 = [16 + 9] + [2(4)(3)] = [25] + [24]$$

Note that the hypotenuse $5^2 = [4^2 + 3^2] = [16 + 9] = 25$ is only valid as a subset of $\#^2 = 49$

Case $(vt') < (ct)$

$$A_i = 1/2(ct)(vt')$$

$$A = A_1 + A_2 + A_3 + A_4 = 4 \left[\left(\frac{1}{2} \right) (ct)(vt') \right]$$



$$\# = (ct') = (ct) + (vt')$$

$$\#^2 = (ct')^2 = (ct)^2 + (vt')^2 + 2(ct)(vt')$$

Trigonometry

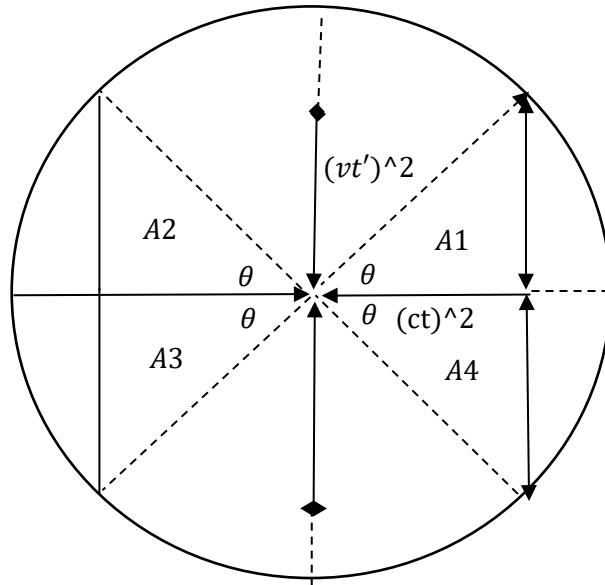
$$ct' = ct + vt'$$

$$(ct')^2 = (ct)^2 + (vt')^2 + 2(ct)(vt')$$

Final State from initial state (ct) is characterized by:

$$A_i = 1/2(ct)(vt')$$

$$A = A_1 + A_2 + A_3 + A_4 = 4 \left[\left(\frac{1}{2} \right) (ct)(vt') \right]$$



$$(ct')^2 = (ct)^2 + (vt')^2 + 2(ct)(vt')$$

$$\frac{(ct')^2}{(ct')^2} = \frac{(ct)^2}{(ct')^2} + \frac{(vt')^2}{(ct')^2} + \frac{2(ct)(vt')}{(ct')^2}$$

$$1_{(ct')^2} = \left(\frac{t}{t'}\right)^2 + \left(\frac{v}{c}\right)^2 + 2\left(\frac{t}{t'}\right)\left(\frac{v}{c}\right)$$

$$\gamma := \left(\frac{t'}{t}\right), \beta := \frac{v}{c}$$

$$1_{(ct')^2} = \left(\frac{1}{\gamma}\right)^2 + (\beta)^2 + 2\left(\frac{\beta}{\gamma}\right)$$

The term $h^2 = 2\left(\frac{\beta}{\gamma}\right) = 2S$ defines Planck's "constant" in terms of the charge to mass ratio $\left(\frac{\beta}{\gamma}\right)$ or

the "spin" S where $S = \frac{h}{\sqrt{2}}$

The expression in radial coordinates becomes:

$\pi 1_{(ct')^2} = \pi\left(\frac{1}{\gamma}\right)^2 + \pi(\beta)^2 + \left(\frac{1}{\gamma}\right)(2\pi\beta)$, where $\left(\frac{1}{\gamma}\right)$ characterizes a radius and $(2\pi\beta)$ a circumference.

It is also possible to identify $\frac{1}{\gamma} = \cos \theta$, $\beta = \sin \theta$ so that

$$1_{(ct')^2} = \left(\frac{1}{\gamma}\right)^2 + (\beta)^2 + \left(\frac{1}{\gamma}\right)(2\beta) = \cos^2 \theta + \sin^2 \theta + 2 \cos \theta \sin \theta$$

$$\pi\left(1_{(ct')^2}\right) = \pi \cos^2 \theta + \pi \sin^2 \theta + (\cos \theta)(2\pi \sin \theta)$$

Case $vt' > ct$

$$ct' = ct + vt'$$

$$(ct')^2 = (ct)^2 + (vt')^2 + 2(ct)(vt')$$

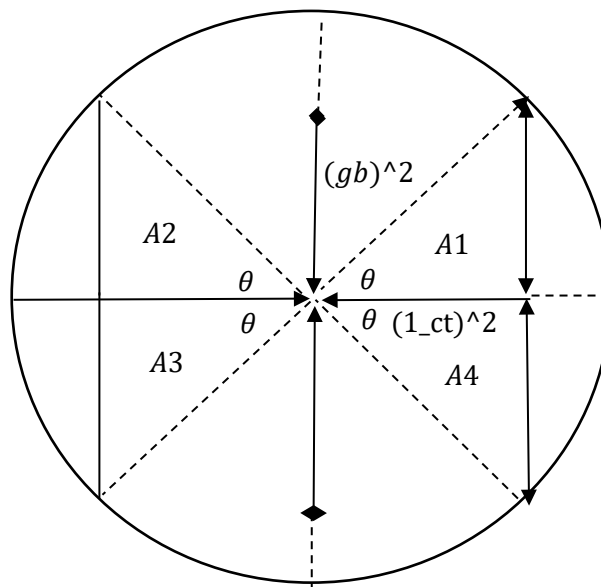
$$\frac{(ct')^2}{(ct)^2} = \frac{(ct)^2}{(ct)^2} + \frac{(vt')^2}{(ct)^2} + \frac{2(ct)(vt')}{(ct)^2}$$

$$\gamma := \left(\frac{t'}{t}\right), \beta := \frac{v}{c}$$

$$\left[\gamma^2 = 1_{(ct)^2} + (\gamma\beta)^2\right] + 2\gamma\beta$$

$$A_i = 1/2(gb)$$

$$A = A_1 + A_2 + A_3 + A_4 = 4 \left[\left(\frac{1}{2}\right) (gb) \right]$$



$$g^2 = 1_{(ct')^2} + (gb)^2 + 2(gb)$$

The term in the brackets can be identified with the hyperbolic identity using complex n

$$\cosh^2 \theta = \left(1_{(ct)}\right)^2 + \sinh^2 \theta$$

$$\psi = \cosh \theta = \left(1_{(ct)}\right) + i \sinh \theta$$

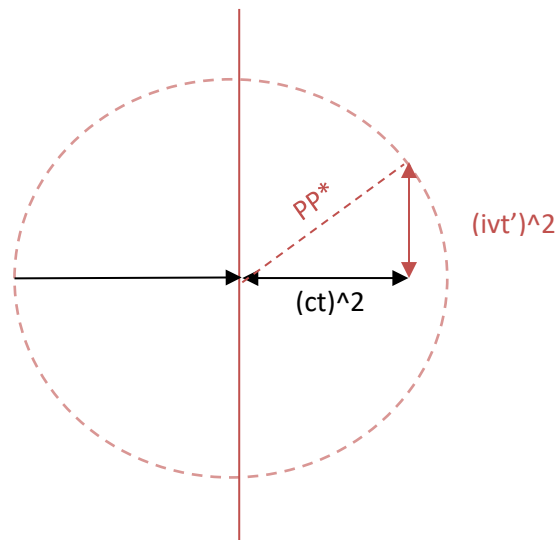
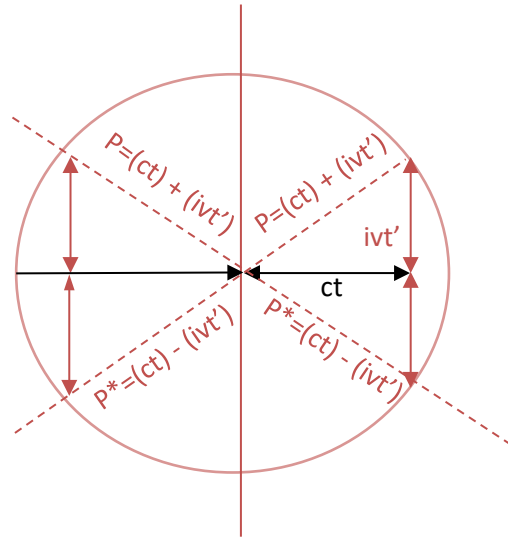
$$\psi^* = \cosh \theta^* = \left(1_{(ct)}\right) - i \sinh \theta$$

$$\psi\psi^* = \cosh^2 \theta = \left(1_{(ct)}\right)^2 + \sinh^2 \theta$$

Wich is untenable, considering the caveats above.

The graphical representation of the complex circle is given below:

The Complex Circle



$$PP^* = (ct)^2 - (ivt')^2$$

$$PP^* = (ct)^2 + (vt')^2 \text{ iff } i = \text{sqr}(-1)$$

$$P := \psi = (ct) + (ivt')$$

$$P^* := \psi^* = (ct) - (ivt')$$

$$PP^* = (ct)^2 + (ivt')(ct) - (ivt')(ct) - i^2 (vt')^2$$

$$\psi\psi^* = (ct)^2 + (vt')^2 \leftrightarrow i = \sqrt{-1}$$

The trigonometric identity is suggested by:

$$1 = \cos \theta + i \sin \theta$$

$$1^* = \cos \theta - i \sin \theta$$

$$(1)(1^*) = \cos^2 \theta + \sin^2 \theta$$

Example (5,4,3) Pythagorean triangle

$$\# := (ct) + (vt')$$

$$\#^2 := [(ct)^2 + (vt')^2] + [2(ct)(vt')]$$

$$(ct) = 4, (vt') = 3 \rightarrow \#^2 = [(5)^2] + \left[4 \left(\frac{1}{2}\right)(12)\right] = [25] + [24] = (4+3)^2 = (7)^2 = 49$$

$$\text{Note that } [25] \neq [25] = (4+3i)(4-3i)$$

That is, $\#^2 \neq \psi\psi^*$; conjugation eliminates the multiplication (interaction) product $[2(ct)(vt')]$

This is a case of Fermat's Last Theorem for $n = 2$

Radial Coordinates:

Note that π only applies in second order, where

$$\pi \#^2 := \pi [(ct)^2 + (vt')^2] + \pi [2(ct)(vt')] \text{ and that}$$

$$\pi [2(ct)(vt')] \equiv (ct)[2\pi(vt')] \equiv (r)(2\pi r') \text{ which is the product of a radius and a circumference.}$$

That is, the interaction area differs by the introduction of π in second order, so that

$$(r)(2\pi r) = 2\pi r^2 \neq (x)(2x) = 2x^2; \text{ that is,}$$

$$(x+x)^2 \neq \pi(r+r)^2$$

Consider the expression:

$$\#^2 = (ct')^2 = [(ct) + (vt')]^2 = (ct)^2 + (vt')^2 + 2(ct)(vt')$$

Compared with the expression:

$$\psi\psi^* = \{(ct')^2 = (ct)^2 + (vt')^2\}$$

Solving this equation yields the so-called “time dilation” equation of Special Relativity:

$$t' = t\Gamma, \Gamma = \frac{1}{\sqrt{1 - \beta^2}}, \beta = \frac{v}{c}$$

To understand the origin of this expression, one must go back to the null results of the Michelson-Morley experiment and Lorentz’s explanation of them. In order to express the null results of the rotation of the two arms of the apparatus in the presents of the atmosphere (“or vacuum”, Lorentz proposed that the length of the arms be characterized by the relation:

(following the analysis in [“Introduction to the Theory of Relativity”](#) by Peter S. Bergmann

$$r := (x - vt)\alpha = c(\gamma x - \beta t)$$

$$r^2 := [(x - vt)\alpha]^2 = [c(\gamma x - \beta t)]^2$$

Note that $\pi r^2 := \pi[(x - vt)\alpha]^2 = \pi[c(\gamma x - \beta t)]^2$ so that if r is defined in Cartesian coordinates (as above)), the expression is an attempt to square the circle).

Expanding the expression $[(x - vt)\alpha]^2 = [c(\gamma x - \beta t)]^2$ and taking the appropriate root results in the two Lorentz transformation equations:

$$x' = (x - vt)\Gamma$$

$$t' = \left(t - \frac{vx}{c^2}\right)\Gamma$$

Einstein then suggested that the speed of light be declared constant (at least on the surface of the earth, etc...): this condition is expressed by the relation $x' = ct' \leftrightarrow x = ct$

Substituting in the Lorentz transforms results in the single expression (actually two identical expressions):

$$x' = ct' = (x - vt)\Gamma$$

The next step is to declare motion in the medium irrelevant by removing the product vt from the equation, so that

$$x' = ct' = (x)\Gamma$$

For $x = ct$, the "time dilation" equation follows immediately:

$$ct' = (ct)\Gamma \text{ and finally } t' = (t)\Gamma$$

Consider the expression $\beta = \frac{v}{c}$ and notice that this is true for any function of mass or charge:

$$\beta = \frac{f(m)v}{f(m)c} = \frac{g(q)v}{g(q)c} = \frac{f(m)v}{f(m)c} = \frac{g(q)v}{g(q)c} \text{ so that relativity is a function of imaginary coordinates only.}$$

Some comments on vectors

Special Relativity (and Quantum Mechanics) are an attempt to express physics in the language of vectors

If \vec{i} and \vec{j} are orthogonal vectors, then their product (either as a dot product or a cross product) is null, since $\cos(0)\sin(\frac{\pi}{2}) = 0$ in terms of the vector components given by the identity matrix:

$$|I| := \begin{vmatrix} \cos(0) & 0 \\ 0 & \sin(\frac{\pi}{2}) \end{vmatrix} = \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} \text{ where } Tr|I| = 2 \text{ and } Det|I| = 1^2$$

Note that $Tr \begin{vmatrix} 1 & 0 \\ 0 & -1 \end{vmatrix} = 0$ which is to be interpreted as $1 - 1 = 0 \leftrightarrow 1 = 1$ since two elements are involved.

(In light of the analysis above, $Tr \begin{vmatrix} 1 & 0 \\ 0 & -1 \end{vmatrix} \neq Tr \begin{vmatrix} 1 & 0 \\ 0 & i^2 \end{vmatrix} = Tr \begin{vmatrix} 1 & 0 \\ 0 & -1 \end{vmatrix}$ since $1 \neq 1 = -i^2$)

That is

$$\psi := (1 + i)$$

$$\psi^* := (1 - i)$$

$$\psi\psi^* = (1 + i)(1 - i) = 1^2 + 1(i) - 1(i) - i^2$$

$$\psi\psi^* = 1^2 + 1 \neq 2(1^2) \text{ Russel's Paradox: } 1^2 \neq 1$$

The Pauli Matrices

The matrix $\sigma_3 := \begin{vmatrix} 1 & 0 \\ 0 & -1 \end{vmatrix}$ is one of the Pauli Matrices of SU(2)

The others are:

$$|\sigma_1| = \begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix}, |\sigma_2| = \begin{vmatrix} 0 & -i \\ i & 0 \end{vmatrix}$$

$$|\sigma_1| + |\sigma_2| = \begin{vmatrix} 0 & 1-i \\ 1+i & 0 \end{vmatrix} = \begin{vmatrix} 0 & \psi^* \\ \psi & 0 \end{vmatrix}$$

$$\text{Det} \begin{vmatrix} 0 & \psi^* \\ \psi & 0 \end{vmatrix} = -\psi^* \psi = -(1-i)(1+i) = 1^2 + 1$$

Interpretation:

If 1 is the electromagnetic force E of a single particle, then 1^2 is an equal and opposite force for each particle at the source and the two sensors, characterized by the matrix

$$\begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix}^2 = \begin{vmatrix} 1^2 & 0 \\ 0 & 1^2 \end{vmatrix} \text{ where again } \text{Tr} \begin{vmatrix} 1^2 & 0 \\ 0 & -1^2 \end{vmatrix} = 1^2 - 1^2 = 0 \text{ is to be interpreted as } 1 = 1 \text{ for two particles}$$

and $\text{Tr} \begin{vmatrix} 1^2 & 0 \\ 0 & 1^2 \end{vmatrix} = 2(1^2)$ characterizes the existence of the two particles. Then $\pm iB$ is the field that

separates the particles at the interaction point in the Stern-Gerlach experiment; since it is imaginary, the final result is again disallowed by the condition that the interaction must be real. Compare this with the result

$$\# = 1 + 1$$

$$\#^2 = (1+1)^2 = [1^2 + 1^2] + [2(1^2)]$$

where $[2(1^2) = 2(1)(1)]$ (that is, $\#^2$ includes both the sum and product group operations.

In Matrix form, this is expressed as

$$\#^2 = \text{Tr} \begin{vmatrix} 1^2 & 0 \\ 0 & 1^2 \end{vmatrix} + \text{Det} \begin{vmatrix} 1 & 1 \\ -1 & 1 \end{vmatrix} = 2_+ (1^2) + 2_x (1^2) = 4$$

This should not be confused with the matrix $\text{Tr}|M| := \text{Tr} \begin{vmatrix} 1^2 & 0 & 0 & 0 \\ 0 & 1^2 & 0 & 0 \\ 0 & 0 & 1^2 & 0 \\ 0 & 0 & 0 & 1^2 \end{vmatrix} = 4(1^2)$ but $\text{Det}|M| = (1^8)$

Compare this with the Minkowski Matrix (a quaternion):

$$|M| := \begin{vmatrix} 1 & 0 & 0 & 0 \\ 0 & i & 0 & 0 \\ 0 & 0 & i & 0 \\ 0 & 0 & 0 & i \end{vmatrix}$$

There is much more to this story, but I don't have the spacetime to write it here.