Modern Physics since Newton

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Forward

When I was a junior at UCSB in the 1960's, I took a course in Modern Physics that described the Special Theory of Relativity in terms of a "time dilation" equation.

I had been in the Navy as an electronics technician, and had taken the undergraduate course in electromagnetism at UC Berkeley, so had some background, but the equation did not make sense if characterized in terms of clocks, rulers, and observers.

The 1960's were a time of turmoil, and although I was accepted into physics graduate school at UCSB, I decided instead to pursue music (Flamenco guitar); to support myself I wrote a correspondence course on Flamenco, which supported me for a brief period. However, with the advent of SDI (and the flood of research dollars to Santa Barbara, and the fact that Reagan wiped out the meager living my wife and I could make via teaching Flamenco at Community Colleges and Extension programs), I re-entered the world of technology vis Santa Barbara Research Center and General Research . There I was exposed to some of the best minds I had ever encountered, who supported my studies and exposed me to perspectives of which I had never imagined; however, I could never resolve the "time dilation" issue to my satisfaction.

I finally retired, and put the project to figure it out in my bucket list (along with my ability to recite "The Cremation of Sam McGee" and "The Shooting of Dan McGrew", which I had learned as a teenager for my allowance. Since over a decade had passed since I had been exposed to physics, I had to re-acquaint myself with the fundamentals (thanks to Wikipedia) – after much searching and temporary madness, I finally focused on Einstein's interpretation of the Lorentz transform and his attempt to resolve his concept of "inertial frames" with Maxwell's derivation of the speed of light and its subsequent explanation of radiation and absorption, which depends on Cartesian Coordinates (along with such concepts of "dot" and "cross" products (Greens and Stokes theorems) in its integral form.

My conclusion is that, in the end, Einstein was attempting to square the circle (with the square postulated as a first order approximation to the circle), to the extent that his analysis is represented by Cartesian coordinates rather than radial coordinates, and so his analysis is inconsistent with the requirement of the conservation of momentum and energy for physical systems.

The conclusion is that it can't be done, no matter how small the square and preserve the constancy of c.

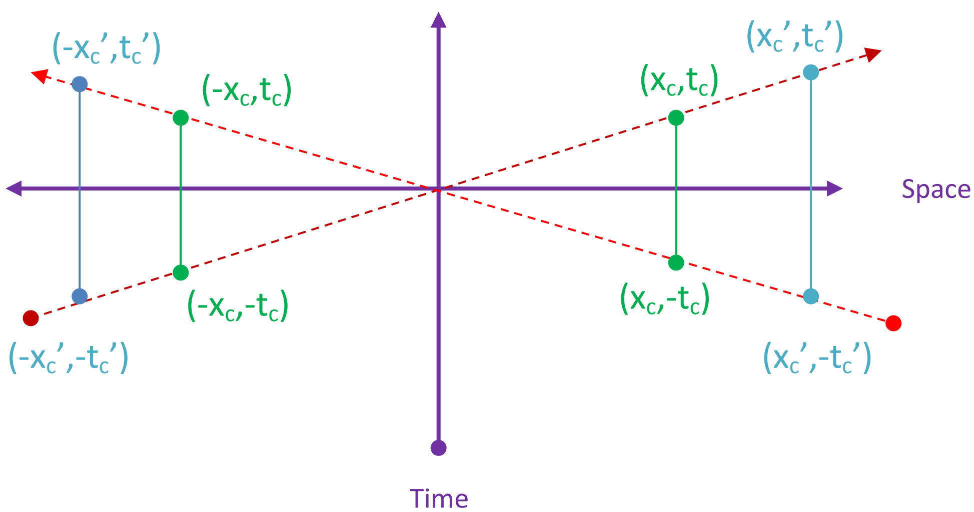
Part I

The Einstein/Bergmann derivation of the Lorentz Transform

I follow the derivation of the Lorentz Transform, following Peter S Bergmann in "Introduction to the Theory of Relativity" After an introductory explanation involving observers, trains, rods and clocks (in Cartesian coordinates), and with the assumption that the speed of light is a constant c, and v defines "inertial motion" relative to c, the actual analysis (pp. 38ff) begins with the hypothesis that the laws of physics are the same in every inertial frame. However, I interpret some of Bergmann's characterizations a bit differently.

The constancy of the speed of light implies there is an absolute coordinate frame in which c is defined as

$x_c = ct_c$ and $x_c = ct_c \Leftrightarrow x_c' = ct_c'$ and $c = \frac{|x_c|}{|t_c|} = \frac{|x'_c|}{|t'_c|}$



In this diagram, time moves up vertically, and the position coordinates of a photon modeled as a coordinate particle move right or left as the time axis crosses their red dotted "world lines". Since the photons do not interact at the point (0,0) shown, they continue their straight world lines after they coincide. For a pure coordinate system, it is immaterial "when" the coordinate photons are created or destroyed as long as their world lines exist at their common point (at that point, they are "simultaneous", at any other instant of time, they are separated.

The velocity of light is then defined as $c = \frac{|x_c|}{|t_c|}$ since c is assumed to be isotropic (independent of direction) and homogeneous. If a medium is assumed, then the velocity of the medium is then assumed to be defined as $v = \frac{x_v}{t_v}$ along some axis passing through the origin at which c is defined …

A Michaelson-Morley apparatus consists of two orthogonal and equal arms which were rotated to test the effect of a motion v in some direction with respect to c in terms of an interference pattern, which would change along the arms as the system was rotated. Since no effect was observed, Lorentz hypothesized that the arms of the apparatus had actually changed to compensate for the interaction of the medium with the apparatus.

Lorentz assumed that the actual length of the arm in the direction of velocity would be changed by a factor linear α that would vary as the system was rotated, causing a variation in space and time by interaction with the medium.

$x' = \alpha(x - vt),\ y' = y = 0,\ z' = z = 0$ (since y is orthogonal to this axis, and z is irrelevant)

In particular, for v = 0 (an orientation orthogonal to the relative velocity of the medium), $x' = \alpha x,\ \ v = 0$. For the orientation in line with the relative velocity of the medium, $x = vt$, so that $x' = 0$, with α to be determined. In the direction orthogonal to the position characterized by α he then assumed that the measurement of time would be affected by a linear factor of both space and time, and replacing "y" with x':

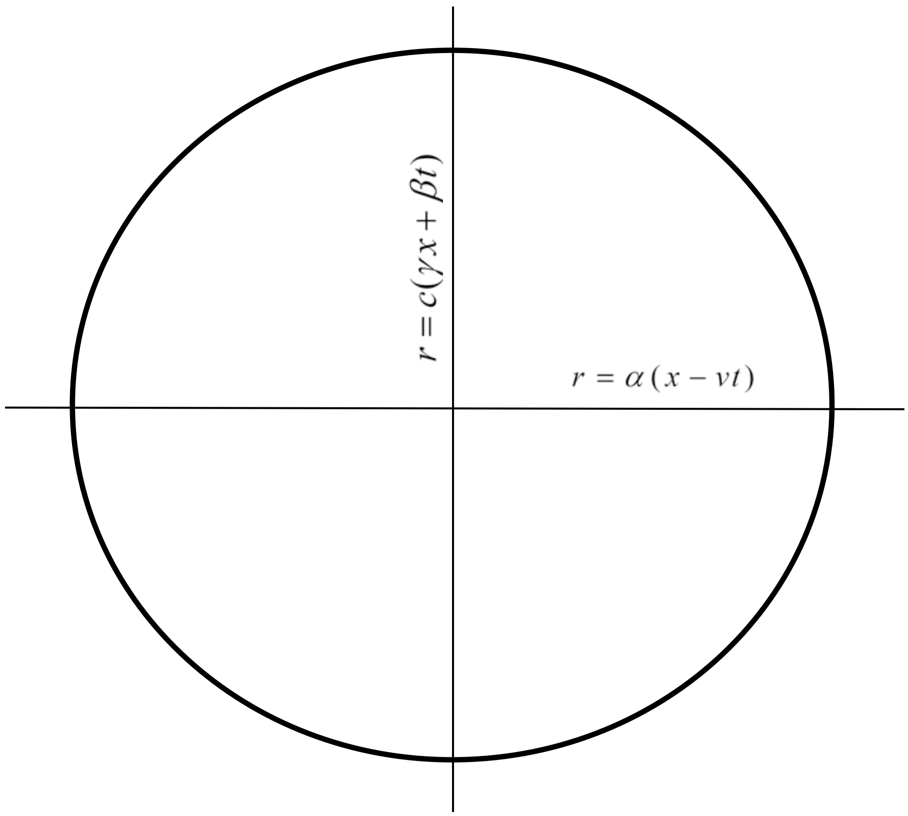
$t' = \gamma x + \beta t$ so that

$x' = ct' = c(\gamma x + \beta t)$

Equating the two characterizations of x', we have the relation:

$\alpha(x - vt) = c(\gamma x + \beta t)$

Since these results apply to arms that are orthogonal, there are no interacting terms, and the apparatus is described by the lengths of the rotating arms in a medium where c is homogeneous and isotropic, and the arm length change as the system is rotated:



The area of the circle can then be calculated by the relation:

$$\pi\alpha^2(x - vt)^2 = \pi c^2(\gamma x - \beta t)^2$$

Eliminating π and expanding and rearranging results in the expression:

$$(c^2\beta^2 - v^2\alpha^2)t^2 = (\alpha^2 - c^2\gamma^2)x^2 + (v\alpha^2 + c^2\beta\gamma)xt$$

Equating the coefficients of x^2 and t^2 (c is a constant independent of area where $s_x = x^2 = xx$ and $s_t = t^2 = tt$) ,and setting the coefficient of xt equal to 0 (no interaction between space and time – i.e., x and t are orthogonal; this is equivalent to using dot and cross products in the integral form of Maxwell's equations, and the elimination of the scalar field to form the EM field tensor in Quantum Electrodynamics) results in the expressions:

$$1:\ c^2\beta^2 - v^2\alpha^2 = c^2$$
$$2:\ \alpha^2 - c^2\gamma^2 = 1$$
$$3:\ v\alpha^2 + c^2\beta\gamma = 0$$

Equating α^2 from 1,2:

$$1:\ \alpha^2 = \frac{c^2(1-\beta^2)}{v^2}$$
$$2:\ \alpha^2 = 1 + c^2\gamma^2$$

$$\frac{c^2(1-\beta^2)}{v^2} = 1 + c^2\gamma^2$$
$$4:\ \gamma^2 = \frac{1}{c^2}\left[\frac{c^2(1-\beta^2)}{v^2} - 1\right]$$

Equating α^2 from (2,3)

$$2:\quad \alpha^2 = 1 + c^2\gamma^2$$
$$3:\quad \alpha^2 = -\frac{c^2\beta\gamma}{v}$$
$$2,3:\ 1 + c^2\gamma^2 = -\frac{c^2\beta\gamma}{v}$$

From 1,3

$$\frac{c^2(1-\beta^2)}{v^2} = -\frac{c^2\beta(\gamma)}{v}$$
$$5:\ \gamma = -\frac{(1-\beta^2)}{v\beta}$$

Squaring 5 and equating with 4:

$$\left(-\frac{(1-\beta^2)}{v\beta}\right)^2 = \frac{1}{c^2}\left[\frac{c^2(1-\beta^2)}{v^2}-1\right]$$

After a bit of algebra, this results in the expression:

$$\beta^2 = \frac{1}{1-\frac{v^2}{c^2}}$$

Substituting in 5:

$$5: \gamma = -\frac{(1-\beta^2)}{v\beta} = -\frac{1}{v}\left[\frac{(1-\beta^2)}{\beta}\right] = -\frac{1}{v}\left[\frac{(1-\frac{1}{1-\frac{v^2}{c^2}})}{\beta}\right]$$

$$1-\frac{1}{1-\frac{v^2}{c^2}} = 1-\frac{c^2}{c^2-v^2} = \frac{c^2-v^2-c^2}{c^2-v^2} = \frac{-v^2}{c^2-v^2}$$

$$\gamma = -\frac{1}{v}\frac{\frac{v^2}{c^2-v^2}}{\beta} = \frac{1}{v}\frac{\frac{v^2}{(c+v)(c-v)}}{\beta} = -\frac{\frac{v}{(c+v)(c-v)}}{\sqrt{\frac{1}{1-\frac{v^2}{c^2}}}} = -\frac{v\sqrt{1-\frac{v^2}{c^2}}}{(c+v)(c-v)} =$$

$$-\frac{v\sqrt{\frac{c^2-v^2}{c^2}}}{(c+v)(c-v)} = \frac{\frac{v}{c}\sqrt{c^2-v^2}}{(c+v)(c-v)} = -\frac{\frac{v}{c}\sqrt{(c+v)(c-v)}}{(c+v)(c-v)} = -\frac{\frac{v}{c}}{\sqrt{(c+v)(c-v)}} =$$

$$-\frac{\frac{v}{c}}{\sqrt{c^2-v^2}} = -\frac{\frac{v}{c}}{c\sqrt{1-\frac{v^2}{c^2}}} = \frac{v}{c^2}\frac{1}{\sqrt{1-\frac{v^2}{c^2}}} = -\frac{v\beta}{c^2}$$

That is, $\gamma = \frac{1-\beta^2}{v\beta} = -\frac{\beta v}{c^2}$

Substituting in Equation 3:

$$3: \quad \alpha^2 = -\frac{c^2\beta\gamma}{v} = \left(-\frac{c^2\beta}{v}\right)\left(-\frac{\beta v}{c^2}\right) = \beta^2$$

These expressions for α , β and γ are then substituted into the original equations

$$x' = \alpha(x - vt) = \beta(x - vt)$$

$$t' = \gamma x + \beta t = -\frac{\beta v}{c^2}x + \beta t = \beta\left[t - \left(\frac{vx}{c^2}\right)\right]$$

$$\beta = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} =$$

We have the Lorentz transforms (from Bergmann):

$$\boldsymbol{x' = \frac{(x - vt)}{\sqrt{1-\frac{v^2}{c^2}}}}$$

$$\boldsymbol{t' = \frac{t - \left(\frac{vx}{c^2}\right)}{\sqrt{1-\frac{v^2}{c^2}}}}$$

Important Note:

Bergmann's parameters γ and β are reversed from that of conventional notation in my experience, and in the following analyses the notation will be reversed as well.

The result is the Lorentz transform:

$$x' = (x - vt)\gamma$$
$$z' = z \ \ (=0)$$
$$t' = (t - \frac{vx}{c^2})\gamma$$
$$with\ \gamma = \frac{1}{\sqrt{1-\beta^2}}, \beta = \frac{v}{c}$$

(where t has replaced y in a spatial conception of the model)

The transform in (x,t) is:

$$x' = (x - vt)\gamma$$
$$t' = (t - \frac{vx}{c^2})\gamma$$
$$with\ \gamma = \frac{1}{\sqrt{1-\beta^2}}, \beta = \frac{v}{c}$$

The "Time Dilation" Equation

Note that for x = ct,

$$t' = (t - \frac{vx}{c^2})\gamma$$
$$x' = ct' = (ct - \frac{vx}{c})\gamma = (ct - vt)\gamma = (x - vt)\gamma$$

That is, from a purely algebraic perspective, under the condition x=ct, the time and space transforms are identical. This is because v has not been specified for both positive and negative values ("parity"), which means we must also consider the relation $x' = (x + vt)\gamma$

Adding these two together gives: $2x' = 2x\gamma$ so that $x' = x\gamma$

For x = ct and x'=ct', this gives the so-called "time dilation" equation (notice that x has been explicitly eliminated by the relations t = x/c and t'=x'/c:

$$t' = \frac{t}{\sqrt{1 - \frac{v^2}{c^2}}} = t\gamma$$

“Time” can then be characterized as a then a scaling factor that describes a velocity v in relation to a velocity c for a given length or radius that relates v to c, with all parameters independent of whether a Galilean or a Radial coordinate system is selected.

I indicate this by using capital letters for these parameters, so that:

$$T' / T = \frac{1}{\sqrt{1 - \frac{V^2}{C^2}}}$$

Time is then said to be covariant with velocity for either coordinate system, since it transforms in direct proportion to velocity.

Part II - The Covariant formulation of STR

The relativistic Equations of Special Theory of Relativity can be derived independently of a coordinate system, where C and V are characterized as mass creation rates and T and T' are considered as creation times, with a change in the total mass of the (single) system from M_0=CT to M'=CT' is characterized by a perturbation VT' where V is defined in terms of C by the relation $\beta = \frac{V}{C}$.

We assume that C is derived from Coulomb's and Ampere's laws by Maxell's equations, so that

$$C^2 = \frac{1}{\sqrt{\varepsilon_0 \mu_0}}$$

If perturbation V is independent of C, so that the system mass before and after the perturbation can be characterized by

$$(CT')^2 = (VT')^2 + (CT)^2 \text{ or}$$

$$(M')^2 = (VT')^2 + (M_0)^2 = (M_V)^2 + (M_0)^2$$

(M_V is the mass added by during the perturbation T' at the creation rate V)

$$(M')^2 \mathrm{C}^4 = (VT')^2 \mathrm{C}^4 + (M_0)^2 \mathrm{C}^4$$

$(VT')^2 \mathrm{C}^4 = (VCT')^2 \mathrm{C}^2 = (M'V)^2 C^2 = P^2 C^2$, where $P = M'V$ is the relativistic momentum.

Then the relativistic energy equation is

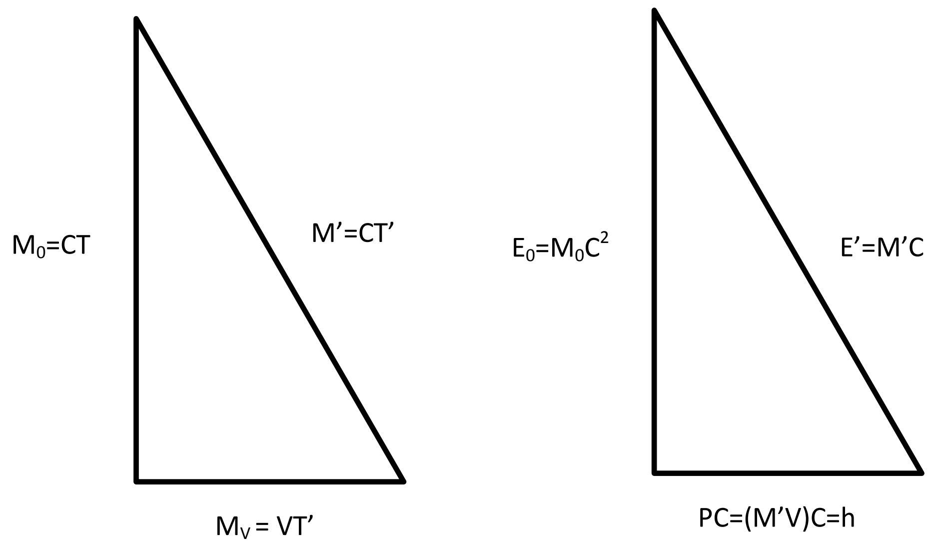
$$(M'C^2)^2 = P^2 \mathrm{C}^2 + (M_0 \mathrm{C}^2)^2$$

$$E'^2 = P^2 C^2 + E_0{}^2$$

The ratio of mass creation times is given by;

$$\frac{T'}{T_0} = \frac{1}{\sqrt{1 - \frac{V^2}{C^2}}}, \quad that\, is\ T' = \frac{T_0}{\sqrt{1 - \frac{V^2}{C^2}}}$$

The following diagrams show these relationships:



I have included h as a precursor to relativistic quantum mechanics. Here the interpretation is that if there is a change of state (from E' to E_0) in a distant source emitting an photon of energy h that Is detected locally, and there is a change in the sensor of the same amount, then nothing has interacted with it along its path. However, if there is a change due to interaction, the observed energy of the photon will decrease, which is observed as a change in h, and is responsible for the red shift.

Note that Doppler (first or second order) is not an issue here, since the coherence length of the photon is tiny compared to the length of the journey.

Part III Co-Variance and Contra-Variance

We have seen that the Lorentz transform can be interpreted as a "time dilation" equation in a space-time coordinate system (x,t) with the assumption of the constant velocity of light c if both positive and negative velocities are included in the analysis (which satisfies the condition of isotropy of both v and c for physical laws), where all other velocities are determined by the relation v/c.

We have also seen that the same equation can be derived independently of a coordinate system in the Energy-Momentum system (E,K) for a single particle (system) with an initial M_0 perturbed by an additional mass M_V to a final mass M', with the transition described by scaling factors (T,T')

The two domains (x,t) and (E,K) are independent of each other; the Lorentz transformation was accomplished without reference to mass, and the Mass-Energy relationships were derived without reference to a coordinate system.

If we set the ratios v/c = V/C with c=C and v = V we can then relate the (E,K) domain to the (x,t) domain by breaking v and c into their space-time components.

The ratio of velocities in Galilean coordinates is: $\frac{v}{c} = \frac{\left(\frac{\Delta x_v}{\Delta t_v}\right)}{\left(\frac{\Delta x_c}{\Delta t_c}\right)} = \frac{\Delta x_v}{\Delta x_c}\frac{\Delta t_c}{\Delta t_v}$, where Δ indicates that discrete lengths and time intervals are defined.

In the relativistic (E,K) domain, the mass creation rates V and C are related by the scaling factors ΔT and ΔT' by:

$$V = C\sqrt{1-\left(\frac{\Delta T_0}{\Delta T'}\right)^2} \;,\; \Delta T' \geq \Delta T_0$$

$$\frac{v}{c} = \frac{\Delta x_v}{\Delta x_c}\frac{\Delta t_c}{\Delta t_v} = \frac{V}{C} = \sqrt{1-\left(\frac{\Delta T_0}{\Delta T'}\right)^2} = \sqrt{1-\left(\frac{\Delta C T_0}{\Delta C T'}\right)^2} = \sqrt{1-\left(\frac{\Delta M_0}{\Delta M'}\right)^2}$$

For a change in v/c, the relation c=C can be preserved for variations in energy provided that one of the parameters C and c have a coordinate in common. That means there are two possible solutions; that in which a common spatial interval is assumed, and that in which a common interval of time is assumed.

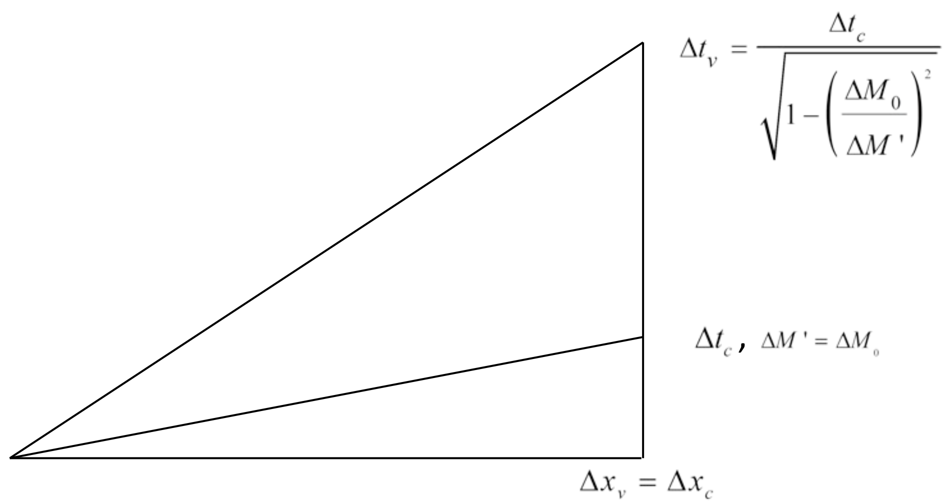
Case 1: $\Delta x_v = \Delta x_c$ (**co-variant** transformation)

In this case, we have $\frac{v}{c} = \frac{\Delta t_c}{\Delta t_v} = \sqrt{1 - \left(\frac{\Delta M_0}{\Delta M'}\right)^2}$, so that

$$\Delta t_v = \frac{\Delta t_c}{\sqrt{1 - \left(\frac{\Delta M_0}{\Delta M'}\right)^2}}$$

For M' = M_0, we have that $\Delta t_v = \Delta t_c$ As M' increases along with V, Δt_v also increases. Since M' in (E,K) is directly proportional to Δt_v in the coordinate system, we say that the transformation is **co-variant**.

(We have ignored parity, since we assume equal values set at the midpoint by squaring the parameters in the solution).



The interpretation of covariance is that an inertial object (for v < c) will take longer to traverse a given distance than a photon, with an increased total mass determined by the ratio M_0/M' .

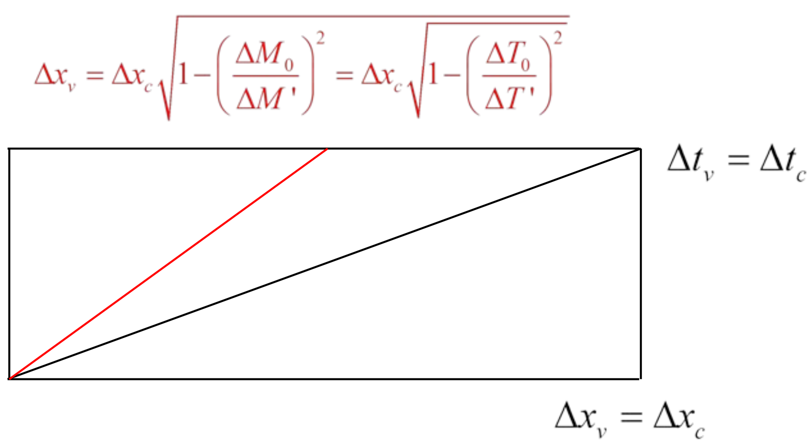
Here the factor $\frac{1}{\sqrt{1 - \left(\frac{\Delta M_0}{\Delta M'}\right)^2}}$ is interpreted as an increased density of the system as a whole for V/C.

Case 2: $t_v = t_c$ (**contra-variant** transformation)

In this case, we have $\frac{v}{c} = \frac{\Delta x_v}{\Delta x_c} = \frac{V}{C} = \sqrt{1-\left(\frac{\Delta M_0}{\Delta M'}\right)^2}$, so that

$$\Delta x_v = \Delta x_c \sqrt{1-\left(\frac{\Delta M_0}{\Delta M'}\right)^2}$$

For $\Delta M' = \Delta M_0$, we have $\Delta x_v = 0$, which means that no additional mass has been added to the rest mass. As $\Delta M'$ increases, Δx_v decreases, meaning that an inertial object for v < c will not travel as far in a given time as a photon. Again, the factor $\sqrt{1-\left(\frac{\Delta M_0}{\Delta M'}\right)^2}$ can be interpreted as a density, which increases as increasing mass (relative to that of a photon); however, since the result is inversely proportional to V/C, the transformation is **contra-variant** with respect to the coordinate system.



Topics:

First Quantization (non-Relativistic)

Second Quantization (Relativistic)

Precursor to GTR

To be developed

Cartesian vs. Radial coordinates

GTR begins where STR leaves off; with $(C\,T)^2 = 1$, which is the final state at CT=1 after all the mass has been accounted for in both its initial state and final state. His goal then is to present a transformation that preserves total energy, but investigates geometries other than circles or spheres. H does this by attempting to provide a tensor density variation around the circumference of the unit circle (sphere).

At this point in the analysis, it is evident that the context is that of an area; either that of a square where each element on the left and right side are interpreted as its sides, or as a circle, in which each side is interpreted as a radius, and:

$$Area_{Square} = s^2 = \alpha^2 (x - vt)^2 = c^2 (\gamma x - \beta t)^2$$

$$Area_{Circle} = \pi r^2 = \pi \alpha^2 (x - vt)^2 = \pi [c^2 (\gamma x - \beta t)^2]$$

The two are, of course, related by π , since

$$r^2 = s^2 = \alpha^2 (x - vt)^2 = c^2 (\gamma x - \beta t)^2$$

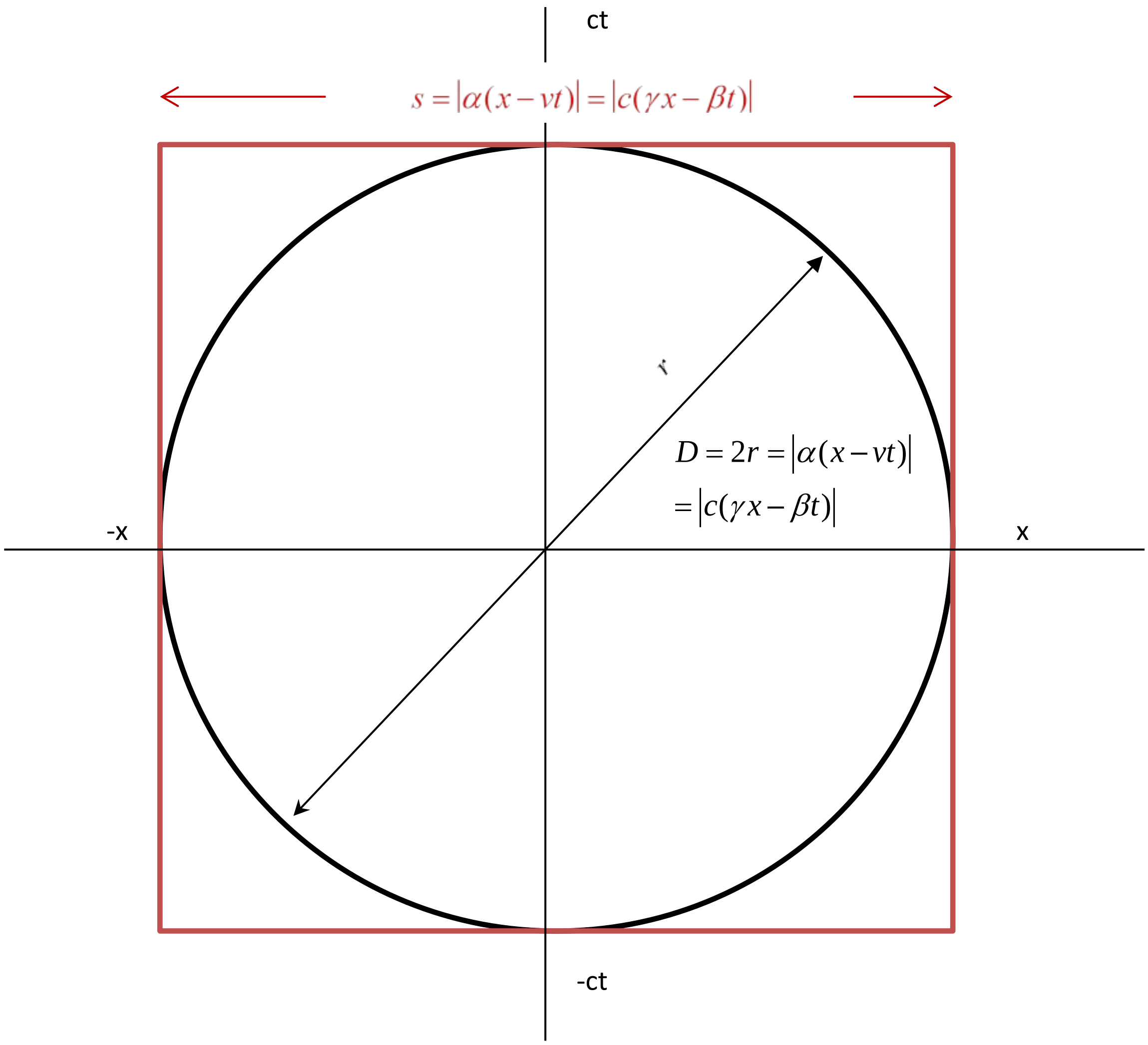
$$\frac{Area_{Square}}{Area_{Circle}} = \frac{\pi s^2}{r^2} = \pi \qquad \text{for } (s = r)$$

Making these areas equal requires morphing the perimeter of the square to that of the circle by adjusting the parameters $\alpha, \beta, and\ \gamma$ so that one can keep the area constant for both diagrams.

This, of course, involves changing the value of π ,which is the relationship of the circumference to the diameter (this procedure is called “squaring the circle”):

$$C_f = 2\pi r = \pi D$$

$$\pi = \frac{C_f}{D} = \frac{C_f}{2r}$$



The “square” analysis is that used by Maxwell in his derivation of the speed of light without directly addressing the force assumptions underlying the permittivity and permeability constants in Coulomb’s and Ampere’s laws; by ignoring this distinction, Einstein “imagineers” (via “thought experiments”) that the Maxwell’s solution (in the integral form of the equations in which dot and cross products are used to describe the “speed of light” in squares, and then assert the relations are true “for any shape, volume, etc…” via Green’s and Stokes’ theorems.

In particular, Maxwell's equations imply a constant speed of light, but say nothing about its energy.

There is much more to say about this, but for now:

Note that for a point on the circumference of the unit circle:

1. The radius, circumference, and area are co-variant:

$M_0(R) = R = CT$ =1

$E_0 = M_0(R)C^2 = C^2$,

2. The density as a function of R is covariant $\rho(R) = \frac{1}{R}$

The dot product of the radius and density = 0 $\left(R \cdot \frac{1}{R} = 0\right)$,

but the cross product of the radius and density = 1 $\left(R \times \frac{1}{R} = 1\right)$, with the result being orthogonal to the plane of the circle (and thus orthogonal). Similarly, the area is given by $\left(R \times R = R^2 = 1\right)$

This is true for any value of R (i.e., M_0(R)=CT=1) That is, if R is interpreted in space time, either Mass is 0, or the relationship is true for any value of R for a given value of $c = \frac{x_c}{t_c}$.

If the energy of the E field is given by $E_{energy} = \varepsilon_0 E$ and the energy of the B field is given by $B_{energy} = \upsilon_0 B$, then if $(EB)_{energy} = (\varepsilon_0 E) \oplus (\upsilon_0 B)$ where $\oplus$ is some operator. If E and B are orthogonal, then

1. if $\oplus$ is a dot product, and E and B are orthogonal, then $(EB)_{energy} = (\varepsilon_0 E) \oplus (\upsilon_0 B) = \varepsilon_0 \upsilon_0 (E \cdot B) = EB \sin\theta = 0,\ for\ \ \theta = 0$, so $\varepsilon_0 \upsilon_0$ is arbitrary, and can have any value, including 0.
2. if $\oplus$ is a dot product, and E and B are orthogonal, then $(EB)_{energy} = (\varepsilon_0 E) \oplus (\upsilon_0 B) = \varepsilon_0 \upsilon_0 (E \times B) = \varepsilon_0 \upsilon_0 EB \cos\theta = \varepsilon_0 \upsilon_0 EB = \varepsilon_0 \upsilon_0 = c^2,\ \theta = 0$, from Ampere's and Coulomb's laws. Of E and B form a unit basis, then $(EB)_{energy} = \varepsilon_0 \upsilon_0 = c^2$, which can be multiplied by any radius r_0 = M_0 so that $(EB)_{energy}(M_0) = M_0 c^2$

In the second case, M_0c^2 (the "rest mass") is orthogonal to the "field space" (E,B) This result suggests that "dot products" of E and B "destroy" mass, and "cross products" of E and B "create mass"; analogous to creation and destruction operators in second quantization.

Note that E and B are orthogonal relationships on any of the coordinate axes, so the result is independent of position, length, and velocity in space-time.

The analysis of the Lorentz transform in Cartesian coordinates violates the assumption of the homogeneity of space and time, since the distance from the origin at (0,0) to the perimeter of the square is only equal to that of the circle at the horizontal and vertical axes of the diagram above (t = t' = 0 or x' = x = 0).

This means that v is only independent of c at these points. However, if c is a constant, then v must be independent of c for all values of v, which can only be true if the relation $x' = \alpha(x - vt), y' = y, z' = z$ is taken to be that of a radius, in which case v is orthogonal to c as a tangent to the circumference.

$$r' = \alpha(r - vt),$$
$$\theta' = \theta = 2\pi,$$
$$\varphi' = \varphi = 2\pi$$
$$t' = \gamma r - \beta t$$

and

$$r = ct \Leftrightarrow r' = ct'$$

The same analysis fan be performed, with the result that

$$r' = ct' = \frac{(r - vt)}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Also, note that v is no longer defined by the relation $v = x/t$ but rather by the relation:

$$v = c\sqrt{1 - \frac{t^2}{t'^2}} = c\sqrt{1 - \frac{(ct)^2}{(ct')^2}} = c\sqrt{1 - \frac{r^2}{r'^2}},\ t' \geq t$$

If v= 0 the denominator (or equivalently, positive and negative radii are included), then

$r' = \frac{r}{\sqrt{1 - \frac{v^2}{c^2}}}$ and r is co-variant with velocity, as are mass and energy. However the density at a point on the circumference is contra-variant with r.

Bottom Line: if you accept Einstein's "trains and rockets" model (space-time definition of velocity), the geometric object is a light cube, not a light sphere. This is a thought experiment intended to reconcile Maxwell's equations with Newtonian mechanics as modeled in Cartesian coordinates.

Outer Product (Cross Product)

In co-variant coordinate space, the "cross-product" $\vec{x} \times \vec{y} = \vec{z}\ for\ |\vec{x}| = |\vec{y}| = |\vec{z}| = |\vec{ct}|$ is the model of a light cube for a fixed value of t = T. In this sense, the outer product creates an additional dimension $(x, y) -> (x, y, z)$ It is a "creation operator".

Inner Product (Dot Product)

In co-variant coordinate space, the "dot product" $\vec{x} \bullet \vec{y} = \vec{x}\ for\ |\vec{x}| = |\vec{ct}| = |\vec{y}|$ The "inner" product "destroys" a dimension.

In a unit matrix, an outer product creates an additional element on the diagonal, an inner product reduces the diagonal by an element.

If the vectors are not unity, the total matrix is multiplied by a quantity describing the change. The unit matrix can always be recovered by an inverse transform. The "trace" of the (diagonlized) matrix gives the total "volume" by squaring each element and adding them. These squares correspond to individual volume elements; in Cartesian coordinates they are cubes, in Spherical coordinates they are spheres (with total angles of 2π)

In General Relativity, the unity matrix is that in which all the elements have been “contracted” to a coordinate point; if Cartesian coordinates are used, the single point has a single “dimension” of length, which actually is a space-time “density” indicating one particle; otherwise there is no coordinate system.

(If there is only three dimensions of space, at any given time, there is only one coordinate system if everything has contracted, and consists of a unity vector multiplied by a constant.

For the solution of Maxwell’s equations in space and time, the speed of light is derived in Cartesian coordinates (with no energy assigned to the E and B fields, charge, or current).

If space-time is isotropic and homogeneous, and c is a constant then the coordinate system must be concentrated at the center of a radial or spherical coordinate system for a measurement by a point particle that travels in time (the journey around the circumference of a circle is different from the periphery square, unless infinite velocity is assumed at the horizontal lengths – which is made by Maxwell in his derivation) for circular coordinates, v is a constant and orthogonal to the radius (tangent to the circumferential world-line).

Co-Variant Formulation

For second quantization, a new particle is created every time VT’=CT as a cross product (since the masses are independent of each other)

$$M_0 = \rho CT,\ \rho = 1$$
$$M_0(1) \times M_0(2) \Rightarrow M_0(3)$$

That is, the cross product of two equal masses generates a third independent mass; the coordinate system is not involved. Similarly, a dot product removes the mass.

If energy is conserved the total energy squared is the sum of the squared values of each of these masses. If all the “V’s” have been removed by contracting tensor elements to their unity value, then the conserved system is described in the same way as above (but can only be covariant in radial coordinates).

(added 10/16/2013)

Coulomb's Law

$$F_E = \frac{1}{\varepsilon_0}\left(\frac{1}{4\pi}\right)\frac{q_1 q_2}{r^2}$$

Note that this relation only depends on space as characterized by r.

Ampere's Law

$F_B = \upsilon_0\left(\frac{2}{4\pi}\right)\frac{I_1 I_2}{r}$, where F_B isthe force per unit length of the wires.

If the "wire" length is defined as r (required for Isotropy)

$$F_B r = \upsilon_0\left(\frac{2}{4\pi}\right)\frac{I_1 I_2}{r}$$
$$F_B(r) = \upsilon_0\left(\frac{2}{4\pi}\right)\frac{I_1 I_2}{r^2}$$

Let

$$q_1 = q_2 = q$$
$$I_1 = I_2 = I$$

Setting the forces equal

$$\frac{1}{\varepsilon_0}\left(\frac{1}{4\pi}\right)\frac{q^2}{r^2} = \upsilon_0\left(\frac{2}{4\pi}\right)\frac{I^2}{r^2}$$

$$\frac{1}{\varepsilon_0}\frac{q^2}{r^2} = 2\upsilon_0\frac{I^2}{r^2}$$

However, both directions for the forces between the static charges in the quantity on the left must be included for the equality to hold, which eliminates the factor of 2.

$$\frac{2}{\varepsilon_0}\frac{q^2}{r^2} = \upsilon_0\frac{2I^2}{r^2}$$
$$\frac{1}{\varepsilon_0}\frac{q^2}{r^2} = \upsilon_0\frac{I^2}{r^2}$$

We now interpret charge as defined in the (E,K) domain, where the rate of charge creation is C and the Time of charge creation is T, so the total charge over the length from T= 0 to T is

$$q = C_q T$$

$$I = \frac{q}{T} = \frac{C_q T}{T} = C_q$$

$$\frac{1}{\varepsilon_0}\frac{q^2}{r^2} = \upsilon_0 \frac{I^2}{r^2} = \frac{C_q{}^2}{r^2}$$

$$\frac{1}{\varepsilon_0} = \upsilon_0 C_q{}^2$$

$$C_q{}^2 = \frac{1}{\varepsilon_0 \upsilon_0}$$

Vector Derivation

The vector relations are given by the "dot" (inner) and the "cross" (outer) product of the vectors.

The Inner Product ("dot" product") •

Note that the dot product reduces the dimension of the vector space $(\vec{E}, \vec{B})$ by 1, so the result is a scalar $|\vec{0}|$. (Note: if this space is a tensor with greater than two dimensions, the dot product multiplies all the values of the reduced tensor, since the latter is multilinear) If any dot product between two such vectors is taken at $\theta = \frac{\pi}{2}$, the resultant is $|\vec{0}|$).

$$\vec{E} \bullet \vec{B} = |\vec{E}||\vec{B}|\sin\theta$$

$$\vec{E} \bullet \vec{B} \Longrightarrow \vec{T}, \vec{T} = \frac{1}{(\upsilon_0 \varepsilon_0)}|\vec{E}||\vec{B}|\sin\theta, \ \theta = \frac{\pi}{2}$$

$$\begin{vmatrix} E & 0 \\ 0 & B \end{vmatrix} \xrightarrow[\Rightarrow]{\bullet} |\vec{T}| = 0, \ \theta = \frac{\pi}{2}$$

The Outer Product ("Cross" product) $\times$

Note that the cross product increases the dimension of the vector space (by the "right hand rule", by 1, so the result adds a dimension to the vector space $\left|C\vec{T}\right|$, so there are now three dimensions $(\vec{E},\vec{B},C\vec{T})$ with $\left|\vec{C}\right|$ orthogonal to the plane $(\vec{E},\vec{B})$

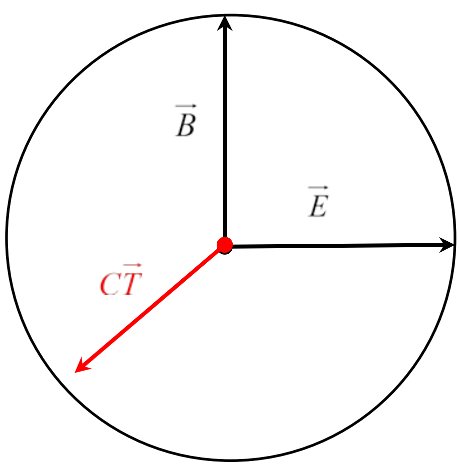
Defining $\vec{C}_B = \frac{1}{\upsilon_0}\vec{B}$ and $\vec{C}_E = \frac{1}{\varepsilon_0}\vec{E}$

We can take the outer product

$$C\vec{T}_E \times C\vec{T}_B = \frac{1}{\varepsilon_0 \upsilon_0}\vec{E}\times\vec{B}$$

If $\vec{E},\vec{B},\vec{T}_E$ and $\vec{T}_B$ are defined as unit vectors $\vec{1}$, then for $\vec{C}_E = \vec{C}_B = \vec{C}$,

$$C^2 = \frac{1}{\varepsilon_0 \upsilon_0}$$



Tensor Derivation

Recall that a tensor function is multilinear (an operation on any element applies to the whole matrix).

$$\begin{vmatrix} E & 0 \\ 0 & B \end{vmatrix} = \begin{vmatrix} \varepsilon_0 C_E & 0 \\ 0 & \upsilon_0 C_B \end{vmatrix} \overset{\times}{\Rightarrow} \begin{vmatrix} \varepsilon_0 C_E & 0 & 0 \\ 0 & \upsilon_0 C_B & 0 \\ 0 & 0 & 1 \end{vmatrix} = \begin{vmatrix} \varepsilon_0 C_E & 0 & 0 \\ 0 & \upsilon_0 C_B & 0 \\ 0 & 0 & 1 \end{vmatrix} = \varepsilon_0 \upsilon_0 \begin{vmatrix} C & 0 & 0 \\ 0 & C & 0 \\ 0 & 0 & \frac{1}{\varepsilon_0 \upsilon_0} \end{vmatrix} = C^2 \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \frac{1}{\varepsilon_0 \upsilon_0} \end{vmatrix}$$

We can contract (inner product) the top two dimensions and multiply by T the result is

$$\varepsilon_0 \upsilon_0 C^2 \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} = \begin{vmatrix} \varepsilon_0 \upsilon_0 C^2 & 0 \\ 0 & 1 \end{vmatrix}.$$

We can multiply the matrix by T^2 so that:

$$T^2 \begin{vmatrix} \varepsilon_0 \upsilon_0 C^2 & 0 \\ 0 & 1 \end{vmatrix} = \begin{vmatrix} \varepsilon_0 \upsilon_0 C^2 T^2 & 0 \\ 0 & 1 \end{vmatrix}$$

If matrix on the right is to form a basis of the vector space, we must have:

$$(CT)^2 = \frac{1}{(\upsilon_0 \varepsilon_0)} = 1$$

For a given $T = T_0 = 1$, $C^2 = \frac{1}{(\upsilon_0 \varepsilon_0)}$

The Lorentz Force Law

The Lorentz Force Law is defined as:

$$F = \frac{Q}{M}(E + v \times B)$$

If Q and M are related relativistically in the (x,t)and (E,K) domain so that:

$$c = C, \quad \frac{v}{c} = \frac{V}{C}$$

Then a relativistic transform ($V \perp C$) in (E, K)) gives

$$Q' = \frac{Q}{\sqrt{1 - \frac{V^2}{C^2}}} = Q\Gamma, \qquad M' = \frac{M}{\sqrt{1 - \frac{V^2}{C^2}}} = M\Gamma$$

where Γ is interpreted as a change in “density” in each case.

Then we have;

$$F = \frac{Q}{M}(E + \left(\frac{v}{c}\right)c \times B) = \frac{Q\Gamma}{M\Gamma}[E + (\beta c \times B)] =$$
$$\frac{Q'}{M'}[E + (\frac{V}{C})C \times B)]$$

That is, the Lorentz force is invariant under a Lorentz transformation (β, Γ) , and only depends on E and B.

Gauge Transformation

Ref: http://en.wikipedia.org/wiki/Gauge_fixing (Gauge Fixing)

$$\vec{E} = -\vec{\nabla}\varphi - \frac{\partial \vec{A}}{\partial t}, \quad \vec{B} = \vec{\nabla} \times \vec{A}$$

Re-define:

$$\vec{E} \to \overrightarrow{E(t)}, \ \vec{A} \to \vec{A}\phi(t),$$
$$\varphi \to \varphi(x,t) = \varphi_x(x)\varphi_t(t)$$
$$\vec{E}(t) = -\vec{\nabla}\left[\varphi_x(x)\varphi_t(t)\right] - \frac{\partial \vec{A}\phi(t)}{\partial t}, \quad \vec{B} = \vec{\nabla} \times \vec{A}(t)$$
$$\vec{E}(t) = -\left[\vec{\nabla}\varphi_x(x)\right]\varphi_t(t) - \vec{A}\frac{\partial \phi(t)}{\partial t}$$

If the transformation

$$\vec{A}\phi(t) \to \left[\vec{A} + \nabla_x \psi(x)\right]\phi(t),$$

$$\vec{E}(t) = -\left[\vec{\nabla}\varphi_x(x)\right]\varphi_t(t) - \left[\vec{A} + \nabla_x \psi(x)\right]\frac{\partial \phi(t)}{\partial t}$$
$$\vec{E}(t) = -\left[\vec{\nabla}\varphi_x(x)\right]\varphi_t(t) - \left[\vec{A}\frac{\partial \phi(t)}{\partial t} + \nabla \psi_x(x)\frac{\partial \phi(t)}{\partial t}\right]$$

If we make another change:

$$\left[\vec{\nabla}\varphi_x(x)\right]\varphi_t(t) \to \left[\vec{\nabla}\varphi_x(x)\right]\varphi_t(t) - \nabla \psi_x(x)\frac{\partial \phi(t)}{\partial t}$$

$$\vec{E}(t) = \left[\vec{\nabla}\varphi_x(x)\right]\varphi_t(t) - \nabla \psi_x(x)\frac{\partial \phi(t)}{\partial t} - \left[\vec{A}\frac{\partial \phi(t)}{\partial t} + \nabla \psi_x(x)\frac{\partial \phi(t)}{\partial t}\right]$$
$$\vec{E}(t) = \left[\vec{\nabla}\varphi_x(x)\right]\varphi_t(t) - \vec{A}\frac{\partial \phi(t)}{\partial t} + \left[-\nabla \psi_x(x)\frac{\partial \phi(t)}{\partial t} + \nabla \psi_x(x)\frac{\partial \phi(t)}{\partial t}\right]$$
$$\vec{E}(t) = \vec{E}(t) = \left[\vec{\nabla}\varphi_x(x)\right]\varphi_t(t) - \vec{A}\frac{\partial \phi(t)}{\partial t}$$

This implies that the functions of space dimension and the time function are independent (functionally orthogonal).

Example:

$$\varphi_x(x) = \alpha x$$
$$\varphi_t(t) = \beta t$$
$$\varphi = \varphi_x(x)\varphi_t(t) = (\alpha x)(\beta t)$$

$$\vec{E} \to \overrightarrow{E(t)},\ \vec{A} \to \vec{A}t,$$
$$\varphi \to (\alpha x)(\beta t)$$
$$\vec{E}(t) = -\vec{\nabla}_x \varphi - \frac{\partial \vec{A}t}{\partial t}, \quad \vec{B} = \vec{\nabla} \times \vec{A}$$
$$\vec{E}(t) = -\vec{\nabla}_x (\alpha x)(\beta t) - \frac{\partial \vec{A}t}{\partial t}$$
$$\vec{E}(t) = -\alpha\beta t \vec{\nabla}_x (x) - \frac{\partial \vec{A}t}{\partial t}$$
$$Since\ \ \vec{\nabla}_x [\alpha x] = \alpha,$$
$$\vec{E}(t) = \alpha\beta t - \vec{A}$$

Let

$$\psi(x) = \alpha' x$$

$$\nabla_x x' = \alpha'$$

$$\vec{A} \to \left[\overrightarrow{At} + \alpha' t\right]$$

$$\vec{E}(t) = -\alpha\beta t - \frac{\partial}{\partial t}\left[\vec{A}t + \alpha' t\right]$$
$$\vec{E}(t) = -\alpha\beta t - \left[\vec{A} + \alpha'\right]$$

Let $t \to t - \alpha'$

$$\vec{E}(t) = -\alpha\beta t - \alpha' - \left[\vec{A} + \alpha'\right]$$
$$\vec{E}(t) = -\alpha\beta t - \vec{A} = \vec{E}$$

Example 1

For $\alpha = \beta = 1$

$$\vec{E}(t) = -t - \vec{A} = \vec{E}$$

Example 2

$\alpha = 1,\ \beta = c,\ \ A = CT,\ T = t$

$\alpha = 1$ implies $x_c = 1$

$$\vec{E}(t) = -ct - \alpha' - \left[\vec{C} + \alpha'\right]$$
$$\vec{E}(t) = -ct - \overrightarrow{CT} = \vec{E}$$

For $c = C$

$$\vec{E}(t) = -c\left(t - \vec{T}\right) = \vec{E}$$

For $\left|\vec{T}\right| = t$, $\left|\vec{E}\right| = 0$

That is, if there is no difference in potential, then $\left|\vec{E}\right| = 0$

(For x_c a length, $\left|\vec{E}\right| = 0$ if the vector time is equal to the scalar time.

Relativity

$x_c = 1$ implies covariant interpretation $x_c = x_v$

$$|T|\Gamma = t\gamma$$
$$|T|' = t' \Leftrightarrow \Gamma = \gamma \Leftrightarrow \frac{v}{c} = \frac{V}{C}$$

Again, $\left|\vec{E}\right| = 0$

Mass Interpretation

Interpret $m_0 = ct = M_0 = CT$ equal to rest mass in both (x,t) and (E,K)

$$\vec{E}(t) = -ct - \overrightarrow{CT} = \vec{E}$$
$$\vec{E}(t) = -m_0 - \overrightarrow{M_0} = \vec{E}$$

(Covariant interpretation)

$m_0 = -\overrightarrow{M_0}$ (add a mass in (x,t)

$x'_c = x_c + m_0$,

subtract mass in (E,K)

$X'_c = X_c - M_0$,

$x'_c = X'_c$, restoring covariance.

(Coordinate interpretation; GTR)

$m_0 = \overrightarrow{M_0} = 0$ (There is no mass, "nothing there", mass as curvature is imaginary function of imaginary coordinate system.

The Bottom Line

Note that

$$m_0 C^2 = \frac{M_0}{(\upsilon_0 \varepsilon_0)}$$
$$C^2 = \frac{1}{(\upsilon_0 \varepsilon_0)},\ m_0 = 1$$

This is Einstein's famous formula for a unit "rest" mass directly from the Lorentz relationships…..

Note: More than one spatial dimension breaks gauge symmetry, since the connection between the vector field X=CT=M_0 and the scalar field x=ct=m_0 is broken. The subject then becomes a many-body problem in both QED and General Relativity, and gets very complex very fast.