Special Theory of Relativity

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Background

Previous to Einstein, physical models rested on the foundations of a coordinate system, which defined a "distance" (or "metric") relative to an origin. The underlying assumption for the coordinate system were that its dimensions should be independent; for example for one dimension each of space and time, the system would be characterized by the orthogonal coordinates (x,t) which defines a system based on an origin at (0,0). Any number of dimensions could be characterized in this way; for example, three dimensions of space (x,y,z) with origin at (0,0,0).

Since the dimensions are independent, they can be considered vectors, with the magnitude equal to the square root of the sum of squares, typified by the relation between radial and Cartesian coordinates where $r=\sqrt{x^2+y^2+z^2}$, where the other dimensions in radial coordinates are angle; in this case assumed to be 0 (or a multiple of 2π $(r,\theta,\psi)=(r,0,0)=(r,n2\pi,n2\pi)$

Note the for the "distance" for the vector $\left|\overrightarrow{(x,t)}\right|=\sqrt{x^2+t^2}$ doesn't make sense, since x and t represent different conceptual entities (space and time), and so are dimensionally inconsistent.

Cartesian Velocity

In order to describe motion, velocity is introduced as a ratio between the distance traveled and the time taken to travel the distance: $x_v=vt_v$; in this system, the speed of light is simply another velocity, defined by $x_c=ct_c$.

Newtonian Mass

For Newton, mass point masses are defined at positions $m=m(x,y,z)$ with the usual relations between energy and momentum; for two particles traveling in equal and opposite directions:

$p=mv+m(-v)=0$ and $T=E_{Kinetic}=mv^2$ (Not that the force is described by $F=ma+m(-a)$) which implies the force is zero for equal and opposite accelerations between the two particles).

Note that the Kinetic energy can have a value even for no momentum. To this was added a potential energy that depends only on the coordinates: $V=E_{potential}=E(x,y,z)$, which are related by the LaGrangian action $L=T-V$. For example, In the absence of kinetic energy, the potential energy at the origin is then represented by $V_0=E(0,0,0)$

Maxwell's Equations

Maxwell's equations draw on the legacy of Coulomb, Gauss, Ampere, and Faraday, which characterizes forces between charges valid for all distances in terms of coupling constants: the permittivity ε_0 and the permeability υ_0 of free space. Maxwell then relates these to the speed of light by combining these laws together with the displacement current to derive the relation $c^2 = \frac{1}{\varepsilon_0 \upsilon_0} \Rightarrow (\varepsilon_0 \upsilon_0) c^2 = 1$ From his derivation from symmetries in the geometries introduced by Green's and Stokes theorems, he interprets c as being a velocity in space-time

Note that this relation can be derived directly from Coulomb's and Ampere's laws (ignoring E and B fields), which suggests that $\varepsilon_0 \upsilon_0$ may be related to force between charges (and thus mass via Newton's laws).

See "The Mass of Light:" for the derivation.

The MM experiment

From these mechanical and electromagnetic models, it wa s assumed that space-time was composed of an underlying medium for propagating the waves generated by the displacement current (across the plates of the capacitor) and so would depend on the direction of Cartesian motion in the appropriate direction x_c that could be detected by interference fringes on a rotating arm structure (the Michaelson- Morley apparatus).

Lorentz Transform

When no such effect was found even through Herculean efforts, Lorentz derived a set of equations that explained the result by adjusting conceptually the arms of the apparatus:

$$x' = \left(\frac{x - vt}{\sqrt{1 - \frac{v^2}{c^2}}} \right) \text{, and } t' = \left(\frac{t - \left(\frac{vx}{c^2} \right)}{\sqrt{1 - \frac{v^2}{c^2}}} \right)$$

(the arms are characterized by a single dimension each of space and time. Note that the equations are equivalent for $x = x_c, t = t_c$ which equates the system when rotated through an angle of $\frac{\pi}{2}$.

However, this "stretching" of spacetime was not consistent with Maxwell's equations, which depends on geometrical symmetries for his derivation and interpretation of c. Also, the transform had two problems:

1. It was independent of the y and z dimensions – this implies an ambiguity between Cartesian and radial coordinate systems, depending on whether the “space” factor was multiplied by π (i.e., considered a radius (circle, sphere) or the half-width of a square or cube).

   The attempt to resolve the difference resulted in the transform being characterized as a ”boost” and “rotation” in hyperbolic coordinates in the Cartesian model to try to account for a “separation” between two observers (with velocities c and v); i.e., ,attempting to retain the concept of c as a velocity to retain the “space-time” interpretation of Maxwell’s “displacement current”.

2. It also was dependent on the factor $(x-vt)$ which meant x' could have different values for $\pm|v|$ unless $x=0$; that is, at a single point in space (the “origin”) .

[Lorentz Transform]()

Einstein's Contribution

Einstein originally developed his laws from the postulate that physical laws should be independent of the underlying coordinate system, and thus made two postulates:

1. The speed of light c is homogeneous and isotropic throughout space-time.
2. The speed of light is independent of all possible velocities in space-time.

Postulate 2 implies that v and c are now independent, and thus have a vector relationship (c, v) Furthermore, in order that the relation be true for all possible values of v, they must be related by scaling factors in this relationship: (ct, vt') The resultant is then calculated by the Pythagorean theorem relating independent quantities to a final result ct':

$(ct')^2 = (vt')^2 + (ct)^2$ Note that this can be thought of as a relation between either the areas of circles in terms of the component length if the expression is multiplied by π .

Solving this equation for t' gives the so-called "time dilation" equation:

$t' = \dfrac{t}{\sqrt{1-\dfrac{v^2}{c^2}}}$ For clarity, rename $t = t_c,\ t' = t_v$, so that

$t_v = \dfrac{t_c}{\sqrt{1-\dfrac{v^2}{c^2}}}$ and note that the "lengths" are no longer mentioned explicitly in this equation.

Note that this relationship destroys the orthogonal relation between space and time (x, t) ot the Galilean system since "time" is now dependent on velocity. Only in the case $v = 0,\ t_v = t_c$ is orthogonality restored (x_c, t_c) In order to defend this perspective, Einstein invented the concept of "inertial frames", and postulated that all physical laws be consistent in all frames, even if distorted by relative velocities.

However, it is still true that these laws obviously still depend on the underlying coordinate system that defines v and c , even if they are implicit. For Einstein, the solution was gravity, which defines mass as a curvature in a world line in such a coordinate system.

[Addendum to Lorentz Transform]

Global Covariance

The goal of "global covariance" is that physical laws be independent of the underlying coordinate system. An alternative to Einstein's approach is to ignore the concept of coordinates altogether in defining the relation (v,c) by defining the mass domain (C,V) (describe by capital letters) by a "mass creation rate" C and a "mass creation time" T so that an initial condition of "rest mass" is defined by $M_0 = M_C = CT$. A ""perturbation" to this mass can be defined by an independent mass creation rate V and creation time T' so that the domain is characterized by (CT,VT'), so that the final result has the relation:

$(CT')^2 = (VT')^2 + (CT)^2$, which is a relation between mass "lengths":

$(M')^2 = (M_v)^2 + (M_c)^2$, The first equation can be solved for T', which gives the relation:

$T' = \left(\dfrac{T}{\sqrt{1-\dfrac{V^2}{C^2}}} \right)$ This equation can then be multiplied by C, so that:

$CT' = \left(\dfrac{CT}{\sqrt{1-\dfrac{V^2}{C^2}}} \right)$, that is, $M' = \dfrac{M_c}{\sqrt{1-\dfrac{V^2}{C^2}}}$

Then we can define Γ_m to be a density that relates the final mass M' to the initial mass M_c in terms of their mass creation rates V and C, Note that the Mass Creation times are not explicit in the final result.

$\dfrac{M_v}{M_c} = \Gamma_m = \dfrac{1}{\sqrt{1-\left(\dfrac{V}{C}\right)^2}}$, and $M_V = M_C \Gamma_m$

This relation satisfies the requirement of "Global Covariance", since the mass relations are not dependent on the dynamics space-time (x,t) in terms of (v,c), but only on the mass creation rates and times in terms of (CT,VT') .

We can then solve for $\frac{V}{C} = \sqrt{1 - \left(\frac{M_C}{M_V}\right)^2}$

$$M_V = \frac{M_C}{\sqrt{1 - \left(\frac{V}{C}\right)^2}}$$

$$\left(\frac{M_V}{M_C}\right)^2 = \frac{1}{1 - \left(\frac{V}{C}\right)^2}$$

$$\left(\frac{M_C}{M_V}\right)^2 = 1 - \left(\frac{V}{C}\right)^2$$

$$\left(\frac{V}{C}\right)^2 = 1 - \left(\frac{M_C}{M_V}\right)^2$$

$$\frac{V}{C} = \sqrt{1 - \left(\frac{M_C}{M_V}\right)^2}$$

Relation between the domains of Space-Time and Mass

Although the previous result in the mass domain is independent of space-time, it can be related to the orthogonal system (v,c) in (x,t) by requiring that the relation $\frac{v}{c}=\frac{V}{C}$ be preserved in the relationship between the space-time and mass domain. In order to do this, we have to deconstruct $\frac{v}{c}$ into its individual space-time components:

$$\frac{\Delta v}{\Delta c}=\frac{\Delta x_v \Delta t_c}{\Delta t_v \Delta x_c}$$

There are two important conditions that preserve the invariant relation: $x_c = ct_c$, a "Time-Like" transformation which equates the spatial components of the relationship, and a "Space-Like" transformation which equates the time components of the relationship.

"Time-Like" Transformation

($\Delta x_v = \Delta x_c$)

Preserving inertial components between space-time (x,t) and mass (C,V) so that $\frac{v}{c}=\frac{V}{C}$, we have:

$$\left(\frac{\Delta v}{\Delta c}\right)_t = \frac{\Delta t_c}{\Delta t_v} = \frac{V}{C} = \sqrt{1-\left(\frac{M_C}{M_V}\right)^2}_T$$

$$\left(\frac{\Delta t_c}{\Delta t_v}\right)^2 = 1-\left(\frac{M_0}{M_v}\right)^2$$

$$\Delta t_v = \frac{\Delta t_c}{\sqrt{1-\left(\frac{M_0}{M_v}\right)^2}} = \Delta t_c \Gamma_m{}'$$

so that $\Delta t_v = \Delta t_c \Gamma_m$

Covariance

Consider the relation:

$\Delta t_c = \dfrac{\Delta t_v}{\Gamma_m}$, and note that Δt_c is an invariant.

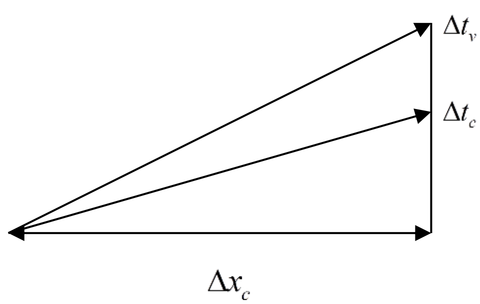
This means that to preserve this invariant, both Δt_v and Γ_m must transform in the same way, so that

$\Delta t_c = \dfrac{r\Delta t_v}{r\Gamma_m}$ for any constant r (or any function of x,y,z,t):

$$\Delta t_c = \frac{f(x,y,z,t)\Delta t_v}{f(x,y,z,t)\Gamma_m}$$

The mass density Γ_m and the time variable Δt_v are said to be "co-variant" with respect to the transformation $t_c = T$ on the space-time and mass domains that preserves the relation $\dfrac{v}{c} = \dfrac{V}{C}$ between velocities and mass creation rates

The space-time "time" affects mass in the same way as the mass creation time. If Γ_m is interpreted as a density that varies with either $\dfrac{v}{c} = \dfrac{V}{C}$, then mass increasing in the mass domain is equivalent to time increasing ("dilating") in the space-time domain for a common "distance" $x_v = x_c$. For example, this implies that a signal traveling at $v < c$ will take longer to traverse this distance than a signal traveling at $v = c$:



$$\boxed{\Delta t_v = \frac{\Delta t_c}{\sqrt{1-\left(\frac{M_0}{M_v}\right)^2}} = \Delta t_c \Gamma_m}$$

“Space-Like” Transformation

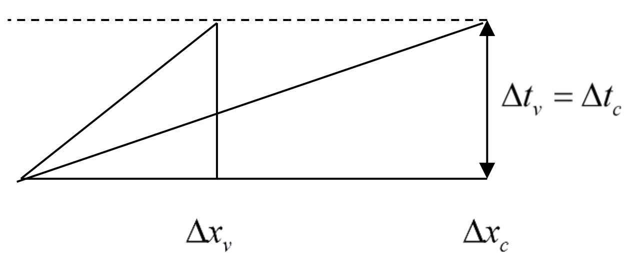
$$\Delta t_v = \Delta t_c$$

$$\left(\frac{\Delta v}{\Delta c}\right)_x = \frac{\Delta x_v}{\Delta x_c} = \frac{V}{C}$$

$$\frac{\Delta x_v}{\Delta x_c} = \sqrt{1 - \left(\frac{M_0}{M_v}\right)^2}$$

$$\Delta x_v = \frac{\Delta x_c}{\Gamma_m}$$

$$\Delta x_c = \Delta x_v \Gamma_m$$



$$\boxed{\Delta x_v = \frac{\Delta x_c}{\Gamma_m}}$$

This means that a signal traveling at $v < c$ will not travel as far in a given amount of time as a signal traveling at $v = c$, since the density increases with V.

Contra-variance

Consider the relation: $\Delta x_c = \Delta x_v \Gamma_m$. Since Δx_c is an invariant, the variables Δx_v and Γ_m must vary inversely:

$\Delta x_c = \left(r\Delta x_v\right)\left(\frac{1}{r}\Gamma_m\right) = \left(\Delta x_v\right)\left(\Gamma_m\right)$ and similarly for any function of x,y,z,t:

$$\Delta x_c = \left(f(x,y,z,t)\Delta x_v\right)\left(\frac{\Gamma_m}{f(x,y,z,t)}\right) = \left(\Delta x_v\right)\left(\Gamma_m\right)$$

The mass density Γ_m and the space variable x_v are said to be "contra-variant" with respect to the transformation x_c on the space-time and mass domains that preserves the relation $\frac{v}{c} = \frac{V}{C}$ between velocities and mass creation rates…

Acceleration

Consider the relations:

$$\begin{aligned} t' &= \Gamma_m t_0 \\ x'\Gamma_m &= x_0 \end{aligned},$$

where (x_c, t_c) has been replaced with (x_0, t_0) to indicate that they are invariants characterizing an initial condition ("proper" ruler, clock).

Then $\frac{x'}{t'} = \frac{x_0}{\Gamma_m}\frac{1}{\Gamma_m t_0} = \frac{x_0}{t_0}\frac{1}{(\Gamma_m)^2}$ and , so that:

$\frac{x'}{t'} = \frac{x_0}{\Gamma_m}\frac{1}{\Gamma_m t_0} = \frac{x_0}{t_0}\frac{1}{(\Gamma_m)^2}$ and $v' = \frac{1}{(\Gamma_m)^2} v_0$

Note that $v' = v$ only if $V = 0$ and that $(\Gamma_m)^2$ is positive definite and $(\rho_m)^2 = \frac{1}{(\Gamma_m)^2} \geq 1$ so that a change in velocity $(\rho_m)^2$ (i.e., an acceleration) acts as a change in energy density in coordinate space.

The Lorentz Transformation and Planck's constant

The Lorentz transform equations are:

$$x' = (x - vt)\gamma = x\gamma - (v\Delta t)\gamma$$
$$t' = (t - \frac{vx}{c^2})\gamma = t\gamma - \left(\frac{v\Delta x}{c^2}\right)\gamma \text{ ,}$$

where the notation Δx and Δt are to be interpreted as discrete "length" and "time" factors.

It is clear that the factor $(v\Delta t)\gamma$ in the "space" transform must be interpreted as "mass" associated with the coordinate system that is factored out if V is independent of C. This implies that $(v\Delta t)\gamma$ corresponds to a mass "separation" associated with the coordinate system. Since $v \neq 0$ and $c \neq 0$, then Einstein's postulate that $v \perp c$ (i.e, (v, c) as a vector) must correspond to setting $\frac{v\Delta x}{c^2} = 0$ in the "time-like" transform, and $v\Delta t = 0$ in the "space-like" transform. Since neither v or c are 0, this means that $\Delta x = 0$ in the former and $\Delta t = 0$ in the latter.

This suggests that there is a relationship to Planck's constant in each of the above cases where $(v\Delta t_h)\gamma = (\Delta x_h)\gamma$; v is the "mass creation rate" and t_h is the "mass creation time" for a coordinate system measured in terms of the "light mass" per unit length when the system is not isotropic and homogeneous. Eliminating these terms (the "separation" in GTR, and the "boost and rotation" in the Lorentz transform – the "rotation" is to the direction you're looking, and the "boost" is to Planck's constant in that direction) ensures global covariance for the fundamental law of physics.

For both cases, if $\Delta x_v = \Delta t_v = 0$, then the time dilation equation, which is covariant and related to the mass transformation which preserves $\left(\frac{v}{c}\right)^2_{(x,t)} = \left(\frac{V}{C}\right)^2_M$ between the space-time and mass domains; however the different terms in the space and time transforms breaks this relationship.

Planck's Constant and the Mass of Light

The "rest mass" of light (the mass of light in our local environment) is given by $M_c = C_h T_h$, where C_h is the "Planck mass creation rate" and T_h is the "Planck mass creation time. This relation is assumed to be valid for all frequencies, so that this mass is independent of the space-time domain. However, if $E_\nu = h\nu$ where ν is the locally observed frequency, then the observed energy will be dependent on both h and ν. If the frequency is assumed to be the same at both the source and sensor, then a change in energy will be due to a change in h.

From Ampere's and Coulomb's laws, the "mass" of light is given by $c^2 = \frac{1}{\varepsilon_0 \upsilon_0}$ so that a unit of rest Energy is $E_{unit} = 1 = (\varepsilon_0 \upsilon_0) c^2 = M_c C^2$

The Rest Mass of Light

If the energy doesn't change during a trip through space, then the energy of a particle won't change; in quantum mechanics, h is introduced as the space "mass" of the journey and the source/sensor, and if it doesn't change, it is divided out when the Energy or Momentum is observed. However, if there has been a change, then it must be due to a photon-on-photon interaction during the trip. (For a one dimensional journey, if "Red Shift" is observed this will affect the ε_0 parameter; if "Einstein rings" are observed, it is the υ_0 factor).

If there has been a change in Planck's constant (e.g., a Red Shift), then the photo will have lost energy. That is, if the mass of the photon at the source/sensor is assumed to be $h = C_h T_h$ then the final mass at the sensor will be less, which means that the interaction must be modeled as an imaginary decrease (since the mass and energy are positive definite.

Then

$(CT')^2 = (CT)^2 + (iVT')^2 = (CT)^2 - (VT')^2$, and

$(h_{sensor})^2 = (h_{source})^2 - (h_{\text{int}eraction})^2$.

The sign of the interaction term would be reversed for a blue shift. Then the light mass (density) of the system observed at the sensor must have increased from that assumed at the source by the interaction along the journey.

Gravity

Consider the product $x't' = x_0 t_0$, so that the mass density vanishes in the determinant of the coordinate matrix

$$A = \begin{vmatrix} x & 0 \\ 0 & t \end{vmatrix}$$

for all values of the coordinates x and t, and in particular, for $r = x = ct$, which implies that from the perspective of STR, mass cannot be associated with curvature in the coordinate system by itself.

That is,

$\left(\frac{x'}{\Gamma_m}\right)(ct'\Gamma_m) = x_0 ct_0 = (x_0)^2$ in the space-like (contra-variant) case, so areas are preserved, and the hyperbolic rotations in space-time are figments of the imagination… (see [squeeze mapping] – the circle is preserved if energy is conserved.)

This implies that GTR must characterize mass as independent of electromagnetic mass (h), so its application must satisfy the vector condition (C_g, C_h),where C_g and C_h are taken to be the mass creation rates for gravity and total (resultant) electromagnetic field ("anywhere", "anywhen"). As with h alone, this is only true for this to be true for all values of gravity and h if they are related by scaling factors T and T', which relate the mass created due to gravity to the mass created by electromagnetism in the mass space $(C_g T, C_h T')$ so we have the resulting expression:

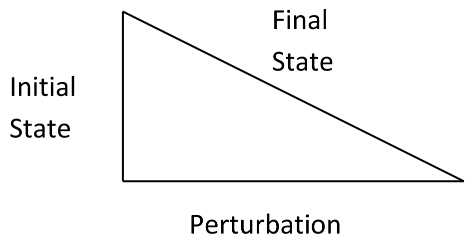
$(C_g T')^2 = (C_h T')^2 + (C_g T)^2$ and $(M')^2 = (M_h)^2 + (M_g)^2$, so that if initial and final conditions via a perturbation are defined, the coordinate system is irrelevant as in the above argument, and the Physical system describing gravity is globally covariant. (For gravity, M_h is infinitesimal compared to M_g , but its effect has been observed (the Casimir effect, due to a surface electric field).

This analysis can be repeated with any other imagined mass (e.g., dark mass, energy) so that mass and energy is always conserved, and the final state can always be calculated if the relation between the initial state and the perturbation are independently specified, and characterized as a vector.

The coordinate characterization by itself is then seen to be a figment of the human imagination, and local to our existence as subjective perception (a very strong anthropic viewpoint, to be sure :) .

Interpretation

Note that the analysis above is for one particle and a perturbation resulting in a final particle, where the concept of "particle " can be replaced by the concept of "environment" in either or all cases. The form of the equation of state is characterized by the relation:



The independent Initial and Perturbation states are then related by the Final State by the Pythagorean Theorem.

The above analysis can then be applied to Relativistic Momentum and Energy:

[Relativistic Energy Derivation]

Since all parameters are positive definite the first order elements can be interpreted as radii, and the second order elements as areas of circles, so that "curvature" defines mass. However, the concept of displacement in tensor analysis is now irrelevant, since mass (including gravitational mass) can be included as a conserved quantity via analogy with classical concepts.

Quantum Triviality

I just discovered that this analysis is analogous to Quantum Trivialty; my Γ_m density is roughly equivalent to the function

$$\Gamma_m \approx g_0 = \left(\frac{g_{obs}}{1-\left(\frac{\beta_2 g_{obs} \ln(A)}{m}\right)} \right)$$ in the document:

[Quantum Triviality] (I KNEW someone else was thinking about this somewhere :)

It should be emphasized that this analysis refers to the total system, so there is no internal structure characterized as a single particle ($M_0 = CT$), which is also the deBrolie wavelength in the "space-like" characterization (period in the "time-like" analysis). However, each time the absorbed energy reaches the point $vt' = ct$, a new particle is created. Since this particle creation is second order, the perturbation approaches the total energy rapidly, and the initial condition becomes vanishingly small (unless the new particle is added to the rest mass); in the limit it disappears.

Maxwell's derivation can then be seen as complementary to mass (since there is no first order element for energy loss), and redshift can now be seen as a quantum process (which changes Planck's constant for photons losing energy over vast distances via interaction with other photons going in the other direction; in this case $h = vt'$ as the connection between STR and quantum mechanics, with ct' becoming the deBroglie wavelength).

The characterization then becomes consistent with quantum field theory, and the problematic "infinities" disappear.

En Fin, there is much more to be said about this approach, and I'm sure it has already been said, since it does seem to be consistent with particle creation/annihilation (one has to use complex numbers to annihilate positive definite quantities); it is just that I haven't seen it in my own limited reading, and hopefully it might clarify some of these issues for those that have had the same confusion as me in looking at the issue….

I currently am active on the forum "Space-Time and the Universe" as "BuleriaChk" and will discuss further issues there for those interested.

For other documents of mine see my Website:

My Take on Relativity

“Flamenco “Chuck”