

The Relation of Light to Mass

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C is conceptualized as the “mass/charge creation rate” of light and T as the time during which mass/charge is created, so that $M_0(CT) = Q_0(CT) = \rho CT$ where $\rho = 1$ is a “density” indicating that only one physical system is involved, interpreted at its center of mass, or equivalently in the “frame” for which C is defined.

The following document is a discussion of:

1. The derivation of C from Coulomb’s and Ampere’s laws, shows the relation in terms of the B and E fields
2. The derivation of C from the E and B fields in terms of vectors and tensors
3. The invariance of the Lorentz force under a Lorentz transform
4. An interpretation of the Gauge transform that clarifies its relation to the scalar for (x,t) and vector field (E,K) and their relation to Special Theory of Relativity.
5. Maxwell’s derivation and its relation to the Lorentz transform. (TBD)

Coulomb's Law

$$F_E = \frac{1}{\epsilon_0} \left(\frac{1}{4\pi} \right) \frac{q_1 q_2}{r^2}$$

Note that this relation only depends on space as characterized by r .

Ampere's Law

$$F_B = \nu_0 \left(\frac{2}{4\pi} \right) \frac{I_1 I_2}{r}, \text{ where } F_B \text{ is the force per unit length of the wires.}$$

If the "wire" length is defined as r (required for Isotropy)

$$F_B r = \nu_0 \left(\frac{2}{4\pi} \right) \frac{I_1 I_2}{r}$$

$$F_B(r) = \nu_0 \left(\frac{2}{4\pi} \right) \frac{I_1 I_2}{r^2}$$

Let

$$q_1 = q_2 = q$$

$$I_1 = I_2 = I$$

Setting the forces equal

$$\frac{1}{\epsilon_0} \left(\frac{1}{4\pi} \right) \frac{q^2}{r^2} = \nu_0 \left(\frac{2}{4\pi} \right) \frac{I^2}{r^2}$$

$$\frac{1}{\epsilon_0} \frac{q^2}{r^2} = 2\nu_0 \frac{I^2}{r^2}$$

However, both directions for the forces between the static charges in the quantity on the left must be included for the equality to hold, which eliminates the factor of 2.

$$\frac{2}{\epsilon_0} \frac{q^2}{r^2} = \nu_0 \frac{2I^2}{r^2}$$

$$\frac{1}{\epsilon_0} \frac{q^2}{r^2} = \nu_0 \frac{I^2}{r^2}$$

We now interpret charge as defined in the (E,K) domain, where the rate of charge creation is C and the Time of charge creation is T, so the total charge over the length from T= 0 to T is

$$q = C_q T$$

$$I = \frac{q}{T} = \frac{C_q T}{T} = C_q$$

$$\frac{1}{\epsilon_0} \frac{q^2}{r^2} = \nu_0 \frac{I^2}{r^2} = \frac{C_q^2}{r^2}$$

$$\frac{1}{\epsilon_0} = \nu_0 C_q^2$$

$$C_q^2 = \frac{1}{\epsilon_0 \nu_0}$$

Vector Derivation

The vector relations are given by the “dot” (inner) and the “cross” (outer) product of the vectors.

The Inner Product (“dot” product) •

Note that the dot product reduces the dimension of the vector space (\vec{E}, \vec{B}) by 1, so the result is a scalar $|\vec{0}|$. (Note: if this space is a tensor with greater than two dimensions, the dot product multiplies all the values of the reduced tensor, since the latter is multilinear) If any dot product between two such vectors is taken at $\theta = \frac{\pi}{2}$, the resultant is $|\vec{0}|$.

$$\vec{E} \cdot \vec{B} = |\vec{E}| |\vec{B}| \sin \theta$$

$$\vec{E} \cdot \vec{B} \Rightarrow \vec{T}, \vec{T} = \frac{1}{(\nu_0 \epsilon_0)} |\vec{E}| |\vec{B}| \sin \theta, \theta = \frac{\pi}{2}$$

$$\begin{vmatrix} E & 0 \\ 0 & B \end{vmatrix} \cdot \vec{T} = 0, \theta = \frac{\pi}{2}$$

The Outer Product ("Cross" product) \times

Note that the cross product increases the dimension of the vector space (by the "right hand rule", by 1, so the result adds a dimension to the vector space $|\vec{C}|$, so there are now three dimensions

$(\vec{E}, \vec{B}, \vec{C})$ with $|\vec{C}|$ orthogonal to the plane (\vec{E}, \vec{B})

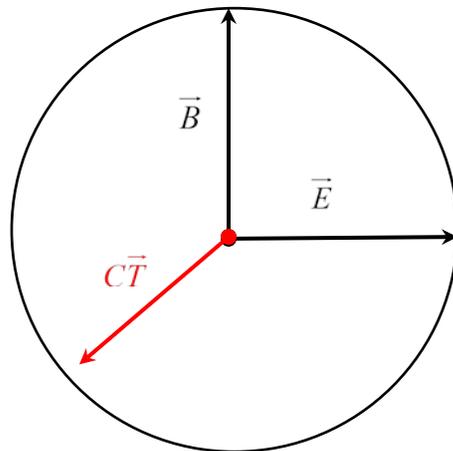
Defining $\vec{C}_B = \frac{1}{\nu_0} \vec{B}$ and $\vec{C}_E = \frac{1}{\epsilon_0} \vec{E}$

We can take the outer product

$$\vec{C}_E \times \vec{C}_B = \frac{1}{\epsilon_0 \nu_0} \vec{E} \times \vec{B}$$

If $\vec{E}, \vec{B}, \vec{T}_E$ and \vec{T}_B are defined as unit vectors $\vec{1}$, then for $\vec{C}_E = \vec{C}_B = \vec{C}$,

$$C^2 = \frac{1}{\epsilon_0 \nu_0}$$



Tensor Derivation

Recall that a tensor function is multilinear (an operation on any element applies to the whole matrix).

$$\begin{vmatrix} E & 0 \\ 0 & B \end{vmatrix} = \begin{vmatrix} \varepsilon_0 C_E & 0 \\ 0 & \nu_0 C_B \end{vmatrix} \xrightarrow{\times} \begin{vmatrix} \varepsilon_0 C_E & 0 & 0 \\ 0 & \nu_0 C_B & 0 \\ 0 & 0 & 1 \end{vmatrix} = \begin{vmatrix} \varepsilon_0 C_E & 0 & 0 \\ 0 & \nu_0 C_B & 0 \\ 0 & 0 & 1 \end{vmatrix} = \varepsilon_0 \nu_0 \begin{vmatrix} C & 0 & 0 \\ 0 & C & 0 \\ 0 & 0 & \frac{1}{\varepsilon_0 \nu_0} \end{vmatrix} = C^2 \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \frac{1}{\varepsilon_0 \nu_0} \end{vmatrix}$$

We can contract (inner product) the top two dimensions and multiply by T the result is

$$\varepsilon_0 \nu_0 C^2 \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} = \begin{vmatrix} \varepsilon_0 \nu_0 C^2 & 0 \\ 0 & 1 \end{vmatrix}.$$

We can multiply the matrix by T^2 so that:

$$T^2 \begin{vmatrix} \varepsilon_0 \nu_0 C^2 & 0 \\ 0 & 1 \end{vmatrix} = \begin{vmatrix} \varepsilon_0 \nu_0 C^2 T^2 & 0 \\ 0 & 1 \end{vmatrix}$$

If matrix on the right is to form a basis of the vector space, we must have:

$$(CT)^2 = \frac{1}{(\nu_0 \varepsilon_0)} = 1$$

For a given $T = T_0 = 1$, $C^2 = \frac{1}{(\nu_0 \varepsilon_0)}$

The Lorentz Force Law

The Lorentz Force Law is defined as:

$$F = \frac{Q}{M}(E + v \times B)$$

If Q and M are related relativistically in the (x,t) and (E,K) domain so that:

$$c = C, \quad \frac{v}{c} = \frac{V}{C}$$

Then a relativistic transform $(V \perp C)$ in (E, K) gives

$$Q' = \frac{Q}{\sqrt{1 - \frac{V^2}{C^2}}} = Q\Gamma, \quad M' = \frac{M}{\sqrt{1 - \frac{V^2}{C^2}}} = M\Gamma$$

where Γ is interpreted as a change in "density" in each case.

Then we have;

$$F = \frac{Q}{M}(E + \left(\frac{v}{c}\right)c \times B) = \frac{Q\Gamma}{M\Gamma}[E + (\beta c \times B)] = \frac{Q'}{M'}[E + \left(\frac{V}{C}\right)C \times B]$$

That is, the Lorentz force is invariant under a Lorentz transformation (β, Γ) , and only depends on E and B .

Gauge Transformation

Ref: http://en.wikipedia.org/wiki/Gauge_fixing (Gauge Fixing)

$$\vec{E} = -\vec{\nabla}\varphi - \frac{\partial \vec{A}}{\partial t}, \quad \vec{B} = \vec{\nabla} \times \vec{A}$$

Re-define:

$$\vec{E} \rightarrow \vec{E}(t), \quad \vec{A} \rightarrow \vec{A}\phi(t),$$

$$\varphi \rightarrow \varphi(x, t) = \varphi_x(x)\varphi_t(t)$$

$$\vec{E}(t) = -\vec{\nabla}[\varphi_x(x)\varphi_t(t)] - \frac{\partial \vec{A}\phi(t)}{\partial t}, \quad \vec{B} = \vec{\nabla} \times \vec{A}(t)$$

$$\vec{E}(t) = -\left[\vec{\nabla}\varphi_x(x)\right]\varphi_t(t) - \vec{A}\frac{\partial\phi(t)}{\partial t}$$

If the transformation

$$\vec{A}\phi(t) \rightarrow \left[\vec{A} + \nabla_x\psi(x)\right]\phi(t),$$

$$\vec{E}(t) = -\left[\vec{\nabla}\varphi_x(x)\right]\varphi_t(t) - \left[\vec{A} + \nabla_x\psi(x)\right]\frac{\partial\phi(t)}{\partial t}$$

$$\vec{E}(t) = -\left[\vec{\nabla}\varphi_x(x)\right]\varphi_t(t) - \left[\vec{A}\frac{\partial\phi(t)}{\partial t} + \nabla\psi_x(x)\frac{\partial\phi(t)}{\partial t}\right]$$

If we make another change:

$$\left[\vec{\nabla}\varphi_x(x)\right]\varphi_t(t) \rightarrow \left[\vec{\nabla}\varphi_x(x)\right]\varphi_t(t) - \nabla\psi_x(x)\frac{\partial\phi(t)}{\partial t}$$

$$\vec{E}(t) = \left[\vec{\nabla}\varphi_x(x)\right]\varphi_t(t) - \nabla\psi_x(x)\frac{\partial\phi(t)}{\partial t} - \left[\vec{A}\frac{\partial\phi(t)}{\partial t} + \nabla\psi_x(x)\frac{\partial\phi(t)}{\partial t}\right]$$

$$\vec{E}(t) = \left[\vec{\nabla}\varphi_x(x)\right]\varphi_t(t) - \vec{A}\frac{\partial\phi(t)}{\partial t} + \left[-\nabla\psi_x(x)\frac{\partial\phi(t)}{\partial t} + \nabla\psi_x(x)\frac{\partial\phi(t)}{\partial t}\right]$$

$$\vec{E}(t) = \vec{E}(t) = \left[\vec{\nabla}\varphi_x(x)\right]\varphi_t(t) - \vec{A}\frac{\partial\phi(t)}{\partial t}$$

Example:

$$\varphi_x(x) = \alpha x$$

$$\varphi_t(t) = \beta t$$

$$\varphi = \varphi_x(x)\varphi_t(t) = (\alpha x)(\beta t)$$

$$\vec{E} \rightarrow \vec{E}(t), \vec{A} \rightarrow \vec{A}t,$$

$$\varphi \rightarrow (\alpha x)(\beta t)$$

$$\vec{E}(t) = -\vec{\nabla}_x \varphi - \frac{\partial \vec{A}t}{\partial t}, \quad \vec{B} = \vec{\nabla} \times \vec{A}$$

$$\vec{E}(t) = -\vec{\nabla}_x (\alpha x)(\beta t) - \frac{\partial \vec{A}t}{\partial t}$$

$$\vec{E}(t) = -\alpha\beta t \vec{\nabla}_x(x) - \frac{\partial \vec{A}t}{\partial t}$$

$$\text{Since } \vec{\nabla}_x[\alpha x] = \alpha,$$

$$\vec{E}(t) = \alpha\beta t - \vec{A}$$

Let

$$\psi(x) = \alpha' x$$

$$\nabla_x \psi = \alpha'$$

$$\vec{A} \rightarrow [\vec{A}t + \alpha' t]$$

$$\vec{E}(t) = -\alpha\beta t - \frac{\partial}{\partial t} [\vec{A}t + \alpha' t]$$

$$\vec{E}(t) = -\alpha\beta t - [\vec{A} + \alpha']$$

Let $t \rightarrow t - \alpha'$

$$\vec{E}(t) = -\alpha\beta t - \alpha' - [\vec{A} + \alpha']$$

$$\vec{E}(t) = -\alpha\beta t - \vec{A} = \vec{E}$$

Example 1

For $\alpha = \beta = 1$

$$\vec{E}(t) = -t - \vec{A} = E$$

Example 2

$\alpha = 1, \beta = c, A = CT, T = t$

$\alpha = 1$ implies $x_c = 1$

$$\vec{E}(t) = -ct - \alpha' - [\vec{C} + \alpha']$$

$$\vec{E}(t) = -ct - \vec{CT} = \vec{E}$$

For $c = C$

$$\vec{E}(t) = -c(t - \vec{T}) = \vec{E}$$

For $|\vec{T}| = t, |\vec{E}| = 0$

That is, if there is no difference in potential, then $|\vec{E}| = 0$

(For x_c a length, $|\vec{E}| = 0$ if the vector time is equal to the scalar time.)

Relativity

$x_c = 1$ implies covariant interpretation $x_c = x_v$

$$|T|\Gamma = t\gamma$$

$$|T|' = t' \Leftrightarrow \Gamma = \gamma \Leftrightarrow \frac{v}{c} = \frac{V}{C}$$

Again, $|\vec{E}| = 0$

Mass Interpretation

Interpret $m_0 = ct = M_0 = CT$ equal to rest mass in both (x,t) and (E,K)

$$\vec{E}(t) = -ct - \vec{CT} = \vec{E}$$

$$\vec{E}(t) = -m_0 - \vec{M}_0 = \vec{E}$$

(Covariant interpretation)

$m_0 = -\vec{M}_0$ (add a mass in (x,t))

$$x'_c = x_c + m_0 ,$$

subtract mass in (E,K)

$$X'_c = X_c - M_0 ,$$

$x'_c = X'_c$, restoring covariance.

(Coordinate interpretation; GTR)

$m_0 = \vec{M}_0 = 0$ (There is no mass, “nothing there”, mass as curvature is imaginary function of imaginary coordinate system.)

The Bottom Line

Note that

$$m_0 C^2 = \frac{M_0}{(v_0 \epsilon_0)}$$

$$C^2 = \frac{1}{(v_0 \epsilon_0)}, m_0 = 1$$

This is Einstein's famous formula for a unit “rest” mass directly from the Lorentz relationships.....

Note: More than one spatial dimension breaks gauge symmetry, since the connection between the vector field $X=CT=M_0$ and the scalar field $x=ct=m_0$ is broken. The subject then becomes a many-body problem in both QED and General Relativity, and gets very complex very fast.