<u>Goldbach's Conjecture and Vectors</u>

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Current Working Document

The Relativistic Unit Circle

Even and Odd Functions

The Derivative (work in progress)

07/04/2020 (Update)

Consider the expression for $\gamma \triangleq \frac{\tau'}{\tau} > 0$:

$\gamma^2 = \left[1\left(\frac{c\tau}{c\tau}\right)\right]^2 + (\gamma\beta)^2 + 2(\gamma\beta)$ Setting $\gamma^2 = 0$ so that for $\left[\mathrm{I}\left(\frac{c\tau}{c\tau}\right)\right] \triangleq \left(\frac{c\tau}{c\tau}\right)^2$

$\left|\left[\mathrm{I}\left(\frac{c\tau}{c\tau}\right)\right]^2 + (\gamma\beta)^2\right| = |2(\gamma\beta)|$, where $\left[\mathrm{I}\left(\frac{c\tau}{c\tau}\right)\right]^2$ is the initial state, and $(\gamma\beta)^2$ is the change from the initial state). This expression is equivalent to the expression $p_1{}^2 + p_2{}^2 = 2(p_1 p_2)$ for the products $p_1 \equiv \mathrm{I}\left(\frac{c\tau}{c\tau}\right) = \frac{c\tau}{c\tau}$ and $(p_1)^2 \equiv \left[\mathrm{I}\left(\frac{c\tau}{c\tau}\right)\right]^2 = \left(\frac{c\tau}{c\tau}\right)^2$ for all (c,τ) and (v,τ') in $\left[(|\mathbb{R}|),(|\mathbb{R}|)\right]$, so that p_1 and p_2 are prime numbers.

For the product $(\beta)(\gamma) = \left(\frac{v}{c}\right)\left(\frac{\tau'}{\tau}\right)$ the terms $x_c' = c\tau'$, $x_v = v\tau'$, and the final state $\left[\mathrm{I}\left(\frac{c\tau'}{c\tau'}\right)\right]^2$ are not defined. This means that p_1 and p_2 cannot be conceptualized as continuous coordinates in "space-time", which has direct relevance to the characterization of DeBroglie and Schroedinger as particle "waves".

Then $\left[1\left(\frac{c\tau}{c\tau}\right)\right]$ and $(\gamma\beta)$ must be an odd prime numbers and therefore $\left[1\left(\frac{c\tau}{c\tau}\right)\right]^2$ and $(\gamma\beta)^2$ as well ,so the sum $\left[1\left(\frac{c\tau}{c\tau}\right)\right]^2 + (\gamma\beta)^2 \equiv \left\{1^2 + \{odd\}^2\right\}$ must also be even.

Note that $\left|2(\gamma\beta)\right| \equiv h^2$

This result shows that increases in entropy to the RUC must be in terms of even numbers for interacting (even) particle pairs (e.g. "bound" prime particles in a potential well) and "free" particles are represented by odd prime numbers.

Matter and Anti-Matter

To express relative change only, one can set

$(v\tau')^2 = h^2$ which eliminates the initial state $(c\tau)^2$ and $(c\tau')^2$ from the full system expressions.

For $\gamma < 1$ ('radiation'), this correspond to the interaction expression $2\left(\frac{\gamma}{\beta}\right) \equiv h^2 = 2\cos\theta\sin\theta$.

Plotting h^2 vs. $2\cos\theta\sin\theta$ shows the relation between "matter" and "anti-matter" (e.g. charges, photons, electrons, etc.)

(This interaction is avoided in the Pauli Spin Matrices, which does not include the "existence" matrix $\begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix}$ as a basis. (It is not included in Lie Groups as well, which forces the "anti-matter" expression to involve complex numbers as a "cross product" in $|\sigma_1| + |\sigma_2| = \begin{vmatrix} 0 & 1-i \\ 1+i & 0 \end{vmatrix}$ where $Tr(\sigma_3) = Tr(\begin{vmatrix} 1 & 0 \\ 0 & -1 \end{vmatrix}) = 1 - 1 = 0$ represents that both matrix elements are equal in first order. The interaction between the E and B fields $h^2 = \pm 2\frac{\beta}{\gamma}$ is eliminated by the conjugation to preserve particle count in the initial and final states).)

For $\gamma > 1$ the interaction is represented by $(\gamma\beta)^2 = 2\beta\gamma \equiv h^2 = 2\cosh\theta\sinh\theta$ which eliminates the initial state $\left[I\left(\frac{c\tau}{c\tau}\right)^2\right] = \left(\frac{c\tau}{c\tau}\right)^2$ and the final state $\gamma^2 = \left[I\left(\frac{c\tau'}{c\tau'}\right)^2\right] = \left(\frac{c\tau'}{c\tau'}\right)^2$ from the expression.

Compare these with $\theta = \pm\frac{\pi}{2}$, where the full expression is

$$\left[(I\left(\frac{c\tau}{c\tau}\right)\right] + \left[(I\left(\frac{c\tau'}{c\tau'}\right)\right])^2 = \left\{\left[(I\left(\frac{c\tau}{c\tau}\right)\right]^2 + \left[(I\left(\frac{c\tau'}{c\tau'}\right)\right]^2\right\}\left(+2\left[(I\left(\frac{c\tau}{c\tau}\right)\right]\left[(I\left(\frac{c\tau'}{c\tau'}\right)\right]\right)$$

$$" \equiv " 1^2 + 1^2 + 2\left(1^2\right) = 4\left(1^2\right)$$

In the general model, the separation coordinate ("curvature") $k^2 = \frac{1}{x^2}$ so that

$$\gamma^2 = \left[1\left(\frac{c\tau}{c\tau}\right)\right]^2 + (\gamma\beta)^2 + \frac{1}{x^2}\left[2(\gamma\beta)\right] = \left[1\left(\frac{c\tau}{c\tau}\right)\right]^2 + (\gamma\beta)^2 + \frac{h^2}{x^2},\quad h^2 \triangleq \left[2(\gamma\beta)\right]$$ to express the interaction decreasing with "distance" from the initial state $x = 1$, which characterizes an "inverse square law". This can also be applied for the "radiation" case where $\frac{\tau'}{\tau} < 1$:

$$\left[1\left(\frac{c\tau'}{c\tau'}\right)\right]^2 = \left(\frac{1}{\gamma}\right)^2 + \beta^2 + \left(\frac{1}{x^2}\right)\left[2\frac{\beta}{\gamma}\right] = \left(\frac{1}{\gamma}\right)^2 + \beta^2 + \left(\frac{h^2}{x^2}\right), h^2 \triangleq \left[2\frac{\beta}{\gamma}\right]$$

(Multiply the second order equations by π for "radial" expressions, where $k^2 = \frac{1}{r^2}$).

Electromagnetism

$$\varphi \triangleq \varepsilon_0 E + \mu_0 (\pm B)$$

$$\varphi^2 \triangleq (\mu_0 E)^2 = (\varepsilon_0 E + \mu_0 B)^2 = (\varepsilon_0 E)^2 + (\mu_0 B)^2 + 2(\varepsilon_0 E)(\mu_0 (\pm B))$$

(Interaction)

For any initial (invariant) value of E, ignoring the initial and final expressions and focusing only on the change as above, consider the following relation (yielding a $k^2 = \frac{1}{r^2}$ as a function of the interaction of photons (electrons, freight trains, galaxies, etc…)

$$h^2 \equiv 2(\varepsilon_0 E)(\mu_0 B) = 2(\varepsilon_0 \mu_0)(EB) = 2\frac{(EB)}{c^2} = 2\frac{(EB)}{(c\tau)^2} = 2\frac{(EB)}{(c\tau')^2} = 2\frac{(EB)}{(r)^2},\ c\tau' = c\tau = r$$

$$(\mu_0 B)^2 \triangleq 2\frac{(EB)}{(r)^2}$$

This means that the interpretation of $m_0 = c\tau_0 \equiv x_0$ (as a mass) where c is an invariant relegates those of us living (and experimenting) on the surface of the earth to interpreting the external universe to the invariants of c and $\pm e$ as global values. (Note that both c and e vary inside matter, and so would vary outside the "matter'' of the atmosphere at sea level, and by extrapolation outside the earth+moon, solar system, galaxy, etc. If equal conditions exist at a far star or galaxy, then any red shift must be attributed to photon matter within the line of sight and/or coming from sources behind us, not as a "spacetime" velocity shift.

Coordinates

If we define the first order distance in vector form, then $r \triangleq x + y + z$, so that

If we define the first order distance in vector form, then

$$r^2 = (x+y+z)^2 = x^2 + y^2 + z^2 + r\,\mathrm{e}\,m(x,y,z,2)$$

However, $r^2 = x^2 + y^2 + z^2 \Leftrightarrow r\,\mathrm{e}\,m(x,y,z,2) = 0$.

By the Trinomial Expansion

$r\,\mathrm{e}\,m(x,y,z,2) = 2xy + 2xz + 2yz$ so that all three coordinates must be equal to 0

Note that radial "coordinates" **in second order** (areas, energy) can be applied as

$$\pi r^2 = \pi(x+y+z)^2 = \pi x^2 + \pi y^2 + \pi z^2 + \pi\left[r\,\mathrm{e}\,m(x,y,z,2)\right]$$

(This can be characterized as the energy distribution for quarks).

Note that a first order transformation from rectangular coordinates to radial (or spherical) coordinates is inconsistent with arithmetic, and can only be imagined if $\left[r\,\mathrm{e}\,m(x,y,z,2)\right] = 0$

This result can be characterized by the Minkowski "metric"

$$|M| = \begin{vmatrix} r & 0 & 0 & 0 \\ 0 & i & 0 & 0 \\ 0 & 0 & j & 0 \\ 0 & 0 & 0 & k \end{vmatrix} \Leftrightarrow |M|^2 = \begin{vmatrix} r & 0 & 0 & 0 \\ 0 & i & 0 & 0 \\ 0 & 0 & j & 0 \\ 0 & 0 & 0 & k \end{vmatrix}^2 = \begin{vmatrix} r^2 & 0 & 0 & 0 \\ 0 & (i)^2 & 0 & 0 \\ 0 & 0 & (j)^2 & 0 \\ 0 & 0 & 0 & (k)^2 \end{vmatrix}$$

where $(i,j,k) = (\sqrt{-1}), \sqrt{-1}, \sqrt{-1}) \equiv (i,j,k)$

The Minkowski metric for a real (positive) unit energy $r = \pm 1$ is then given by:

$$|M|^2 \equiv \begin{vmatrix} 1^2 & 0 & 0 & 0 \\ 0 & (i)^2 & 0 & 0 \\ 0 & 0 & (j)^2 & 0 \\ 0 & 0 & 0 & (k)^2 \end{vmatrix} = \begin{vmatrix} 1^2 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{vmatrix}$$

If the imaginary "axis" is aligned with the real axis, then the j and k axes are unobservable (to your left, right, up an down) along with the i axis (behind, in front of you), and the only observable is $h^2 \triangleq 1^2$ (at your focal plane). If electromagnetism is included, then the factor $\frac{1}{r^2}$ is included as a "distance" (in terms of energy) from the source, so $h_r^{\ 2} \triangleq \left(\frac{1}{r^2}\right)1^2,\ r \geq 1$

In the expansion $E^2 = (1 + i + j + k)^2$ expressed as a quaternion, the interacting factors are removed by conjugation (elimination of "cross products" by exchanging imaginary elements) so that the only real energy is that which is observed $(h_r)^2 = \left(\frac{1}{r^2}\right)(1)^2,\ r \triangleq 1$

In this context, the energy is due to distance (since it is actually radiated in all directions at the source). Additional losses in energy may occur by subsequent interactions along the "geodesic" path.

Source Creation and Destruction

The position of a Source is given by the coordinate system as a point at (x, y, z) when observed from a position at $(0,0,0)$ and the observed energy is the change of state at the source as $E_\delta = h^2 = 2\left(\frac{\pm\beta}{\gamma}\right)$ (the radiated interaction energy there), where the total energy at radiation is given by:

$$E_f = E_0 + E_\Delta + E_\delta$$

If it is assumed that an observer can observer only one position at a time, the observed Source is observed as "created" when it moves into an empty position at (x, y, z) and is "destroyed" when it moves away from its current position. (Wherever you look, that's what you see)

For a real source, the observed energy at $(0,0,0)$ is modified (as a first approximation) by an inverse square law so that $E_{\text{observed}} = \frac{1}{r^2}h^2 = \frac{1}{r^2}E_\delta$

Charge and Mass (07/09/2020)

Let $(c\tau')$ be the total mass of a system consisting of a boson $(c\tau)$ and a photon $(v\tau')$

Let $(c\tau')^2$ be the total energy of the system, $(c\tau)^2$ be the energy of a boson, and $(v\tau')^2$ be the energy of a photon with positive and negative spin polarization. Then

$(c\tau')^2-(c\tau)^2=(v\tau')^2+2(c\tau)(v\tau')$, where $v\triangleq\sin\theta$ so that

$h^2\triangleq 2(c\tau)([\pm v]\tau')\equiv 2\cos\theta\sin(\pm\theta)=\pm 2\cos\theta\sin(|\theta|)$

Plotting h^2 against $2\cos\theta\sin\theta$ in the first and fourth quadrants of the RUC $-\frac{\pi}{2}<\theta<\frac{\pi}{2}$ represents the interaction between photon and boson, and thus charge. If the "coordinate" Inverse Square law is included as a factor from Electromagnetism the interaction between the B and E fields then the interaction between the photon and electron can be modified as

$h^2\triangleq 2(c\tau)([\pm v]\tau')\equiv 2\cos\theta\sin(\pm\theta)=\frac{1}{r^2}\left[\pm 2\cos\theta\sin(|\theta|)\right],\ r\geq 1$ where

$r^2=\frac{1}{k^2}=(c\tau)^2=\sqrt{(\varepsilon_0\mu_0)\tau}\Rightarrow\frac{1}{r^2}=\frac{1}{(c\tau)^2}$ and represents a "separation" between mass and charge, which is practically unobservable (I think).

Distinguishable particles

Consider the interacting case where

$c\tau=(v+\delta)\tau',\ c=(v+\delta)$

So that

$$(c\tau')=(c\tau)+(c\tau)$$
$$(c\tau')^2=\left[(c\tau)+(c\tau)\right]^2=(c\tau)^2+(c\tau)^2+2(c\tau)(c\tau)=2(c\tau)^2+2(c\tau)(c\tau)$$

Then the interaction energy can be represented as:

$$\Delta^2\triangleq(c\tau')^2-\left[(c\tau)^2+(c\tau)^2\right]=2(c\tau)(c\tau),\ \Delta^2>0$$

So that $\Delta^2=2(c\tau)(c\tau)=2(c\tau)(v+\delta)\tau'=2(c\tau)(v\tau')+2(c\tau)(\delta\tau')$

If the interaction is between a proton (boson) and an electron, this separation (in energy) can represent a interaction δ which can be positive or negative. However, the initial equation cant be then recast as a system of three particles:

$$v' = v + \delta$$
$$(c\tau') = (c\tau) + (v'\tau')$$
$$(c\tau')^2 = (c\tau)^2 + \left[(v+\delta)\tau'\right]^2 + 2[c\tau]\left[(v+\delta)\tau'\right]$$

The expression $v'\tau' \triangleq (v+\delta)\tau'$ then results in

$$(c\tau')^2 = (c\tau)^2 + (v'\tau')^2 + (h_r)^2$$
$$2(c\tau)\left(\frac{v'}{r}\tau'\right) \triangleq (h_r)^2 \quad ,$$
$$(c\tau')^2 - \left[(c\tau)^2 + (v'\tau')^2\right] = (h_r)^2$$

where $\delta \to 0$ as $r \to \infty$ and $c' \to c$ as $1 \leftarrow r$, $1 < r < \infty$.

Note that for $r \to \infty$, the interaction energy falls as $\frac{1}{(r)^2}$ so that at "infinity" there is no interaction.

For the original system, before radiation δ characterizes a "binding energy"; after radiation, the system reverts to a system of two non-interacting particles:

$$\begin{vmatrix} c\tau & 0 \\ 0 & v\tau' \end{vmatrix}.$$

This analysis can then be applied to the hydrogen atom, which includes the proton $p \triangleq c\tau$, the electron $e \triangleq (v\tau')$ and the neutrino $\eta \triangleq \delta$ from the previous characterization.

Imaginary Interactions in linear systems

For $\frac{\tau'}{\tau} < 1$ (radiation), one can include the final state as a result of a linear interaction, provided that:

$$I\left(\frac{\tau'}{\tau}\right) \triangleq \beta + i\left(\frac{1}{\gamma}\right)$$
$$\left[I\left(\frac{\tau'}{\tau}\right)\right]\left[I\left(\frac{\tau'}{\tau}\right)\right]^* = \left[\beta + i\left(\frac{1}{\gamma}\right)\right]\left[\beta - i\left(\frac{1}{\gamma}\right)\right] = \beta^2 + \left(\frac{1}{\gamma}\right)^2$$

One can pretend that $\left(\frac{1}{\gamma}\right)^2$ is an imaginary "perturbation" to β^2 and thus represents an imaginary "acceleration", which is then represented by $h^2 = 2\beta\left(\frac{1}{\gamma}\right)$. A similar analysis applies for $\frac{\tau'}{\tau} > 1$ (absorption).

However, this imaginary acceleration cannot be assigned to space or time (represented by first order real numbers), since the complex plane is an imaginary) order; (a dimension below unity), where

$$2(1_i) = \log_i i^2 = \log_i(-1) = 1 \neq 2(1_1) = \log_1(1)^2 = 2$$

(i.e., the complex plane does not interact with positive real numbers.)

This is interpreted as a linear representation of a real physical system in Special Relativity and Quantum Mechanics if one sets

$1^2 = \beta^2 + \left(\frac{1}{\gamma}\right)^2$ or $1^2 = \left(\frac{1}{\gamma}\right)^2 + \beta^2$ and interprets 1^2 as a "normalization". This interpretation is real only for

$I\left(\frac{\beta}{\beta}\right)^2 = \frac{\beta^2}{\beta}$ or $I\left(\frac{\gamma}{\gamma}\right)^2 = \left(\frac{\gamma}{\gamma}\right)^2$, $\theta = \pm\frac{\pi}{2}$ or $\theta = 0$, respectively.

(Imaginary numbers are complex only for those who think they are somehow real).

Note: From the RUC, the (trigonometric) ratio of the perturbation (change) $v\tau'$ to the initial condition $c\tau$ is:

$$\tan\theta = \frac{v\tau'}{c\tau} = \frac{\sin\theta}{\cos\theta} = \frac{\sinh\theta}{1} = \beta\gamma$$

This expression is first order, and in particular, does not include the interaction energy (entropy) $h^2 = 2\beta\gamma = 2S^2 = 2\sin\theta\cos\theta$. This means the classic "slope" of the derivative in a first order coordinate system is only a change in first order, but not in second order, where

$$\varphi = x + vt$$
$$\varphi^2 = (x)^2 + (vt)^2 + 2x(vt)$$
$$\pi\varphi^2 = \pi(x)^2 + \pi(vt)^2 + x(2\pi vt)$$

Goldbach's conjecture ("Every (positive) even number is the sum of two primes.")

Update: 5/5/20

Consider the two dimensional matrix with two arbitrary numbers, n_1 and n_2:

$\begin{vmatrix} n_1 & 0 \\ 0 & n_2 \end{vmatrix}^2 = \begin{vmatrix} (n_1)^2 & 0 \\ 0 & (n_2)^2 \end{vmatrix}$ If all common elements (k^2) are taken outside the matrix, the matrices then represent prime numbers:

$k^2 \begin{vmatrix} p_1 & 0 \\ 0 & p_2 \end{vmatrix}^2 = k^2 \begin{vmatrix} (p_1)^2 & 0 \\ 0 & (p_2)^2 \end{vmatrix}$, where all numbers have a unique representation by the fundamental theorem of arithmetic.

Note that the single valued expression $Tr\begin{vmatrix} n_1 & 0 \\ 0 & n_2 \end{vmatrix}^n = Tr\begin{vmatrix} (n_1)^n & 0 \\ 0 & (n_2)^n \end{vmatrix} = (n_1)^n + (n_2)^n$ implies that $Det\begin{vmatrix} (n_1)^n & 0 \\ 0 & (n_2)^n \end{vmatrix} = (n_1)^n (n_2)^n$ is not defined (as in Fermat's Expression), corresponding to the elimination of such products by complex conjugation. The vectors $\begin{vmatrix} (n_1)^n \\ 0 \end{vmatrix}$ and $\begin{vmatrix} 0 \\ (n_2)^n \end{vmatrix}$ are then affine (no common connection, independent). In particular, the "spin" vectors $\begin{vmatrix} 1 \\ 0 \end{vmatrix}$ and $\begin{vmatrix} 0 \\ 1 \end{vmatrix}$ are similarly affine under this prescription, where one defines **The Unity Function** as

$$\mathrm{I}(a) = \frac{a}{a}$$

Note that for $\varphi = 1+1 = 2$ in first order, if there is no interaction the elements can be counted and $\varphi^2 = 1^2 + 1^2 = 2(1^2)$. If there is interaction, then $\varphi^2 = (1+1)^2 = 1^2 + 1^2 + 2(1^2) = 4$. However, note that $1 = \log_1(1) \neq 2 = \log_1(1^2)$

Since each of the constituents of k^2 also in the product $n_1 n_2$ they can be factored out of the product, so that $n_1 n_2 = k^2 (p_1 p_2)$, where p_1 and p_2 are prime.

Then

$$(k^2) Tr\left(\begin{vmatrix} (p_1)^2 & 0 \\ 0 & (p_2)^2 \end{vmatrix}\right) = k^2\left[(p_1)^2 + (p_2)^2\right]$$
$$= \left(Det\begin{vmatrix} kp_1 & kp_1 \\ -kp_2 & kp_2 \end{vmatrix}\right) = \left((k^2) Det\begin{vmatrix} p_1 & p_1 \\ -p_2 & p_2 \end{vmatrix}\right) = (k^2)\left[2(p_1)(p_2)\right]$$

$$L = \left[(p_1)^2 + (p_2)^2\right] = 2(p_1)(p_2) = 2N$$

$2(p_1)(p_2)$ is an even number, and $(p_1)^2 + (p_2)^2$ is the sum of two primes. The expression is complete for even numbers by the Prime Factorization Theorem (the Fundamental Theorem of Arithmetic); the expression is then true for all even numbers except for the multiplicative factor k^2.

However, it would seem that Goldbach's conjecture could be wrong, since $L' = k^2 L$ is no longer the sum of two primes.

However, note that the "interaction" term $h^2 \triangleq (n_1)(n_2)$ is not defined for the complete system for non-interacting numbers $(n_1)^2 + (n_2)^2$; one must include the term n in the defining relation $n = n_1 + n_2$ where

$$n^2 = (n_1 + n_2)^2 = (n_1)^2 + (n_2)^2 + 2(n_1)(n_2)$$

Is now complete, and includes all multiples where $2(n_1)(n_1)$ is defined. The parameterization of $n \triangleq c\tau',\ n_1 \triangleq c\tau,\ n_2 \triangleq v\tau'$ where in this context $\{c, \tau, v, \tau'\}$ can be considered positive integers.

$$c\tau' = c\tau + v\tau'$$
$$(c\tau')^2 = (c\tau)^2 + (v\tau')^2 + 2(c\tau)(v\tau')$$

ensures that the relation $(c\tau)^2 + (v\tau')^2 = 2(c\tau)(v\tau')$ characterizes the even prime numbers as a subset of $(c\tau')^2$ which includes both even and odd numbers with both multiplication and division defined.

The proof of Goldbach's conjecture divides the positive real numbers into two disjoint sets;

Even numbers $\{E\} \triangleq \left\{|\mathbb{R}| : L = (p_1)^2 + (p_2)^2 = 2(p_1)(p_2) = 2N\right\}$ and

Odd numbers: $\{O\} \triangleq x : x \notin \{E\}$

Where

$\{|\mathbb{R}|\} = \{O\} \cup \{E\} = \left\{(c\tau')^2\right\}$, and where $\{E\}$ represents interacting numbers and $\{O\}$ represents odd numbers.

For Goldbach's conjecture, the proof depends on accepting that p_1 and p_2 are prime numbers $\sqrt{p_1}$ and $\sqrt{p_2}$ are prime numbers, and the same for 1 and $\sqrt{1}$.

Then setting

$\sqrt{p_1} \in \{P\} \Leftrightarrow (p_1)^2 = \left(\sqrt{p_1}\right)^2 = p_1 \in \{O\} \in \{P\}$ (where the set of prime numbers is represented by $\{P\}$)

and

$$p_2 \triangleq \sqrt{1} \in \{P\} \Leftrightarrow (p_2)^2 = \left(\sqrt{1}\right)^2 = 1 \in \{O\} \in \{P\}$$

$\left(\sqrt{p_1}\right)^2 + \left(\sqrt{p_2}\right)^2$ is the sum of two prime numbers iff $(p_1) + (p_2)$ is the sum of two prime numbers.

Then (e.g.) $3 + 1 = 2(2)(1) = 4$ where $\{3, 1, 2, 1\}$ are all prime numbers. (Note that if these conditions are not accepted, then the above is a counter example).

In general, if $p_1^{\,2} + 1^2$ is the sum of two primes, then so is $p_1 + 1$ so the equation

$\left(\sqrt{p_1}\right)^2 + \left(\sqrt{1}\right)^2 = 2(p_1)(1) = 2N$ for all $\left\{\left(\sqrt{p_1}\right)^2, \left(\sqrt{1}\right)^2\right\} \in \{P\}$ and $2N \in \{E\}$

Since all prime numbers > 2 are odd, adding the prime number 1 to such a prime number makes it even.

$$L = Tr\begin{vmatrix} \left(\sqrt{p_1}\right)^2 & 0 \\ 0 & \left(\sqrt{1}\right)^2 \end{vmatrix} = Tr\begin{vmatrix} p_1 & 0 \\ 0 & 1 \end{vmatrix} = p_1 + 1 = Det\begin{vmatrix} p_1 & p_1 \\ -1 & 1 \end{vmatrix} = 2(p_1)(1) = 2N$$

i.e., $p_1 + 1 = 2N$

From a "relativistic" perspective

$$c\tau' = c\tau + v\tau'$$

$$(c\tau')^2 = (c\tau + v\tau')^2 = (c\tau)^2 + (v\tau')^2 + 2(c\tau)(v\tau')$$

$$(c\tau')^2 - \left[(c\tau)^2 + (v\tau')^2\right] = 2(c\tau)(v\tau') = h^2 = 2S^2,\ 2(c\tau)(v\tau') \triangleq h^2 = 2S^2,\ S = \frac{h}{\sqrt{2}}$$

$$(c\tau')^2 = 0 \Leftrightarrow \left|(c\tau)^2 + (v\tau')^2\right| = \left|2(c\tau)(v\tau')\right|$$

Note: here $+$ is a group operation under addition "existence" and juxtaposition $(c\tau)(v\tau')$ is the group operation under multiplication ("interaction")

$(c\tau')^2 = 0$ means that there is no multiplicative interaction (and therefore no division) between the parameters c and τ', and thus $(c\tau)^2$ and $+(v\tau')^2$ are prime numbers. That is, on the l.h.s, $c = 0$ or $\tau' = 0$, but not both.

If p is a prime number, then so are $(p)^2$, $\sqrt{(p)}$,and in general $(p)^n$ and $\sqrt[n]{p}$ are prime numbers as well.

$$\left|(c\tau)^2 + (v\tau')^2\right| = \left|2(c\tau)(v\tau')\right|$$

$$p_1 \triangleq (c\tau), p_2 \triangleq (v\tau')$$

$$p_1 \triangleq (c\tau),\ p_2 \triangleq (v\tau')$$

$$(p_1)^2 + (p_2)^2 = 2(p_1)(p_2)$$

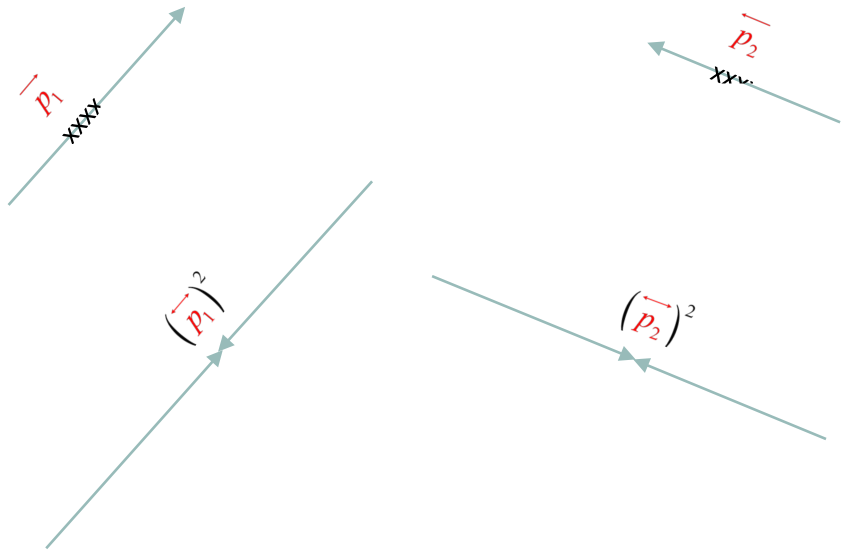
$(p_1)^2 + (p_2)^2 = 2(p_1)(p_2) \neq (p_1)^2 + (p_2)^2$ (since (p_1) and (p_2) are prime. Note that the terms $L = (p_1)^2 + (p_2)^2$ and $2N = 2(p_1)(p_2)$ are not prime

$$(1_{p_1}) = \log_{p_1}(p_1)^2$$

$$\neq (1_{p_1}) = \log_{p_1}(p_1)$$

$$\therefore (p_1)^2 \neq (p_1)$$

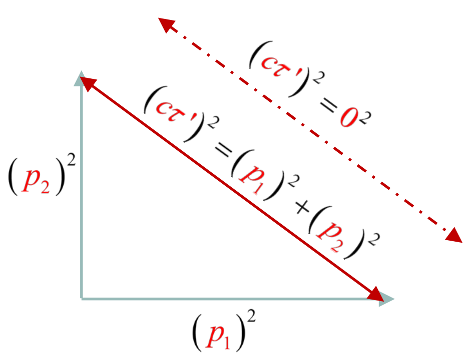
Addition of Prime Numbers



$$|p| \triangleq \begin{vmatrix} p_1 & 0 \\ 0 & p_2 \end{vmatrix}$$

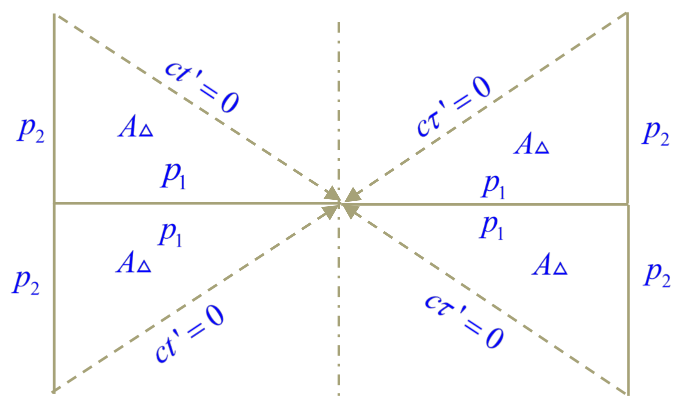
$$|p|^2 = \begin{vmatrix} p_1 & 0 \\ 0 & p_2 \end{vmatrix}^2 = \begin{vmatrix} (p_1)^2 & 0 \\ 0 & (p_2)^2 \end{vmatrix}$$

$$Tr\left(|p|^2\right) = (p_1)^2 + (p_2)^2$$



From the Relativistic Unit Circle with no (i.e., interacting) resultant; $(c\tau')^2 = 0^2$ is the magnitude of an affine (not connected) vector; as such, it is the definition of prime numbers $(p_1)^2$ and $(p_2)^2$

Multiplication of Prime Numbers



$$A_{\triangle} = \frac{1}{2}(p_1 p_2)$$
$$A = 4[A_{\triangle}] = 2p_1 p_2$$

$$L \triangleq (c\tau')^2 \triangleq (p_1)^2 + (p_2)^2,\ (c\tau')^2 = 0^2$$
$$N \triangleq (c\tau)(v\tau') = p_1 p_2,\ \sqrt{(p_1)^2 + (p_2)^2} = \sqrt{(c\tau)^2 + (v\tau')^2} = 0$$

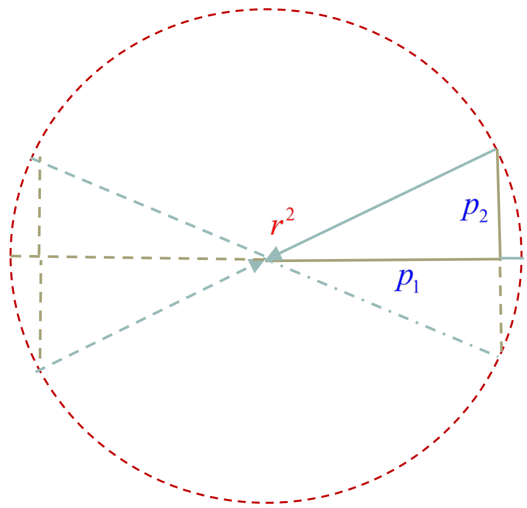
$$2(c\tau')^2 = 4(c\tau')\ (=0)$$
$$(c\tau')^2 = 2(c\tau')\ (=0)$$
$$L = 2N$$

$$L = Tr\left(\begin{vmatrix} (p_1)^2 & 0 \\ 0 & (p_2)^2 \end{vmatrix}\right) = Tr\left(\begin{vmatrix} (p_1) & 0 \\ 0 & (p_2) \end{vmatrix}\right)^2 = (p_1)^2 + (p_2)^2$$
$$2N = Det\left(\begin{vmatrix} p_1 & p_1 \\ -p_2 & p_2 \end{vmatrix}\right) = 2(p_1)(p_2)$$

L is even and the sum of two primes. The system is complete under addition, multiplication, and power of $n=2$ for parameterized nomials as prime numbers

$$0=(c\tau')^2 \triangleq |L| = \left|(c\tau)^2+(v\tau')^2\right| = 2|N| = 2(c\tau)(v\tau'):$$

$$\{c,\tau,v,\tau'\}\in\mathbb{R}^+,\{c\tau,v\tau',c\tau,v\tau'\}\triangleq\{p_1,p_2,p_1,p_2\}=\{\mathrm{P}^+\}$$



$$r^2 = L = (p_1)^2+(p_2)^2 = 4\left(\frac{1}{2}p_1p_2\right)\pi = 2p_1p_2 = 2l$$

$$\pi r^2 = (p_1)(2\pi p_2)$$

This ensures that the count of even numbers is consistent for Addition and Multiplication of positive prime (non-interacting) numbers.

Vector Dot and Cross Products

(I am in the process of expanding this section to include complex numbers and their interpretation in physical contexts). Stay tuned….

From the The Relativistic Unit Circle:

$$h^2 = 2\sin\theta\cos\theta$$

$$\cos\theta = \cos(-\theta)$$
$$\pm\sin\theta = \sin(\pm\theta)$$

Dot Product

$$\vec{A} \triangleq 2\overrightarrow{\sin\theta}$$
$$\vec{B} \triangleq \overrightarrow{\cos\theta}$$
$$\vec{A}\cdot\vec{B} \triangleq 2\left|\overrightarrow{\sin\theta}\right|\left|\overrightarrow{\cos\theta}\right| = \cos\theta(2\sin\theta) = h^2$$
$$h^2 \triangleq 2(p_1p_2)$$

Here the dimension of one vector is "projected" on the dimension of the other in the interaction term, so that the product is the "entropy" of the interaction for $v\tau' \neq 0$

Cross Product

$$\vec{A} \triangleq 2\overrightarrow{\cos\theta}$$
$$\vec{B} \triangleq \overrightarrow{\sin\theta}$$
$$\vec{A}\otimes\vec{B} \triangleq 2\left|\overrightarrow{\sin\theta}\right|\left|\overrightarrow{\cos\theta}\right| = \pm\sin\theta(2\cos\theta) \sim 2(p_1p_2) = -2(p_2p_1) \sim \vec{B}\otimes\vec{A}$$

Here $+\sin\theta$ and $-\sin\theta$ are mutually exclusive, and so $\vec{A}\otimes\vec{B}$ and $\vec{B}\otimes\vec{A}$ must be referred to a different "third dimension" where the dimensions of $\vec{A}$ and $\vec{B}$ vanish, and in particular $\overrightarrow{A^2}$ and $\overrightarrow{B^2}$

So that $h^2 = 0$ and $0 = A^2 + B^2 + A\otimes B$

This means that for the interaction term, either both A and B are both zero, or the condition for prime numbers must obtain so that

$$A^2 + B^2 = -A\otimes B$$
$$\left|A^2 + B^2\right| = \left|A\otimes B\right|.$$

Goldbach's conjecture ("Every (positive) even number is the sum of two primes.") is then proved.

(Q.E.D)

From the RUC radiation model $\gamma \triangleq \frac{\tau'}{\tau},\ \beta \triangleq \frac{v}{c}$ from the single valued function

$(c\tau') = (c\tau) + (v\tau')$ and squaring (so the l.h.s. represents an equal and opposite force as a final state, where

$$(c\tau')^2 = (c\tau)^2 + (v\tau')^2 + 2(c\tau)(v\tau')$$

Radiation

Dividing by $(c\tau')^2$ result in the expression

$$[1(c\tau')]^2 = \frac{(c\tau')^2}{(c\tau')^2} = \left(\frac{1}{\gamma}\right)^2 + (\beta)^2 + 2\left(\frac{\beta}{\gamma}\right),\ \gamma \triangleq \frac{\tau'}{\tau}, \beta \triangleq \frac{v}{c}$$

In this context of the RUC, $\left(\frac{1}{\gamma}\right) \triangleq \cos\theta,\ \beta \triangleq \sin\theta$, so the equation beomes

$$[1(c\tau')]^2 = (\cos\theta)^2 + (\sin\theta)^2 + 2\sin\theta\cos\theta$$

The term $(\cos\theta)^2 + (\sin\theta)^2$ can be expressed as

$1 = \cos\theta + i\sin\theta \triangleq e^{i\theta}$ where

$$\psi\psi^* \triangleq 1(e^{i\theta}) = \frac{e^{i\theta}}{e^{i\theta}} = [\cos\theta + i\sin\theta][\cos\theta - i\sin\theta] = 1(e^{i\theta}) = \cos^2\theta + \sin^2\theta$$

Here the complex axis is independent of the real axis, so the real and complex bases is represented by the matrix $\mathbb{C} \triangleq \begin{vmatrix} 1 & 0 \\ 0 & i \end{vmatrix}$ so that $\mathbb{C}^2 \triangleq \begin{vmatrix} 1^2 & 0 \\ 0 & i^2 \end{vmatrix} = \begin{vmatrix} 1 & 0 \\ 0 & i \end{vmatrix}^2$ and (in general) $\mathbb{C}^n \triangleq \begin{vmatrix} 1^n & 0 \\ 0 & i^n \end{vmatrix} = \begin{vmatrix} 1 & 0 \\ 0 & i \end{vmatrix}^n$

Then $1(e^{i\theta}) = \cos^2\theta + \sin^2\theta$ is represented by the matrix $|\psi\psi^*| \triangleq \begin{vmatrix} \cos^2\theta & 0 \\ 0 & \sin^2\theta \end{vmatrix} = \begin{vmatrix} \cos\theta & 0 \\ 0 & \sin\theta \end{vmatrix}^2$ so that $Tr|\psi\psi^*| = \cos^2\theta + \sin^2\theta$ and $Det|\psi\psi^*| = 0$ because $\cos\theta \perp \sin\theta$ (i.e., they exist on perpendicular axes on the complex plane.

Thus complex conjugation eliminates the interaction term $2\sin\theta\cos\theta$ in the original expression.

This term can be represented in matrix format by $h(\theta)^2 = Det\begin{vmatrix} \cos\theta & \cos\theta \\ -\sin\theta & \sin\theta \end{vmatrix} = 2\cos\theta\sin\theta$

If $h(\theta)$ is taken to be a constant (representing a constant entropy) then plotting it against the interaction term gives the plot:

(rough – should be plotted with limits from 0 to $2n\pi$)

graph.pdf

,

which shows the periodic nature of the function in the positive and negative domain due to the action of $\sin \pm \theta = \pm \sin \theta$

Physically, at the final state, this can represent two particles of opposite spin that are now "free" of the term $\cos^2 \theta + \sin^2 \theta$ since they no longer interact with it.

Physically, then the equation represents a constant input (depending on θ) and the interaction $2\sin\theta\cos\theta$ (also dependent on θ)

$$[1(c\tau')]^2 = (\cos\theta)^2 + (\sin\theta)^2 + 2\sin\theta\cos\theta$$

(the initial and final states are defined by $1\left(\frac{\tau}{\tau}\right)$ and $1\left(\frac{\tau'}{\tau'}\right)$, respectively.

(For radiation, the "spins" are in opposite direction at the juncture of the radius and the circumference; for absorption the "spins" are directed against each other. This is called (I think) "chirality"

Absorption

For **absorption**, the relation $\left(\frac{c\tau'}{c\tau}\right)^2 = \left[1(c\tau)\right]^2 + \left(\frac{v\tau'}{c\tau}\right)^2 + 2\left(\frac{v\tau'}{c\tau}\right)$ becomes

$\gamma^2 = [1(c\tau)]^2 + (\gamma\beta)^2 + 2(\gamma\beta)$ so that (analogously to the above) the relation can be made:

$(\cosh\theta)^2 = [1(c\tau)]^2 + (\sinh\theta)^2 + 2(\cosh\theta)(\sinh\theta)$ and the same complex analysis can be performed.

Note that $\tan\theta = \gamma\beta$ for both radiation and absorption in the RUC.

Complex Conjugation and Quantum Mechanics (Radiation)

Consider the expression

$$\varphi = (ct') = (ct) + (v\tau')$$
$$\varphi^2 = (ct')^2 = (ct)^2 + (v\tau')^2 + 2(c\tau)(v\tau')$$

Now form the expression

$$\alpha = \sqrt{(ct)^2 + (v\tau')^2} + i\sqrt{2(c\tau)(v\tau')}$$
$$\alpha\alpha^* = \left\{\left[(ct)^2 + (v\tau')^2\right] + i2\sqrt{(c\tau)(v\tau')}\right\}\left\{\left[(ct)^2 + (v\tau')^2\right] - i2\sqrt{(c\tau)(v\tau')}\right\} = \left[(ct)^2 + (v\tau')^2\right] + 2(c\tau)((\pm v)\tau')$$

where $h^2 = 2(c\tau)((\pm v)\tau')$

This is an expression of the concept that $\left[(ct)^2 + (v\tau')^2\right]$ and h^2 do not interact (since the "imaginary" and real axis are orthogonal, and so independent). In that case one can define the function

$\psi\psi^* = h^2$ and "imagine" it is the Hamiltonian corresponding to the Schrodinger equation where

$\psi\psi^* \triangleq h^2 = i\hbar \frac{d}{d\tau} | \Psi(\tau) >= \widehat{H}\Psi(\tau)$, noting that when the derivative is taken in the classical wave format $\Psi^*(\tau) = \exp(\frac{i}{\hbar}[Px - E(\tau)])$ the factor $\frac{i}{\hbar}$ cancels out in the expression $\Psi\left(\frac{d}{d\tau}\right)\Psi^* = E$ and similarly for the expression $\Psi^*(x) = \Psi\left(\frac{d}{d\tau}\right)\Psi^* = P$. One can imagine that E and P are somehow real, so that $\widehat{H}\Psi(\tau)$ is real as well.

One can swap the imaginary value in the expression α so that

$$\alpha = \sqrt{2(c\tau)(v\tau')} + i\sqrt{(c\tau)^2 + (v\tau')^2} +$$
$$\alpha\alpha^* = \left\{2\sqrt{(c\tau)(v\tau')} + i\left[(c\tau)^2 + (v\tau')^2\right] + \right\}\left\{\sqrt{(c\tau)(v\tau')} - i2\left[(c\tau)^2 + (v\tau')^2\right]\right\} = 2(c\tau)((\pm v)\tau') + \left[(c\tau)^2 + (v\tau')^2\right]$$

$h^2 = 2(c\tau)((\pm v)\tau')$ is real, by ignoring the factor $\left[(c\tau)^2 + (v\tau')^2\right]$

(as an exercise, make the substitutions for $\frac{\tau'}{\tau} = \frac{1}{\gamma} = \cos\theta$ and $\beta = \frac{v}{c} = \sin\theta$ as in the RUC.

Waves and Particles

Consider the expression for radiation from the RUC:

$$\left[I(\frac{c\tau'}{c\tau'})\right]^2=\left[\left(\frac{1}{\gamma}\right)^2+(\beta)^2\right]+\left[2\frac{\beta}{\gamma}\right]=\left[\left\{\sqrt{\left[\left(\frac{1}{\gamma}\right)^2+(\beta)^2\right]}\right\}^2+\left\{\sqrt{(\beta)}\right\}^2\right]+\left[\sqrt{\left[2\frac{\beta}{\gamma}\right]}\right]^2$$

$$\triangleq(\Psi)^2+(\mathrm{P})$$

Where $(\Psi)^2=\left[\left(\frac{1}{\gamma}\right)^2+(\beta)^2\right]$ represents a "wave" characterization of two elements in terms of sums of powers (of 2), and $(\mathrm{P})=\left\|\left[2\frac{\beta}{\gamma}\right]\right\|=\left|2(\beta)\left(\frac{1}{\gamma}\right)\right|$ represents a "particle" characterization of two elements in terms of powers (of 1). (In the RUC these elements are not parameterized in terms of "space" and "time", but in terms of mass.) In this case any negative sign refers to a rotation of θ but is positive in each quadrant, so that $\sin\theta=|\sin\theta|$ and the expression $\sin(-\theta)=-\sin(\theta)$ is not permitted in the full expansion. That is, $(\mathrm{P})<\left[I(\frac{c\tau'}{c\tau'})\right]^2$ for $v\neq 0$

Note that this expression is equivalent to

$$\left[I(\frac{c\tau'}{c\tau'})\right]=\left(\frac{1}{\gamma}\right)+\beta\Leftrightarrow\left[I(\frac{c\tau'}{c\tau'})\right]^2=\left[\left(\frac{1}{\gamma}\right)^2+(\beta)^2\right]+\left[2\frac{\beta}{\gamma}\right]$$

However, **if there is no interaction** between "waves" and "particles", then the expression must include imaginary numbers so that

$I_\psi\left(\left(\frac{1}{\gamma}\right)+\beta\right)=\left(\frac{1}{\gamma}\right)+i\beta$ or $I_\psi\left(\left(\frac{1}{\gamma}\right)+\beta\right)=\beta+i\left(\frac{1}{\gamma}\right)$ where in the latter case, the "rest" state configuration $(v=0)$ implies that the initial state is imaginary, so there is nothing to begin with.

With complex conjugation, the former becomes:

$$\left(I_{\psi}\right)\left(I_{\psi}\right)^{*}=\left(\left(\frac{1}{\gamma}\right)+i\beta\right)\left(\left(\frac{1}{\gamma}\right)-i\beta\right)$$

$$=\left(\frac{1}{\gamma}\right)^{2}+\left(\beta\right)^{2}$$

Note that imaginary numbers only appear in second order to indicate a negative product where

$-(ab)=(ia)(ib)=i^{2}(ab)$, so that $(ab)-(ab)=0 \Leftrightarrow -(ab)=i^{2}(ab)$

Then setting $\left(I_{\psi}\right)\left(I_{\psi}\right)^{*}=0 \Leftrightarrow \left(\frac{1}{\gamma}\right)^{2}=\left(\beta^{2}\right)$ which can only be true for $\beta=0$ so that $v=0$

And $\left(I_{\psi}\right)\left(I_{\psi}\right)^{*}=\left(\frac{1}{\gamma}\right)^{2}=\left[I\left(\frac{\tau}{\tau}\right)\right]^{2}=\left[I\left(\frac{\tau'}{\tau'}\right)\right]^{2}\triangleq\cos^{2}\theta,\ \theta=0$

For

$$\left[I\left(\frac{c\tau'}{c\tau'}\right)\right]^{2}=\left[\left(\frac{1}{\gamma}\right)^{2}+\left(\beta\right)^{2}\right]+\left[2\frac{\beta}{\gamma}\right]$$

$$\left[I\left(\frac{c\tau'}{c\tau'}\right)\right]^{2}=\left[\left(\frac{1}{\gamma}\right)^{2}+\left(\beta\right)^{2}\right]-\left[2\left(i\beta\right)\left(i\frac{1}{\gamma}\right)\right]=\left[\left(\frac{1}{\gamma}\right)^{2}+\left(\beta\right)^{2}\right]+\left[2\left(\beta\right)\left(\frac{1}{\gamma}\right)\right]$$

so

$$\left[I\left(\frac{c\tau'}{c\tau'}\right)\right]^{2}=0 \Rightarrow \left[\left(\frac{1}{\gamma}\right)^{2}+\left(\beta\right)^{2}\right]-\left[2\left(\beta\right)\left(\frac{1}{\gamma}\right)\right]=0$$

And

$$\left[\left(\frac{1}{\gamma}\right)^{2}+\left(\beta\right)^{2}\right]=\left[2\left(\beta\right)\left(\frac{1}{\gamma}\right)\right]$$

This expression corresponds in form to that of Goldbach’s conjecture, where

$$\left[\left(p_{1}\right)^{2}+\left(p_{2}\right)^{2}\right]=\left[2N\right]=\left\{e\right\}$$

but is imaginary because it only characterizes even numbers. The full expression becomes positive if odd numbers are included, where

$\left[I(\frac{c\tau'}{c\tau'}) \right]^2 = \{e\} + \{o\}$ where the sets $\{e\}$ and $\{o\}$ are both positive.

Note that the odd set corresponds to the "wave" characterization" for

$$1 \triangleq \left(\frac{1}{\gamma}\right) + (i\beta)$$
$$1^2 \triangleq (1)(1)^* = \left(\frac{1}{\gamma}\right)^2 + (\beta)^2 \triangleq \cos^2\theta + \sin^2\theta$$

So that setting the odd and even sets equal implies Goldbach's conjecture so that all particles are counted under both the sum and multiplier operations, where

$$\varphi = 1 + 1$$
$$\varphi^2 = (1+1)^2 = 1^2 + 1^2 + 2(1^2) = 4(1^2)'$$

corresponding to the four quadrants of the RUC.

This means the "metric" of the positive definite numbers (i.e., imaginary numbers excluded) is characterized by $(1,1,1,1)$ instead of Dirac's $(1,-1,1,-1)$ or Minkowski's $(1,-1,-1,-1)$ where $1^2 + 1^2 = 0 \Rightarrow 1^2 = -1^2 \Rightarrow 1^2 - 1^2 = 0$

Quaternions

$$\vec{i} \otimes (\vec{k} \otimes \vec{j}) \triangleq ikj = 1$$
$$\vec{i} \otimes (\vec{j} \otimes \vec{k}) \triangleq ijk = -1$$

$$(\vec{i} \otimes \vec{j}) = \vec{k}$$
$$(\vec{j} \otimes \vec{i}) = -\vec{k} = \overleftarrow{k}$$
$$(\vec{i} \otimes \vec{j}) + (\vec{i} \otimes \vec{j}) = \vec{k} - \vec{k} = \vec{k} + \overleftarrow{k} = \vec{0}$$

$$\vec{i} \otimes \left(\vec{j} \otimes \vec{k} \right)$$

$$\left(\vec{j} \otimes \vec{k} \right) = \vec{l}$$

$$\left(\vec{k} \otimes \vec{j} \right) = -\vec{l} = \overleftarrow{l}$$

$$\left(\vec{j} \otimes \vec{k} \right) + \left(\vec{k} \otimes \vec{j} \right) = \vec{l} - \vec{l} = \vec{l} + \overleftarrow{l} = \overleftrightarrow{0}$$

$$(\psi_i) = r_i + \vec{i} \otimes \left(\vec{j} \otimes \vec{k} \right)$$

$$(\psi_i)^* = r_i - \vec{i} \otimes \left(\vec{k} \otimes \vec{j} \right)$$

$$(\psi_i)(\psi_i)^* = (r_i)^2 + r_i \left(\vec{i} \otimes \left(\vec{j} \otimes \vec{k} \right) \right) - r_i \left(\vec{i} \otimes \left(\vec{j} \otimes \vec{k} \right) \right) - (r_i)^2 \left[i \otimes \left(\vec{j} \otimes \vec{k} \right) \right]^2 = (r_i)^2 - \left[\vec{i} \otimes \left(\vec{j} \otimes \vec{k} \right) \right]^2$$

$$(R_i)^2 \triangleq (\psi_i)(\psi_i)^* = (r_i)^2 - \left[\vec{i} \otimes \left(\vec{j} \otimes \vec{k} \right) \right]^2$$

$$\left[(R_i)^2 \right]^* \triangleq \left[(\psi_i)(\psi_i)^* \right]^* = (r_i)^2 - \left[\vec{i} \otimes \left(\vec{j} \otimes \vec{k} \right) \right]^2$$

$$\left[(R_i)^2 \right] \left[(R_i)^2 \right]^* = \left\{ (r_i)^2 + \left[\vec{i} \otimes \left(\vec{j} \otimes \vec{k} \right) \right]^2 \right\} \left\{ (r_i)^2 - \left[\vec{i} \otimes \left(\vec{j} \otimes \vec{k} \right) \right]^2 \right\} = \left[(r_i)^2 \right]^2 \left\{ \left[\vec{i} \otimes \left(\vec{j} \otimes \vec{k} \right) \right]^2 \right\}^2$$

$$= \left[(r_i)^2 \right]^2 \left\{ \left[\vec{i} \otimes \vec{l} \right]^2 \right\}^2$$

$$\left[(r_i)^2 \right]^2 \left\{ \left[\vec{i} \otimes \vec{l} \right]^2 \right\}^2$$

$$\vec{i} \perp \vec{l} \Rightarrow \left[\vec{i} \otimes \vec{l} \right] = 0 \Rightarrow \cos\left(\theta_{il} \right) = 0$$

$$\Rightarrow (R_i)^2 = (\psi_i)(\psi_i)^* = (r_i)^2$$

$$\left(R_{ijk} \right)^2 = (r_i)^2 + \left(r_j \right)^2 + \left(r_j \right)^2$$

Quaternions eliminate the interaction between the imaginary axes in 3-space, leaving only the real radii.

For vectors then

$$\begin{vmatrix} r & 0 & 0 & 0 \\ 0 & i & 0 & 0 \\ 0 & 0 & j & 0 \\ 0 & 0 & 0 & k \end{vmatrix} \otimes \begin{vmatrix} r & 0 & 0 & 0 \\ 0 & i & 0 & 0 \\ 0 & 0 & j & 0 \\ 0 & 0 & 0 & k \end{vmatrix} = \left(R_{ijk}\right)^2 = \left(r_i\right)^2 + \left(r_j\right)^2 + \left(r_j\right)^2 \Leftrightarrow \text{r real (positive definite), } \vec{i}, \vec{j}, \vec{k} \text{ imaginary}$$

$$\vec{i} \triangleq \left(\sqrt{-1}\right)\vec{i},\ \vec{j} \triangleq \left(\sqrt{-1}\right)\vec{j},\ \vec{k} \triangleq \left(\sqrt{-1}\right)\vec{k}$$

$$\pi\left(R_{ijk}\right)^2 = \pi\left(r_i\right)^2 + \pi\left(r_j\right)^2 + \pi\left(r_j\right)^2 = \pi\left(\left(r_i\right)^2 + \left(r_j\right)^2 + \left(r_j\right)^2\right)$$