Electron Spin

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Website

Relativity (and other Topics)

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Updated

04/20/2018 08:30 Included Degeneracy

04/22/2018 07:47 Included Pauli σ_3 Matrix

04/23/2018 09:24 update and clarification to Pauli σ_3 Matrix

05/14/2018 A major revision, focusing on the Pauli Matrices, with the original material to be revised and added back in subsequent updates.

05/15/2018 Added "Spin Up" and "Spin Down" derivation

05/16/2018 Bug fixes

Work in Progress – (Reload and Refresh your browser for most recent version.

Spin (Physics) (Wikipedia)

Stern-Gerlach Experiment (Wikipedia)

Pauli Matrices (Wikipedia)

Dirac Matrices (Wikipedia)

The Relativistic Unit Circle and Proof of Fermat's Last Theorem (my pdf)

Proof of Goldbach's conjecture (a slight digression which you may enjoy)

Summary – This shows the relation of Electron Spin to particle count, and why linear systems cannot preserve energy if particle count is conserved via the Binomial Expansion as a single valued function relating particle count, particularly in the case of $n=2$. In particular, the multiplication and addition

rules of arithmetic and vectors do not apply when particle count $n=1$ is conserved, which leads to a discussion of entropy (available on request). The relation to gravity is quick and obvious (just ask me).

The first step is to derive the Pauli and Dirac relations for Electron Spin from first principles; classical Electromagnetism in the Pauli relation, and the Dirac's interpretation in terms of the Special Theory of Relativity as an extension of the Pauli Analysis.

The Pauli Spin Matrices

Maxwell's Equations

Maxwell's derived the speed of light in terms of Ampere and Coulomb's force laws via the coordinate concept of displacement current in the integral solution using Green's and Stokes theorems (a linear system in terms of areas and volumes using dot and cross products to model the relation of charge to current), with the result that $c=\frac{1}{\sqrt{\varepsilon_0\mu_0}}$, where ε_0 and μ_0 are the permittivity and permeability constants, respectively.

Special Relativity

Special Relativity eliminates that analysis by simply setting c to be a universal constant, so that one might as well assign $c=1$, in which case the distinction between Newton's laws and electromagnetism (with its coordinate system) become moot in terms of energy, since

$E=mv^2=m(\frac{v}{c})^2c^2=\left(mc^2\right)\beta,\ \beta=\frac{v}{c}=v$ Therefore, the Theory of Special Relativity is a theory of mass and energy, where reference to explicit coordinate systems is replaced by the concept of "Inertial Frames". (Einstein the re-introduces coordinate systems in his General Theory as an attempt to provide a model of gravity that is independent of Electromagnetism).

Spin vs. Orbital Angular Momentum

Orbital angular momentum refers to the classical concept of a single physical system; a top, an electron in orbit around an atom, or a boy twirling a ball at the end of a rope. In this case, the attractive force is modeled at the center of the circumference of a circle.

In contrast, (very roughly), spin is a model of two orbital systems which are only in instantaneous contact at a point on their circumferences; it the spins are the same, then the systems merge (i.e. stop) so angular momentum is zero and the systems lose their identity and are unobservable, or if the spins are opposite, the repel each other with the interaction modeled as $2h^2$ (which will be explained in the discussion that follows). Electron spin then refers to elements that retain their identity but have equal and opposite energies; since this is true for all possible masses, the "radius" of each electron can be assigned the value of unity, so only the interaction is relevant.

In this context the relativistic description of the system (as a whole) means that all the elements are positive definite, so the concept of positive and negative charge is irrelevant. The concept of "negative mass" in the popular literature is explained by rotation of the gamma vector from a pre-existing initial condition below.

In the Relativistic interpretation, spin is a rotation of the world gamma vector $\gamma = \frac{\tau'}{\tau}$ as a result of removing degeneracy, where $\tau' \neq \tau$, and is a result of Dirac's relativistic development from Pauli's classical interpretation of the Stern-Gerlach experiment.

Classical (Pauli) derivation of the Electron Spin

The three Pauli matrices $(\sigma_1, \sigma_2, \sigma_3)$ are an interpretation of Electron Spin in classical terms from charge q, velocity v, Electric field E and Magnetic field B, conforming to the Lorentz force law

$mA = q(E + v \otimes B)$ for a unit mass $m = 1$, so that the starting point is $A = q(E + v \otimes B)$. The model then assumes that both the kinetic energy ($k.e. = mv^2$) of both of the charged particles interact with both the E and B fields, but that the E and B fields do not interact with each other.

The Pauli σ_3 Matrix

The σ_3 matrix represents the initial state of the Stern-Gerlach experiment before the B field is turned on, where

$$\varphi_{(mv,q,E)} = \begin{vmatrix} q & 0 \\ 0 & -q \end{vmatrix} \begin{vmatrix} E^2 & 0 \\ 0 & E^2 \end{vmatrix} \begin{vmatrix} mv^2 & 0 \\ 0 & mv^2 \end{vmatrix} = \begin{vmatrix} q & 0 \\ 0 & -q \end{vmatrix} \begin{vmatrix} (\pm E)^2 (mv^2) & 0 \\ 0 & (\pm E)^2 (mv^2) \end{vmatrix}$$ and

$\sigma_3 = \begin{vmatrix} 1 & 0 \\ 0 & -1 \end{vmatrix}$ for $q = \pm 1$, $m = 1$, $v^2 = 1^2$ and $E^2 = 1^2$ where the velocities are in the same direction from the source $v > 0$ and $m > 0$, so that $\theta = 0$ from the source to the interaction point when the B field is turned on.

Note that $E^2 = (\pm E)^2 \cos^2(\theta) = (E^2 \cos^2 \theta)$, $\theta = 0$ as a representation of the field energy in the initial state. Since there is no B field, $B^2 = (\pm B)^2 \sin^2(\theta) = (B \sin \theta)^2 = 0$, $\theta = 0$

The Pauli σ_1 and σ_2 matrices

When the B field is turned on, the purpose of the Pauli matrices is to model the interaction between the charges, interpreted as spin by preserving the E field in the σ_1 matrix and the interaction between the velocity and B field in the σ_2 but eliminating any interaction between the E and B field, since they are orthogonal (independent) in Maxwell's equations. This is accomplished through conjugation of the independent fields.

Note that setting $m=1$ means the charge to mass ratio is characterized by $\frac{\pm q}{m}=\pm q,\ m=1$.

Let $|\psi|=q\left[E+ivB\right]$

Then $|\psi||\psi|*=|\psi|*|\psi|=(\pm q)^2\left[E+ivB\right]\left[E-ivB\right]=(\pm q)^2\left[E^2+v^2B^2\right]$

so that

$$|\psi||\psi|*=(\pm q)^2\left[E^2+(v\otimes B)^2\right]=\left[(\pm q)^2E^2+(\pm q)^2(v\otimes B)^2\right]$$

Then

$$|\psi||\psi|*=q^2E^2+q^2\left(vB^2\right)=q^2\left[E^2+v^2B^2\right]$$

Note that for $E=\cos\theta$ and $B=B\sin\theta$,

$$|\psi||\psi|*=q^2\left[E^2+(vB)^2\right]=q^2\left[\sin^2\theta+\cos^2\theta\right]=q^2,\ v^2=(\pm 1)^2$$

$$|\psi|=\begin{vmatrix}0 & (\pm q)E\\ (\pm)E & 0\end{vmatrix}+\begin{vmatrix}0 & i(-q)vB\\ i(q)vB & 0\end{vmatrix}=\begin{vmatrix}0 & (\pm q)[E-ivB]\\ (\pm q)[E+ivB] & 0\end{vmatrix}=\begin{vmatrix}0 & (\pm q)\\ (\pm q) & 0\end{vmatrix}\begin{vmatrix}0 & E-ivB\\ E+ivB & 0\end{vmatrix}$$
$$=\begin{vmatrix}0 & (\pm q)\\ (\pm q) & 0\end{vmatrix}\left\{\begin{vmatrix}0 & E\\ E & 0\end{vmatrix}+\begin{vmatrix}0 & -ivB\\ ivB & 0\end{vmatrix}\right\}=$$
$$=\begin{vmatrix}0 & (+q)\\ (+q) & 0\end{vmatrix}\left\{\begin{vmatrix}0 & E\\ E & 0\end{vmatrix}+\begin{vmatrix}0 & -ivB\\ ivB & 0\end{vmatrix}\right\}(\vec{i})+\begin{vmatrix}0 & (-q)\\ (-q) & 0\end{vmatrix}\left\{\begin{vmatrix}0 & E\\ E & 0\end{vmatrix}+\begin{vmatrix}0 & -ivB\\ ivB & 0\end{vmatrix}\right\}(\vec{j})$$
$$=\left|S^+\right|\vec{i}+\left|S^-\right|\vec{i}$$

Since the E and B fields are independent (i.e., do not interact), the determinants of the components of the (S) matrices are taken individually.

$$|S^+| = \begin{vmatrix} 0 & (+q) \\ (+q) & 0 \end{vmatrix} \left\{ \begin{vmatrix} 0 & E \\ E & 0 \end{vmatrix} + \begin{vmatrix} 0 & -ivB \\ ivB & 0 \end{vmatrix} \right\} = \begin{vmatrix} 0 & qE \\ qE & 0 \end{vmatrix} + \begin{vmatrix} 0 & -iqvB \\ iqvB & 0 \end{vmatrix}$$

$$= |qE| + |qvB|$$

$$Det(|qE|) + Det(|qvB|) = -(qE)^2 + (qvB)^2$$

$$|S^-| = \begin{vmatrix} 0 & (-q) \\ (-q) & 0 \end{vmatrix} \left\{ \begin{vmatrix} 0 & E \\ E & 0 \end{vmatrix} + \begin{vmatrix} 0 & -ivB \\ ivB & 0 \end{vmatrix} \right\} = \begin{vmatrix} 0 & -qE \\ -qE & 0 \end{vmatrix} + \begin{vmatrix} 0 & +iqvB \\ -iqvB & 0 \end{vmatrix}$$

$$Det|S^-| = (qE)^2 + (qvB)^2$$

$$Det(|-qE|) + Det(|qvB|) = (-qE)^2 + (qvB)^2$$

The Classical Definition of Spin (Pauli Derivation)

Finally, the spin s is defined by the relation

$$
\begin{aligned}
&Det\left(\left|qE\right|\right)+Det\left(\left|-qE\right|\right)+Det\left(\left|qvB\right|\right)+Det(\left|qvB\right|)\\
&=(qE)^2-\left(qE\right)^2+2\left(qvB\right)^2=2\left(qvB\right)^2\\
&\left(h_{vB}\right)^2=2\left(qvB\right)^2=2s^2,\ s=\left(qvB\right)\\
&s=\frac{\left(h_{vB}\right)}{\sqrt{2}}
\end{aligned}
$$

Note that in this interpretation, Planck's constant h is a function of the interaction between the velocity v and the perturbing B field that interacts with the moving charges, where the instant before the B field is turned on the velocities are the same even though the charges are opposite in parity, but the result of the B field is that the charges are now the same, but the velocities (as classical energies $E=\pm mv^2,\ m=1$) are now opposite). Since the charges are the same, they have the same effect on the sensors on either side of the split path. (Note that Energy parity can be expressed by $\pm m=\pm 1$in first order.)

Setting each of the variables equal to unity invokes the matrices in terms of their energy bases so they can be evaluated for different experimental parameters. This yields the Pauli σ_1 and σ_2 matrices in addition to the σ_3 matrix presented earlier. Then for

$$q=\pm 1,\ E^2=v^2=B^2=1^2, m=1,$$

$$\left|S^+\right|=\begin{vmatrix}0 & qE^2\\ qE^2 & 0\end{vmatrix}+\begin{vmatrix}0 & -iq\left(vB\right)^2\\ iq\left(vB\right)^2 & 0\end{vmatrix}=\begin{vmatrix}0 & 1^2\\ 1^2 & 0\end{vmatrix}+\begin{vmatrix}0 & -i(1^2)\\ i(1^2) & 0\end{vmatrix}=\begin{vmatrix}0 & 1^2\\ 1^2 & 0\end{vmatrix}+\begin{vmatrix}0 & -i\\ i & 0\end{vmatrix}=\left|\sigma_1\right|+\left|\sigma_2\right|$$

$$\left|\sigma_1\right|=\begin{vmatrix}0 & 1^2\\ 1^2 & 0\end{vmatrix}$$

$$\left|\sigma_2\right|=\begin{vmatrix}0 & -i(1^2)\\ i(1^2) & 0\end{vmatrix}$$

$$\left|S^-\right|=\begin{vmatrix}0 & -qE^2\\ -qE^2 & 0\end{vmatrix}+\begin{vmatrix}0 & \left(-q\right)\left(-i\left(vB\right)^2\right)\\ \left(-q\right)i\left(vB\right)^2 & 0\end{vmatrix}=\begin{vmatrix}0 & -1(1^2)\\ -1(1^2) & 0\end{vmatrix}+\begin{vmatrix}0 & i(1^2)\\ -i(1^2) & 0\end{vmatrix}=-\left(\sigma_1+\sigma_2\right)$$

Then

$$\left|S^{-}\right| = \begin{vmatrix} 0 & -qE^2 \\ -qE^2 & 0 \end{vmatrix} + \begin{vmatrix} 0 & iq(vB)^2 \\ -iq(vB)^2 & 0 \end{vmatrix}$$

$$-\left|S^{-}\right| = \begin{vmatrix} -1 & 0 \\ 0 & -1 \end{vmatrix} \left\{ \begin{vmatrix} 0 & -(1)^2 \\ -(1)^2 & 0 \end{vmatrix} + \begin{vmatrix} 0 & i(1)^2 \\ -i(1)^2 & 0 \end{vmatrix} \right\} = \begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix} + \begin{vmatrix} 0 & -i \\ i & 0 \end{vmatrix} = (\sigma_1 + \sigma_2)$$ in the negative energy basis.

Note that

$\left(\left|S^{+}\right| \otimes \left|S^{-}\right|\right) + \left(\left|S^{-}\right| \otimes \left|S^{+}\right|\right) = \left|S\right|^2 \left|\vec{0}\right|$, representing equal and opposite reaction of spin represented as positive and negative energy. That is, chirality is represented by the null vector of the interaction of equal and opposite spins .

The three Pauli matrices in the unity energy bases are then:

$$\left|\sigma_1\right| = \begin{vmatrix} 0 & 1^2 \\ 1^2 & 0 \end{vmatrix} ,$$

$$\left|\sigma_2\right| = \begin{vmatrix} 0 & -i\left(1^2\right) \\ i\left(1^2\right) & 0 \end{vmatrix} = \begin{vmatrix} 0 & -i \\ i & 0 \end{vmatrix} ,$$

and $$\sigma_3 = \begin{vmatrix} 1\left(1^2\right) & 0 \\ 0 & -1\left(1^2\right) \end{vmatrix} = \begin{vmatrix} 1 & 0 \\ 0 & -1 \end{vmatrix}$$

One final note: in the expression ivB the only variant is B which is turned on at the point of action in the model; v is simply split into different signs (directions), represented by the interaction of the charges with B . Therefore, it is the B field that is responsible for the interaction, and therefore the imaginary number i should be specifically assigned to the B field, so the interaction matrix is

$$\sigma_2 = \begin{vmatrix} 0 & -v^2(iB^2) \\ v^2(iB^2) & 0 \end{vmatrix} = \begin{vmatrix} 0 & -iB^2 \\ iB^2 & 0 \end{vmatrix}$$ for $v^2 = 1^2$, and the "action" is completely characterized by the introduction of the B field as the "perturbation" or the "change" agent.

Spin "Up" and Spin "Down"

$$\varphi^+ = q(E + vB)$$
$$q = 1,\ v = \pm 1$$
$$(\varphi^+)^2 = (E + vB)^2 = [E^2 + v^2B^2] + 2E(+vB)$$
$$\varphi^- = q(E - vB)$$
$$(\varphi^-)^2 = (E - vB)^2 = [E^2 + (vB)^2] - 2E(vB)$$

$E^2 = E\vec{i} \cdot E\vec{i} = E^2(\vec{i} \cdot \vec{i})$, $B^2 = B\vec{j} \cdot B\vec{j} = B^2(\vec{j} \cdot \vec{j})$, and where $\pm v = \pm 1$ represents parity as the interaction between the E and B fields where

$$(E\vec{i} \otimes B\vec{j}) = (EB)\vec{k}$$
$$-(B\vec{j} \otimes E\vec{i}) = -(BE)\vec{k} = (EB)(\overrightarrow{-k})$$
$$(EB)\vec{k} + (EB)(\overrightarrow{-k}) = 2EB(\vec{0})$$

And the null vector $(\vec{0})$ represents the opposed interaction

$$\begin{vmatrix} (\varphi^+)^2 & 0 \\ 0 & (\varphi^-)^2 \end{vmatrix} = \begin{vmatrix} [E^2 + B^2] & 0 \\ 0 & [E^2 + B^2] \end{vmatrix} + \begin{vmatrix} 2EB & 0 \\ 0 & -2EB \end{vmatrix}$$

Setting $EB = 1^2$ models the interaction as equal and opposite. Setting

$4s^+ = 2(1^2)$ and $4s^- = -2(1^2)$ corresponding to the 4 quadrants of the unit circle, where the interaction in each quadrant is $s = \pm\frac{1}{2}(1^2)$. Then traversing ccw from $\theta = 0$, the interaction in quadrants 1 and 3 will be $s^+ = \frac{1}{2}(1^2)$ and that in quadrants 2 and 4 will be $s^- = -\frac{1}{2}(1^2)$

$$\begin{vmatrix} s^+ & 0 \\ 0 & s^- \end{vmatrix} = \begin{vmatrix} \frac{1}{2} & 0 \\ 0 & -\frac{1}{2} \end{vmatrix} = \begin{vmatrix} \frac{1}{2} \\ 0 \end{vmatrix} + \begin{vmatrix} 0 \\ -\frac{1}{2} \end{vmatrix} = \begin{vmatrix} s^+ \\ 0 \end{vmatrix} + \begin{vmatrix} 0 \\ s^- \end{vmatrix}$$

The column vectors $\begin{vmatrix} \frac{1}{2} \\ 0 \end{vmatrix}$ and $\begin{vmatrix} 0 \\ -\frac{1}{2} \end{vmatrix}$ are referred to as "Electron Spin", representing the normalized interaction between the E and B fields. Then the half quadrants of the unit circles are represented by the Electron Spin matrices, where

$s\uparrow = \begin{vmatrix} s\uparrow \\ 0 \end{vmatrix} = \begin{vmatrix} \frac{1}{2} \\ 0 \end{vmatrix}$ is referred to as "spin up" and $s\downarrow = \begin{vmatrix} 0 \\ s\downarrow \end{vmatrix} = \begin{vmatrix} 0 \\ -\frac{1}{2} \end{vmatrix}$ is referred to as "spin down",

This result can then be arranged in row or column vectors depending on whether or not it is characterized in a dual space.