Fermat’s Last Theorem and Complex Numbers

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There are two “theorems” that urban logic considers extraordinarily difficult to prove: “Goldbach’s Conjecture“ and “Fermat’s Last Theorem”. The proofs of these theorems are actually almost trivial but have far-reaching consequences in the application of mathematics to theoretical physics (and many other disciplines). Complex Numbers form the basis of most of modern physics, including The Theory of Relativity (both Special and General), Quantum Mechanics, among many other disciplines.

I begin with proofs of Goldbach’s Conjecture and Fermat’s Last Theorem (I have not seen such proofs in anything I have read to date). It is hard to believe that they haven’t been proven long ago by Math “C” students within a month after their appearance, who were then whisked away by men in black (or maybe white) coats, never to be heard from again.)

Links to Relevant Documents on my Site

Proof of Fermat’s Theorem for Multinomials (and other topics)

Interaction Examples (in progress, adding all the time)

Pauli Spin (in progress)

The Relativistic Unit Circle

Assumption: All numbers are positive definite $n := \left(\sqrt{n}\right)^2 = \sqrt{n^2}$ Note that

$$\{a,b,c\} > 0$$
$$-c = a-b,\ b > a \leftrightarrow b-c = a > 0$$

(i.e., negative numbers only exist as differences between positive numbers.)

Imaginary Numbers (in Magenta)

$$i := \sqrt{-1}$$
$$i^2 := \left(\sqrt{-1}\right)\left(\sqrt{-1}\right) = \sqrt{(-1)(-1)} = \sqrt{(1)(1)} = \sqrt{(1)^2} = 1$$
$$i^2 \neq -1$$

Proof of Goldbach’s Conjecture: “Every even number is the sum of two primes.”

Every number can be characterized as prime relative to its own base by the relation:

$n = n(\frac{n}{n}) = n\left(1_n\right)$ where $\left(1_n\right) := \frac{n}{n}$ and where $\left(1_m\right) = \left(1_n\right) \leftrightarrow m = n$. Proof of Goldbach's Conjecture follows immediately: $n + n = 2n$ (QED)

Proof of Fermat's Last Theorem: $c^n \neq a^n + b^n \ \forall \ \{a,b,c,n > 0,\ n > 2\}$

Note: the proof will be for $n > 1$ from which $n > 1$ follows automatically. The proof relies on the Binomial Expansion, proved by Newton long ago.

$$\begin{aligned}
&c := a + b \\
&c^n := (a+b)^n = \left[a^n + b^n\right] + f(a,b,n) \quad \text{(Binomial Expansion)} \\
(n=2)\ &c^n = \left[a^n + b^n\right] \leftrightarrow f(a,b,n) = 0 \\
&f(a,b,n) \neq 0 \\
&\therefore c^n \neq \left[a^n + b^n\right] \qquad (\text{QED})
\end{aligned}$$

Circles and the Pythagorean Triple

Note that $c^2 = \left[a^2 + b^2\right]$ is the equation of a circle, and is the foundation of the vector "metric" where c^2 is the resultant (squared hypotenuse) of the Pythagorean right triangle (c,a,b), $c = \sqrt{c^2}, a = \sqrt{a^2}, b = \sqrt{b^2}$ In the discussion below, the Pythagorean Triple $(5,4,3)$ will be used in our example, which will be valid for all such triples.

For the right triangle, the "metric" is defined as the relation:

$$\begin{aligned}
&c^2 = a^2 + b^2 \\
&5^2 = 4^2 + 3^2 = 25
\end{aligned}$$

Note that this relations violates the result of Fermat's Last Theorem and is the subject of much of the following discussion.

Prime Numbers

If the numbers a and b ($a \neq b$) are prime, the relations of addition and multiplication c an be expressed by matrices:

$$c = a + b = Tr\begin{vmatrix} a & 0 \\ 0 & b \end{vmatrix}, (a)\vec{i} \cdot (b)\overleftarrow{j} = \begin{vmatrix} a \\ 0 \end{vmatrix} \cdot \begin{vmatrix} 0 \\ b \end{vmatrix} = 0 \text{ where } a = \cos(0) = 1,\, b = \sin\left(\frac{\pi}{4}\right) = 1$$

Note that $Det\begin{vmatrix} a & 0 \\ 0 & b \end{vmatrix} = ab$, but is irrelevant since scalar multiplication $ab = (ab)\dfrac{ab}{ab} = (ab)(1_{ab})$ is not defined as prime.

Powers of c (c^n) are defined as $c^n = a^n + b^n = Tr\begin{vmatrix} a^n & 0 \\ 0 & b^n \end{vmatrix} = Tr\begin{vmatrix} a & 0 \\ 0 & b \end{vmatrix}^n$ where again scalar multiplication $(a^n b^n) = (ab)^n \dfrac{(ab)^n}{(ab)^n} = (ab)^n \left(1_{(ab)^n}\right)$ is not defined.

That is, interaction (multiplication product ab) between the two elements a and b is not defined. The term $a + b$ represents the existence of the two elements and $\# := c = a + b$ represents their sum (i.e., count) of the sets $\{a\} + \{b\}$.

Note that in general, $\#^n = c^n \neq a^n + b^n \leftrightarrow c \neq \sqrt[n]{a^n + b^n}$; for the case $n = 2$ equality appears to apply only for Pythagorean Triples, e.g. $5^2 = 4^2 + 3^2 = 25$ **but there are many triples of numbers that are not Pythagorean Triples**. This anomaly will be discussed in the following development.

The Interaction Equation

The “Interaction” expression is defined by

$$\begin{aligned} \# &:= c = a+b \\ \#^2 &= c^2 = (a+b)^2 = \left[a^2+b^2\right]+\left[2ab\right] \end{aligned}$$

where $\left[a^2+b^2\right]$ represents the existence of two interacting particles/fields and $\left[2ab\right]$ represents their multiplicative interaction. For the case $a=4,\ b=3$ the expression results in

$$7^2=\left(4+3\right)^2=\left[16+9\right]+\left[2(4)(3)\right]=\left[25\right]+2(12)=\left[25\right]+\left[24\right]=49$$

This relation can be expressed in matrices as:

$$\#^2 = Tr\begin{vmatrix} a^2 & 0 \\ 0 & b^2 \end{vmatrix} + Det\begin{vmatrix} a & a \\ -b & b \end{vmatrix} = \left[a^2+b^2\right]+\left[2ab\right]$$

Multiplying by π results in $\pi\left(\#^2\right)=\pi\left[a^2+b^2\right]+\pi\left[2ab\right]$. Note that the “existence” term $\left[a^2+b^2\right]=\left[\pi a^2+\pi b^2\right]$ is analogous to the sum of the areas of two circles while the interaction term $\pi\left[2ab\right]=a\left(2\pi b\right)=aC_b$ is analogous to a radius a times a circumference $C_b=2\pi b$.

Note that for $b=a$

$\#^2=c^2=(a+a)^2=\left[a^2+a^2\right]+\left[2a^2\right]$ but that this is not equivalent to the matrix

$$|a| := \begin{vmatrix} a & 0 & 0 & 0 \\ 0 & a & 0 & 0 \\ 0 & 0 & a & 0 \\ 0 & 0 & 0 & a \end{vmatrix},\ |a|^2 := \begin{vmatrix} a & 0 & 0 & 0 \\ 0 & a & 0 & 0 \\ 0 & 0 & a & 0 \\ 0 & 0 & 0 & a \end{vmatrix}^2 = \begin{vmatrix} a^2 & 0 & 0 & 0 \\ 0 & a^2 & 0 & 0 \\ 0 & 0 & a^2 & 0 \\ 0 & 0 & 0 & a^2 \end{vmatrix}$$ even though

$Tr|a|^2=a^2+a^2+a^2+a^2=4a^2$ which characterizes the existence of 4 non-interacting particles instead of two (equal) interacting particles:

$$\#=a+a$$

$$\#^2=\left[a^2+a^2\right]+\left[2a^2\right].$$

This suggests that the interaction term is related to binding the two existing particles together for $b>a$.

Complex Conjugation

Consider the expression:

$$i := \sqrt{-1}$$
$$\left(\sqrt{\psi}\right) := \sqrt{a} + i\sqrt{b}$$
$$\left(\sqrt{\psi}\right)^* := \sqrt{a} - i\sqrt{b}$$
$$\left(\sqrt{\psi}\right)\left(\sqrt{\psi^*}\right) = \left(\sqrt{\psi\psi^*}\right)^2 = a + \left[+\left(i\sqrt{b}\right) - \left(i\sqrt{b}\right)\right] - (i)^2 b$$

That is

$$\left(\sqrt{\psi}\right)\left(\sqrt{\psi^*}\right) = \left(\sqrt{\psi\psi^*}\right)^2 = a + \left[+\left(i\sqrt{b}\right) - \left(i\sqrt{b}\right)\right] - (i)^2 b$$
,
$$\psi\psi^* = a - \left(i^2\right)b = a + b$$

Then $(\psi\psi^*)^2 = (a+b)^2 = \left[a^2 + b^2\right] + [2ab]$, which is the Interaction equation above. Note that the "imaginary" number is irrelevant to the definition of addition in positive real numbers $c = a + b$..

Pythagorean Triples ($\left(5,4,3\right)$)

Let $a \geq b$ so that the difference $c = a - b \geq 0$

Consider the expression:

$$\psi := a + ib$$
$$\psi^* := a - ib$$
$$\psi\psi^* = a^2 + [iba - iab] - (ib)^2 = a^2 + b^2$$

Setting $a = 4$, $b = 3$ results in: $\psi\psi^* = a^2 + b^2 = 16 + 9 = 25$.

$\psi\psi^* = a^2 + b^2$ is the "resultant" of the vector relation $a^2 + b^2$, where $5 = \sqrt{\psi\psi^*}$ is the "hypotenuse" of the right triangle $\{5, 4, 3\}$

Note that the area of the triangle $A_\triangle = \frac{1}{2}ab$ is not defined.

Inserting in the interaction equation yields

$$\#^2 = 7^2 = [25] + [24] = [25] + [24] = [\psi\psi^*] + [24] = 49$$

That is, the complex conjugate defined as a relation between real and imaginary numbers becomes real in the context of the Interaction equation:

$$\#^2 = \left[a^2 + b^2\right] + \left[2ab\right] = \left[\psi\psi *\right] + \left[2ab\right] == \left[a^2 + b^2\right] + \left[2ab\right]$$