# The Binomial Expansion, The Relativistic Unit Circle, Quantum Spin, and Fermat's Theorem 

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## The Relativistic Unit Circle (Reference)

Mathematical Foundation (Reference)
The Pauli/Dirac Matrices (Reference - relationship of null vector)
Quantum Triviality (Wiki Article)
Consider the Binomial Expansion the case $n=2$ :
$c^{2}=(a+b)^{2}=a^{2}+b^{2}+2 a b$

Note that it has two parts, a "circle" part $R^{2}=a^{2}+b^{2}$ and a "rectangle" part $A=2 a b$.

One can think of $R^{2}$ as a "non-interacting" energy and $s^{2}$ as an interacting energy.
By interpreting this as the final state using the "Time Dilation" equation of relativity (via the relativistic unit circle), the "circle" part can be characterized as
$\psi^{2}=\gamma^{2}+\beta^{2}=1^{2}=\cos ^{2} \theta+\sin ^{2} \theta$

If we assume that the characterization is that of energy, then the "interaction" energy $h=2 a b$ is missing from the equation (it is internal to the circle); if parametrized, it is "created" and "destroyed" with each rotation of the real axis. The system thus has two dimensions, and is a description of a physical system with an initial state $\gamma$ and a "perturbation" state $\beta$

If there is no initial state, then $\gamma=0$ and $\beta$ is irrelevant (one can't perturb a non-existent system)

Then $\psi=0$ (in STR, this is the equivalent of saying that a non-interacting photon has no mass. This is the "Wow" of physics - observation of photons without "feeling" then' the "Wow" of physics. That is there is no "initial" condition except that we "observe" the world only through light which we cannot feel (unless we get a sunburn - the "Ow" of physics. This subjective interpretation also includes the assumption that the speed of light is instantaneous (one can't go "faster" than the speed of light because it would violate "causality") and that the "null" vector of light does not describe an equal and opposite interaction, but is merely a "simultaneous" observation assuming one can look in both directions at once.

If the photon has mass, then the photon state is completely described by $\psi^{2}=1^{2}$ over the area of the unit circle, but the initial state is "destroyed" $\gamma=0$ at $\theta= \pm \frac{\pi}{2}$ where $\beta= \pm 1$ and then "created" again at $\theta=0, \pm \pi$.

If one stresses that the interaction operates on the "imaginary" axis (pun intended), then one can express this relation as $1^{2}=(\gamma+i \beta)(\gamma-i \beta)=\gamma^{2}+\beta^{2}+i \beta \gamma-i \beta \gamma=\gamma^{2}+\beta^{2}$, so that $\psi=e^{i \beta}=\cos \beta+i \sin \beta=e^{i \theta}$, where (geometrically) $\beta=\theta$ for $\beta$ expressed in radians.

If the photon is assumed to exist but not interact, then its total energy is given by $\psi^{2}=1^{2}$ as the final condition, and also the initial condition (in relativity, $\left(c t^{\prime}=c t\right) \Leftrightarrow\left(t^{\prime}=t\right)$

If the energy of the unit circle is taken into account, but all four quadrants are taken to be positive definite, then the energy $\delta=\frac{1}{2} \gamma \beta$ is the energy for a given value of $\theta$ (i.e., $\theta$ is an invariant) in each quadrant of the relativistic unit circle; four quadrants $\delta=4\left(\frac{1}{2} \gamma \beta\right)=2 \gamma \beta$, which establishes the relation of the interaction energy to the Binomial Theorem, since now the total energy is $E_{\text {total }}=\left(\gamma^{2}+\beta^{2}\right)+2 \gamma \beta$, where the "circle" has no energy, since its total energy is described by $2 \gamma \beta$. In particular, note (again) that $A=2 \gamma \beta$ is the equation of a rectangle, not a circle. However, one can "imagine" an equivalent circle by specifying $h^{2}=2 \gamma \beta$ (where $h$ is Planck's constant for a photoequivalent electron, and $h^{2}=\left(m c^{2}\right)^{2}$ corresponds to the total relativistic energy of a photonequivalent physical system), so that $s^{2}=\gamma \beta=\frac{h^{2}}{2}$, where $s^{2}$ is the energy/area of an equivalent square, so that $s=\frac{h}{\sqrt{2}}$. Note that the distinction between the square and circle is irrelevant for $n=2$ for the "equivalent square", since one has $\pi s^{2}=\pi(\gamma \beta)=\pi\left(\frac{h^{2}}{2}\right)$.

For $n$ such interactions, then $s_{n}=\frac{n h}{\sqrt{2}}$
From a different perspective, if we assume $\gamma=1$ as an initial condition, then solution of the "Time Dilation" equation is $\left(\psi^{\prime}\right)^{2}=\left(\gamma^{\prime}\right)^{2}=1^{2}+(\gamma \beta)^{2}=1^{2}+1^{2} \beta^{2}$. For $\beta=1$, $\left(\gamma^{\prime}\right)^{2}=1^{2}+1^{2}=(\sqrt{2})^{2}=2(1)^{2}$, which specifies that $\left(\gamma^{\prime}=\gamma=1\right) \Leftrightarrow \beta=0$, in which case ( $n \gamma^{\prime}=n \gamma=n$ ) Therefore, if $\beta \neq 0, \gamma^{\prime}$ cannot be an integer.

If it is assumed that the system is characterized by two particles, $(\gamma=\beta=1)$, then the total system including the interaction is given by $\psi^{2}=\gamma^{2}+\beta^{2}+2 \gamma \beta$. If $\gamma$ is assumed to be invariant, then frpm $\psi^{2}=\gamma^{2}+\beta^{2}+2 \gamma \beta=\gamma^{2}+\delta_{\gamma \beta}$ and focusing attention on the interaction $\delta_{\gamma \beta}$ only; we have $\delta_{\gamma \beta}=\beta^{2}+2 \gamma \beta$, where the expression includes the "bare perturbation" $\beta^{2}$ as well as the interaction between $\gamma$ and $\beta$. This can also be characterized as the "spin" perturbation to $\gamma=1$ where $s_{\beta}=\delta_{\gamma \beta}=\beta^{2}+2 \beta, \gamma=1$ For $n \beta=\frac{n v}{c}=n,(v=c) \Leftrightarrow \gamma=1, \beta=1$, then $s_{\beta}=\delta_{\gamma \beta}=\left(\beta^{2}+2 \beta\right), \gamma=1$

This equation assumes that the spin is equal and opposite (i.e. a "null" vector) where $s_{\beta}(\overrightarrow{0})=\delta_{\gamma \beta}(\overrightarrow{0})=\left(\beta^{2}+2 \beta\right)(\overrightarrow{0})$ (i.e. $\beta=\sqrt{\beta^{2}}$ ) if parity is included, then there are two equations:
$\left(\delta_{\gamma \beta}{ }^{+}\right) \vec{r}=\left(\beta^{2} \vec{k}+\beta^{+} \vec{i}\right)$ and $\left(\delta_{\gamma \beta}{ }^{-}\right) \vec{r}=\left(\beta^{2} \vec{k}+\beta^{-} \vec{j}\right)$, where
$\left|\begin{array}{cc}\beta^{+} & 0 \\ 0 & \beta^{-}\end{array}\right| \rightarrow\left(\beta^{+}\right)\left(\beta^{-}\right)(\overrightarrow{0})=\beta^{2}(\overrightarrow{0}),\left|\beta^{+}\right|=\left|\beta^{-}\right|$
Then for each spin taken separately,
$n^{+} \beta_{n}^{+}=n^{+}, \beta_{n}^{+}=1, s_{\beta}^{+}=\left(n^{+}\right)^{2}+n^{+}=\left(n^{+}\right)\left(n^{+}+1\right)$
$n^{-} \beta_{n}^{-}=n^{-}, \beta_{n}^{-}=-1, s_{\beta}^{-}=\left(n^{-}\right)^{2}+n^{-}=\left(n^{-}\right)\left(n^{-}-1\right)$
If only the $1^{\text {st }}$ and the $4^{\text {th }}$ quadrants of the relativistic unit circle is relevant (so that "spin" is never destroyed; i.e. $\beta=\sqrt{ \pm \beta^{2}}$, then $2 s_{|\beta|}(\overrightarrow{0})=2[|n|(|n|+1)](\overrightarrow{0})$; i.e. $s_{|\beta|}(\overrightarrow{0})=[|n|(|n|+1)](\overrightarrow{0})$

But since $\gamma^{2}=1, \psi^{2}=\gamma^{2}+n(n+1)=1^{2}, n>0, \beta \geq 0$, so that $\psi^{2}=2 \gamma^{2}+n(n+1)$ where the spin is the interaction for a two particle system of "rest energy" $\gamma^{2}$, and $s_{n}=n(n+1)=\frac{n h^{2}}{\sqrt{2}}$. Note in particular that $s_{n}=0$ if $n=0$ or $h=0$, so that there is either no interaction $n=0$, or no interaction energy $h=0$ and that $\psi^{2}=1^{2} \Leftrightarrow s_{\gamma \beta}=0$.

It is clear that $2 \gamma \beta$ is the interaction "energy" of a physical system consisting of photon-equivalent particles:

1. One system "photon-equivalent particle": $\psi^{2}=1^{2}=\gamma^{2}+\beta^{2}$ (the relativistic unit circle $n=0, \gamma \beta<1, \gamma<1, \beta<1)$ where $\psi$ can be an integer only if $\beta=0, \gamma=1$ so that $n \psi^{2}=n\left(1^{2}\right)=\gamma^{2}$
2. two unperturbed unit circles (photon-equivalent particles)
$\left(\psi^{\prime}\right)^{2}=\left(\gamma^{\prime}\right)^{2}=1^{2}+1^{2}=2 \gamma, \beta=0, \psi^{\prime}=\gamma^{\prime}=\sqrt{2}$, so that $\psi^{\prime}$ cannot be an integer since $n \gamma^{\prime}=n \psi^{\prime} \sqrt{2} \quad$ is an integer only if $\psi^{\prime}=\frac{1}{\sqrt{2}}$
3. a single perturbed physical system + a perturbation where $\gamma=|\gamma|>0, \beta=|\beta|>0$, so that $\psi^{2}=\left(\gamma^{\prime}\right)^{2}=\gamma^{2}+\left(\beta^{2}+2 \gamma \beta\right)=\gamma^{2}+\left(s_{n}\right)^{2}$
a. $\psi^{2}=\gamma^{2}+n^{2}\left(\frac{h^{2}}{\sqrt{2}}\right)^{2}$
b. $\quad \psi^{2}=\gamma^{2}+(\sqrt{n(n+1)})^{2}$. Note that. $\psi_{n}=\lim _{n \rightarrow \infty}\left(n(n+1)=n^{2}\right.$. If $n$ is a radius at $\infty$, then $k=\frac{1}{n}=0$ is the curvature. But $n=0$ means there is no interaction, so $\psi^{2}=\gamma^{2}$, and the question is moot. That means that for any (positive definite) interaction $\left(\psi_{n}\right)^{2}=(\sqrt{n(n+1)})^{2}$ is not an integer, so $\psi^{2}=\gamma^{2}+(\sqrt{n(n+1)})^{2}=\gamma^{2}+\left(\psi_{n}\right)^{2}=1^{2}+\left(\psi_{n}\right)^{2}$ is not an integer.

This characterization can be expanded to include the proof of Fermat's Last Theorem from the Binomial Theorem, where $\left(\psi^{\prime}\right)^{n}=\left(\gamma^{\prime}\right)^{n}=(\gamma+\beta)^{n}=\gamma^{n}+\beta^{n}+\operatorname{rem}(\gamma, \beta, n)$
where $\left(\psi^{\prime}\right)^{n}=\left(\gamma^{\prime}\right)^{n}=(1)^{n}$ is an integer only if $\beta=0$, in which case $n\left(\psi^{\prime}\right)^{n}=n\left(\psi^{\prime}\right)^{n}=n\left(1^{2}\right)=n,\left(\psi^{\prime}\right)^{n}=(\psi)^{n}=\cos ^{n}(0)$ and $\operatorname{rem}(\gamma, \beta, n)=0$.

Since the relativistic unit circle applies to all independent real numbers $(\gamma, \beta)$ which have equal and opposite interactions (i.e., the common "origin" $(0,0)$ is defined, the vectors $\vec{\gamma}$ and $\vec{\beta}$ are connected at the origin. For the case $\mathrm{n}=2$ (Pythagorean triples), $\psi^{2}=\left(\gamma^{2}+\beta^{2}\right)=(\gamma+\beta)^{2}$ only by the prescription $\psi^{2}=(\gamma+i \beta)(\gamma-i \beta)=\left(\gamma^{2}+\beta^{2}\right)$, where in any case, $\psi$ cannot be an integer for the vector space.

In only one dimension $c \vec{i}=(a+b) \vec{i}$, both sides of the equation refer to the same unique coefficient, so that $c \vec{i}=c \vec{i}$ is only true if $c \gamma \vec{i}=c \gamma \vec{i}$ (there is no interaction, or the interaction is equal and opposite in terms of the null vector for two dimensions, where the zero point is the center of the positive and negative metric.

For a further discussion of the null vector in this context, see "Pauli/Dirac" matrices

