## Spin Quantization

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The Relativistic Unit Circle

## Spin Quantization (ct Invariant Final Condition)

$$
\begin{aligned}
& \left|\begin{array}{ll}
\gamma & 0 \\
0 & \gamma
\end{array}\right| \rightarrow \gamma^{2}(\ddot{0}) \\
& \left|\begin{array}{ll}
\beta & 0 \\
0 & \beta
\end{array}\right| \rightarrow \beta^{2}(\overrightarrow{0}) \\
& 1^{2}(\overrightarrow{0})=\left(\gamma^{2}+\beta^{2}\right)(\overrightarrow{0})=\left[\cos ^{2} \theta+\sin ^{2} \theta\right](\overrightarrow{0})
\end{aligned}
$$

$$
s_{1}=\left|\begin{array}{cc}
\gamma & 0 \\
0 & \beta
\end{array}\right|=\gamma \vec{i} \otimes \beta \vec{j}=\gamma \beta(\vec{i} \otimes \vec{j})=\gamma \beta \vec{k}
$$

$$
s_{3}=\left|\begin{array}{cc}
-\gamma & 0 \\
0 & -\beta
\end{array}\right|=(-\gamma) \vec{i} \otimes(-\beta) \vec{j}=\gamma \beta(\vec{i} \otimes \vec{j})=\gamma \beta \vec{k}
$$

$$
s_{2}=\left|\begin{array}{cc}
-\gamma & 0 \\
0 & \beta
\end{array}\right|=(-\gamma) \vec{i} \otimes(\beta) \vec{j}=-\gamma \beta(\vec{i} \otimes \vec{j})=\gamma \beta(-\vec{k})=\gamma \beta(\overleftarrow{k})
$$

$$
s_{4}=\left|\begin{array}{cc}
\gamma & 0 \\
0 & -\beta
\end{array}\right|=(\gamma) \vec{i} \otimes(-\beta) \vec{j}=-\gamma \beta(\vec{i} \otimes \vec{j})=\gamma \beta(-\vec{k})=\gamma \beta(\overleftarrow{k})
$$

$$
s_{1}+s_{3}=2 \gamma \beta \vec{k}
$$

$$
s_{2}+s_{4}=2 \gamma \beta(\overleftarrow{k})
$$

$$
\gamma=h=h_{\gamma}, \quad \beta=h=h_{\beta}
$$

$$
\left(s_{1}+s_{3}\right)+\left(s_{2}+s_{4}\right)=2 \gamma \beta \vec{k}+2 \gamma \beta(\overleftarrow{k})=2 \gamma \beta(\ddot{0})=2 h^{2}
$$

$$
s_{1}=s_{2}=s_{3}=s_{4}=s^{2}
$$

$$
4 s^{2}=2 h^{2}
$$

$$
s=\frac{h}{\sqrt{2}}
$$

Note that $1^{2}=\left(h_{\gamma}\right)^{2}+\left(h_{\beta}\right)^{2} \Leftrightarrow h_{\gamma}=1, h_{\beta}=0$.

## Quantization by $\mathbf{n}$ (ct' invariant Initial Condition)

$\left(\gamma^{\prime}\right)^{2}=1^{2}+(\gamma \beta)^{2}=\gamma\left(\gamma+\beta^{2}\right)$
$\gamma=n, \beta^{2}=1^{2}$
$\left(E_{n}^{\prime}\right)^{2}=\left(\gamma^{\prime}\right)^{2}=n\left(n+(1)^{2}\right)$
This shows that the effect of $\beta$ on the resultant energy $E_{n}{ }^{\prime}$ decreases with increasing $n$, so that $\lim _{n \rightarrow \infty}\left(E_{n}^{\prime}\right)=n^{2}$

The total energy of the single physical system as a photon consists of the energy of the relativistic unit circle characterized as $E_{\gamma}=\gamma^{2}$ plus the energy of the perturbation $E_{n}^{\prime}$ so that $E_{(\gamma+\beta)}=E_{\gamma}+E_{\gamma \beta}=E_{\gamma}+\lim _{n(n+1) \rightarrow n^{2}}=1^{2}+n_{\gamma \beta}{ }^{2}$ which can only be the integer $1^{2}$ if $\left(n_{\gamma \beta}\right)^{2}=0$ (I.e., there is only the single photon - there may be other photons in the Universe, which is, of course, a quibble... :)

## Consequences of the Binomial Theorem

From the Binomial Theorem, $\left(h_{\tau}\right)^{n}=\left[\left(h_{\gamma}\right)+\left(h_{\beta}\right)\right]^{n}=\left(h_{\gamma}\right)^{n}+\left(h_{\beta}\right)^{n}+\operatorname{rem}\left(h_{\gamma}, h_{\beta}, n\right)$.
If $h_{\gamma}=1$, then $\left(h_{\tau}\right)^{n}=\left(h_{\gamma}{ }^{n}\right)+\left(h_{\beta}{ }^{n}\right)+\operatorname{rem}\left(h_{\gamma}, h_{\beta}, n\right)$ cannot be an integer multiple of $\left(h_{\gamma}{ }^{n}\right)$ unless $\left(h_{\beta}{ }^{n}\right)=0$, so $\operatorname{rem}\left(h_{\gamma}, h_{\beta}, n\right)=0$ and $E_{n}=\left(n h_{\tau} v\right)^{n}=\left(n \frac{h_{\gamma} c}{\lambda}\right)^{n}=\left(\frac{n h_{\gamma}}{\tau}\right)^{n}=\left(\frac{n}{\tau}\right)^{n}=\left[\left(m_{0}\right)_{\gamma, \tau}\right]^{n} c^{2}$, where $\left[\left(m_{0}\right)_{\gamma, \tau}\right]=\frac{1}{\sqrt{\left(\varepsilon_{0}\right)_{\gamma, \tau}\left(\mu_{0}\right)_{\gamma, \tau}}}$ and $\left(\varepsilon_{0}\right)_{\gamma, \tau} \perp\left(\mu_{0}\right)_{\gamma, \tau}$ [i.e. $\left(m_{0}\right)_{\gamma, \tau}=\left(\frac{n}{\tau}\right), h_{\gamma}=1$

## Impulse Response

(Final State invariant)
Note that the area (i.e., "energy") of the relativistic unit circle is independent of the "parity" of the "momenta", since
$r^{2}=1^{2}=\left(\gamma^{2}+\beta^{2}\right)=(\gamma+i \beta)(\gamma-i \beta)$,
and
$r^{2}=( \pm \gamma)^{2}+( \pm \beta)^{2}=( \pm \gamma)^{2}+(\mp \beta)^{2}=(\mp \gamma)^{2}+( \pm \beta)^{2}$
(Initial State invariant)
This is not true of a system with "spin" however, since $\left(\gamma^{\prime}\right)^{2}=1^{2}+2 \gamma \beta$ so that either $\gamma$ or $\beta$ can be negative.

If $\gamma \beta>0$ That means that a system originally having energy of $E_{0}=1^{2}$ has been increased to $E^{\prime}=1^{2}+2 \gamma \beta=E_{0}+2 \gamma \beta$, corresponding to an absorption of $\Delta E=2 \gamma \beta$

If $\gamma \beta<0$ That means that a system originally having energy of $E_{0}=1^{2}$ has decreased to $E^{\prime}=\left(\gamma^{\prime}\right)^{2}=1^{2}-2 \gamma \beta=E_{0}-2 \gamma \beta$, corresponding to radiation of $\Delta E=2 \gamma \beta$

In both cases the sensor or source (absorbing or emitting $\Delta E$, respectively, is not specified.
For $\Delta E=2 \gamma \beta, h^{\prime}=\gamma \beta \Rightarrow \Delta E=2\left(h^{\prime}\right)^{2}$, so $h^{\prime}=\frac{\Delta E}{\sqrt{2}}$
Then for $E_{0}=h=1^{2}$, the total energy is given by $\left(E_{\text {total }}\right)^{2}=\left(E_{0}\right)^{2} \pm(\Delta E)^{2}$
Lorentz Transform

For the Lorentz transform, this corresponds to
$\gamma^{\prime}=c t^{\prime}=(c t) \gamma \pm\left(v t^{\prime}\right)=(c t) \gamma \pm\left(\beta c t^{\prime}\right) \gamma, \beta^{\prime}= \pm\left(\frac{v^{\prime}}{c}\right), \gamma=1$
$t^{\prime}=t \pm\left(\beta t^{\prime}\right)=t+\left( \pm \beta t^{\prime}\right)$
$\left(t^{\prime}\right)^{2}=(t)^{2}+\left( \pm \beta t^{\prime}\right)^{2},\left|\beta t^{\prime}\right|<|t|$

Suppose a system is characterized by the Binomial Expansion for the basis $\vec{\gamma}=\gamma \vec{i}, \gamma=1$ $\backslash$
$(\gamma+\beta)^{n}=(\gamma+\beta)^{2}(\gamma+\beta)^{n-2}=\left(\gamma^{2}+\beta^{2}\right)(\gamma+\beta)^{n-2}+2 \gamma \beta(\gamma+\beta)^{n-2}$
$(\gamma+\beta)^{n}-2 \gamma \beta(\gamma+\beta)^{n-2}=\left(\gamma^{2}+\beta^{2}\right)(\gamma+\beta)^{n-2}$
$(\gamma+\beta)^{n-2}\left(\gamma^{2}+\beta^{2}\right)=(\gamma+\beta)^{n-2}\left(1^{2}\right)=(\gamma+\beta)^{n}-2 \gamma \beta(\gamma+\beta)^{n-2}$
Therefore, the system $(\gamma+\beta)^{n-2}$ multiplied by the impulse $1^{2}=\left(\gamma^{2}+\beta^{2}\right)$ yields the system response $(\gamma+\beta)^{n}-2 \gamma \beta(\gamma+\beta)^{n-2}$; i.e, $(\gamma+\beta)^{n-2}\left(1^{2}\right)=(\gamma+\beta)^{n}-2 \gamma \beta(\gamma+\beta)^{n-2}$. This is possible only if $\beta=0$, so that $(\gamma+\beta)^{n-2}\left(1^{2}\right)=\left(\gamma^{n-2}\right)\left(\gamma^{2}\right)=\gamma^{n}$ (Ins) and $(\gamma+\beta)^{n}-2 \gamma \beta(\gamma+\beta)^{n-2}=\gamma^{n}$ (rhs)

$$
\begin{aligned}
& r^{2}=(\gamma+i \beta)(\gamma-i \beta)=( \pm \gamma)^{2}+( \pm \beta)^{2}=1^{2} \\
& =( \pm \gamma)^{2}+(\mp \beta)^{2}=(\mp \gamma)^{2}+( \pm \beta)^{2}
\end{aligned}
$$

Consider the equation
$(\gamma+\beta)^{n}=(\gamma+\beta)^{2}(\gamma+\beta)^{n-2}=\left(\gamma^{2}+\beta^{2}+2 \gamma \beta\right)(\gamma+\beta)^{n-2}=1^{2}+2 \gamma \beta(\gamma+\beta)^{n-2}$ so that
$1^{2}=(\gamma+\beta)^{n}-2 \gamma \beta(\gamma+\beta)^{n-2}$
Then $1^{2}$ is the impulse for any linear system $\Psi(\gamma, \beta, n)$ where the system response is $\varphi(\gamma, \beta, n)$; that is $[\Psi(\gamma, \beta, n)]\left[(\gamma+\beta)^{n}-2 \gamma \beta(\gamma+\beta)^{n-2}\right]=\varphi(\gamma, \beta, n)$

Suppose a system is given by the Binomial Expansion for the basis $\vec{\gamma}=\gamma \vec{i}, \gamma=1$ $\psi(\gamma, \beta, n)=\psi(1, \beta, n)=(\gamma+\beta)^{n}=(1+\beta)^{n}$

This can only be true if $[\Psi(\gamma, \beta, n)]=1^{n-2}$ so that $\varphi(\gamma, \beta, n)=1^{n}=\left(\gamma^{2}+\beta^{2}\right)^{n}$, that is, $2 \gamma \beta=0$, which means that $\gamma=0, \beta=0$, or $\gamma \vec{i} \cdot \beta \vec{j}=\gamma \beta(\vec{i} \cdot \vec{j})= \pm \sin \theta= \pm 0$ and therefore $\theta=0$ or $\theta=\pi$ in the relativistic unit circle ( $\beta= \pm 0$ ).
$\left|\begin{array}{cc}\beta & 0 \\ 0 & \beta\end{array}\right|=\left|\begin{array}{cc} \pm 0 & 0 \\ 0 & \pm 0\end{array}\right| \rightarrow( \pm 0)^{2}(\overrightarrow{0})$ so the $\vec{j}$ dimension does not exist (it has no metric length).

## Vector spaces

A vector space is formed for each Lorentz rotation of the relativistic unit circle where either $m_{0}=c t=1$ (no perturbation $v t^{\prime}=0$ ) or $m_{0}{ }^{\prime}=c t^{\prime}=1$ is taken as the final state, and constitutes a dimension, where the dimension of the system is $n=n(c t)$ or $n=n\left(c t^{\prime}\right)$

That gives a set of positive integers $1^{2}, 2\left(1^{2}\right), 3\left(1^{2}\right), \ldots ., n\left(1^{2}\right)$; if no integers are created at all, then $c t=0$ or $c t^{\prime}=0$. Within the relativistic unit circle, equal and opposite integer creation rates and times yield the null vector for that dimension, where
$\left|\begin{array}{cc}c t & 0 \\ 0 & c t\end{array}\right| \rightarrow( \pm c t) \otimes( \pm c t) \rightarrow(c t)^{2}(\overline{0})$ or $\left|\begin{array}{cc}c t^{\prime} & 0 \\ 0 & c t^{\prime}\end{array}\right| \rightarrow\left( \pm c t^{\prime}\right) \otimes\left( \pm c t^{\prime}\right) \rightarrow\left(c t^{\prime}\right)^{2}(\overline{0})$
This forms the basis set $I_{n}=\left|\begin{array}{cccccc}1^{2} & 0 & 0 & 0 & 0 & 0 \\ 0 & 1^{2} & 0 & 0 & 0 & 0 \\ 0 & 0 & 1^{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & 1^{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & \ldots . . & 0 \\ 0 & 0 & 0 & 0 & 0 & 1^{2}\end{array}\right|$, where $\operatorname{Tr}\left(I_{n}\right)=n\left(1^{2}\right)$ and
$\operatorname{det}\left(I_{n}\right)=\left(1^{2}\right)^{n}$. Each dimension represented by a basis $1^{2}$ can be multiplied by a metric length $x$, where the scalar $x=0$ is taken to be the midpoint of all possible metric lengths in that dimension. Each dimension is then represented by the symbols $\vec{i}, \vec{j}, \vec{k}, \ldots$, so that a vector is represented by $\vec{x}=x \vec{i}=x=0\left(1^{2}\right) \vec{i}$ in that dimension. For $x=0$, there is no metric length associated with the dimension, so that if a common metric for all dimensions is assumed, the trace of the system is $n=0$. Setting $x=0$ for a single dimension removes that dimension from the system (a tensor "contraction), so the dimension is then $n-1$.

This results when the "dot" product of two basis vectors are taken $\vec{i} \cdot \vec{j}=0$. Conversely, an additional dimension is added when the "cross" product of two basis vectors is taken: $\vec{i} \otimes \vec{j}=\vec{k}, \vec{j} \otimes \vec{i}=(-\vec{k})=(\bar{k})$, where the reverse arrow indicates an equal and opposite orientation of the $( \pm k)=(\vec{k})$ vectors. Adding these vectors yields the "null" vector representing an equal and opposite "force" in physics):
$(\vec{i} \otimes \vec{j})+(\vec{j} \otimes \vec{i})=\vec{k}+\overleftarrow{k}=(\overline{0})$, where a scalar multiple of the null vector gives the magnitude of the equal and opposite "force" $\vec{x}=x(\overrightarrow{0})$. If $x=0$, the magnitude of this force is equal to zero, so that $\overrightarrow{0}=0(\overrightarrow{0})$. Which is a contraction of the system; if such a vector is retained in the system, then the determinant is zero for the whole system.

If interactions $\overrightarrow{\Delta v t^{\prime}}=\left(v t^{\prime}\right) \vec{s}$ are included in the system for the metrics represented by (1) $(\overrightarrow{c t})=(1)\left(1^{2}\right) \vec{i}$, then if it is independent of the dimension, it is characterized by a second dimension $\vec{s}$ so that $\vec{c}=c(\vec{r})=c(\vec{i} \otimes \vec{s})=c\left(\overrightarrow{k_{s}}\right)$ where $\vec{r}=\overrightarrow{k_{s}}$ is not a member of the original system, since it is not a basis vector unless explicitly included; it is therefore orthogonal to the hyperplane of the original system.

If the three vectors $\overrightarrow{(c t)}, \overrightarrow{\left(c t^{\prime}\right)}$, and $\overrightarrow{\left(v t^{\prime}\right)}$ have no common metric, then they are "affine" i.e., their individual metrics have no common origin, or unrelated (in the case of integers, these vectors form a Pythagorean triple $(\vec{a}, \vec{b}, \vec{c})$ with no common origin, and hence no common metric; in particular, there is no common null vector for the vector space.

If they have a common origin (so that the midpoints of all possible lengths in each dimension coincide, then the vector relation is given by $\left(c t^{\prime}\right) \vec{r}=\left(v t^{\prime}\right) \vec{s}+(c t) \vec{i}$. If it is understood that $\vec{r}=\vec{s}$ and $\vec{k}$ represents vectors not in the hyper plane given by $\left(\overrightarrow{i_{n}}, \overrightarrow{j_{n}}\right)$ the relation can then be characterized by the relation $\left(c t^{\prime}\right) \vec{r}=\left(v t^{\prime}\right) \vec{j}+(c t) \vec{i}$ This relation can also be characterized as an initial condition $\overrightarrow{c t}$, a "perturbation" $\overline{\left(v t^{\prime}\right)}$ and a final condition $\overrightarrow{\left(c t^{\prime}\right)}$, where $\left(v t^{\prime}\right)=0$ means that $\overrightarrow{c t^{\prime}}=\overrightarrow{c t}$ so that $t^{\prime}=t$ for the common "metric creation rate" $c$.

## (ct') as Invariant Final Condition

The relativistic unit circle is then derived by considering the two "inputs" (ct) and ( $v t^{\prime}$ ) in terms of the final "output" $\left(c t^{\prime}\right)$, so that dividing by the scalar $\left(c t^{\prime}\right)$ yields the relation (1) $\vec{r}=\left(\frac{v}{c}\right) \vec{j}+\left(\frac{t}{t^{\prime}}\right) \vec{i}=\beta \vec{i}+\gamma \vec{j}$. The magnitude of the vector $\vec{r}$ is given by the relation $1^{2}=r^{2}=\gamma^{2}+\beta^{2}$ so that $1=\sqrt{\gamma^{2}+\beta^{2}}$
For Fermat's Theorem that $n=n\left(\sqrt{\gamma^{2}+\beta^{2}}\right)$ cannot be an integer unless either $\gamma^{2}=0$ or $\beta^{2}=0$.

Since the metric relation given by the scaling factors $t$ and $t^{\prime}$, the relation $1^{2}=r^{2}=\gamma^{2}+\beta^{2}$ includes all possible relations in the vector space defined by $\vec{r}$.

Therefore, consider the relation $\left(r^{\prime}\right)^{2}=(\gamma+\beta)^{2}=\gamma^{2}+\beta^{2}+2 \gamma \beta=1^{2}+2 \gamma \beta$ For the relativistic unit circle, $\gamma \beta$ must be equal to zero, so that $\gamma \cdot \beta=0$, which means that $\beta \perp \gamma$ ( $\gamma$ and $\beta$ are orthogonal, so $\gamma$ is independent of any value of $\beta$, and vice versa. This also means
that the interaction $\gamma \beta$ has no common metric, so its interaction "energy" is "virtual", or imaginary. It thus can be removed from $(\gamma+\beta)^{2}$ by the prescription $r 2=\left(\gamma^{2}+\beta^{2}\right)=(\gamma+i \beta)(\gamma-i \beta)$, where $i=\sqrt{-1}$

## (ct) as Invariant Initial Condition

If, however, the equation $\left(c t^{\prime}\right) \overrightarrow{r^{\prime}}=\left(v t^{\prime}\right) \vec{s}+(c t) \vec{i}$ is characterized by the "initial" condition $c t$ , then $\left(\frac{t^{\prime}}{t}\right) \vec{r}=\left(\frac{v}{c} \frac{t^{\prime}}{t}\right) \vec{s}+(1) \vec{i}$, so that $\left(\frac{t^{\prime}}{t}\right)^{2}=\left(\frac{v}{c} \frac{t^{\prime}}{t}\right)^{2}+\left(1^{2}\right)=(\beta \gamma)^{2}+\left(1^{2}\right)$ In terms of the state as the final condition, this can only be valid if $v=0$ so that $t^{\prime}=t$. However, if a second "interactive" metric is applied so that $\left(1^{\prime}\right)=\left(\gamma^{\prime}\right)^{2}=(\gamma \beta)+1^{2}$, then this characterizes a new "basis" for the dimension characterized by $\vec{r}$. For any integer n , if $n^{2}=\gamma \beta$ so that $\gamma=1, \beta=n$, then the new basis is given by $\left(1^{\prime}\right)^{2}=\left(\gamma^{\prime}\right)^{2}=(\gamma \beta)+1^{2}$, so
$\left(1^{\prime}\right)^{2}=\left(\gamma^{\prime}\right)^{2}=(n \gamma)+1^{2}=(n \gamma)+\gamma^{2}=\gamma(n+1) \quad$ Finally, if $\gamma=n$, then $\left(1^{\prime}\right)^{2}=\left(\gamma^{\prime}\right)^{2}=S=n(n+1)$ . This new basis is then characterized as "spin", consisting of the energy of the relativistic unit circle plus the interaction energy $\gamma \beta$ for "momentum" $\beta=n$, so that $S^{2}=[n(n+1)]^{2}$.

Note that for $\beta=1, S^{2}=2=2\left(\gamma^{2}\right)$, so $S=\sqrt{2} \gamma$ This corresponds to the equal and opposite additional interaction between $\pm S$, characterized by $\left|\begin{array}{ll}S & 0 \\ 0 & S\end{array}\right| \rightarrow S^{2}(\overrightarrow{0})$

If $\gamma$ has the quantized energy $h$,so that $\gamma=h=1$ then for a single value of interaction energy, $S=\left(1^{\prime}\right)=h \sqrt{2}$, so that $\gamma^{\prime}=\frac{h}{\sqrt{2}}$ For $n$ such spins, $n S=n \gamma^{\prime}=\frac{n h}{\sqrt{2}}$ in the dimension $\vec{r}=\vec{k}$, where $\overrightarrow{c^{\prime}}=c^{2} \overrightarrow{r^{\prime}}=\left(\gamma^{2}+\beta^{2}\right)+\frac{n}{\sqrt{2}}=1^{2}+\frac{n}{\sqrt{2}}$
(For Fermat's Theorem, this means that $c^{2}$ cannot be an integer unless $n=0$ (i.e., $\beta=0$ for $\gamma=1$ ), so $c=\sqrt{c^{2}}$ cannot be an integer.

If c cannot be an integer for the case $n=2$ for either an invariant initial condition or invariant final condition (as an integer), then it can only be an integer if $v t^{\prime}=0$; i.e. $\beta=0$ where $c t$ and $c t^{\prime}$ have a common metric. (Again, if there is no common metric, then Pythagorean triples indicate that the vectors are affine, and the iondependent triples result from the base of the number systems as counted in either dimension - but not both).

Then $c^{n}=(\gamma+\beta)^{n}=\gamma^{n}+\beta^{n}+\operatorname{Re} m(\gamma, \beta, n)=(\gamma+\beta)^{2}(\gamma+\beta)^{(n-2)}$ (where $\operatorname{Re} m(a, b, n)$ consiss of all terms in the Binomial Expansion that are not $\gamma^{n}$ or $\beta^{n}$ ) cannot be an integer unless either $\gamma=0$ or $\beta=0$, since all terms in $\operatorname{Rem}(\gamma, \beta, n)$ must be positive for $\gamma>0$ and $\beta>0$

In addition, note that $(\gamma+\beta)^{2}(\gamma+\beta)^{(n-2)}=\gamma^{n}+\beta^{n}+2 \gamma \beta \operatorname{Re} m(\gamma, \beta, n-2)$, which shows the relation of Spin and the elements of relativistic (photon or photo-electron equivalent mass) interaction (in powers of $n$ ) to the Binomial Theorem...

Taking the cross product of two dimensions

If the scalar 0 is taken to be the midpoint of all possible metric lengths, then each of these basis vectors can then be

From the relativistic unit circle (in two dimensions), for $c t^{\prime}$ selected as the final state ((which is equivalent to the axiom of choice), the $t^{\prime}=t \gamma, \gamma=\frac{1}{\sqrt{1-\beta^{2}}}$ yields $1^{2}=\gamma^{2}+\beta^{2}$, so that $1=\sqrt{\gamma^{2}+\beta^{2}}$ where both $\gamma<1$ and $\beta<1$ for the circle, so that $c=c \sqrt{\gamma^{2}+\beta^{2}}$ can only be an integer if either $\gamma=0$ or $\beta=0$ (i.e., the system is one dimensional).
(The analysis also applies to Cartesian coordinates for $\gamma$ and $\beta$ characterized as lengths (i.e., metrics of real number lines).

For two integers $(a, b)$, the relation for the Binomial Theorem for $n=2$ is given by ( $a t, b t^{\prime}$ ), so that if $a$ is considered the "reference" (the smallest integer (which amounts to the axiom of choice i.e., as the initial condition), the relation is
$t^{\prime}=t \gamma, \gamma=\frac{1}{\sqrt{1-\beta^{2}}}, \beta=\frac{b}{a} \quad$ If $c$ is chosen as the final condition, then $\left(c t^{\prime}\right)^{2}=\left(b t^{\prime}\right)^{2}+(a t)^{2}$

Consider $\gamma=c t=c t^{\prime}=1$ to be the initial condition, so $t=t^{\prime}$, so $\gamma=1 c t=c t^{\prime}, \gamma=1$. Then $\left(\gamma^{\prime}\right)^{2}=\gamma^{2}+\gamma \beta=1^{2}+(1) \beta, \beta=\frac{v}{c} \quad$ (A second dimension is being created for $0 \leq \beta<1$, independent of $\gamma$ ) When $v$ increases to $v=c,\left(\gamma^{\prime}\right)^{2}=1^{2}+1^{2}=2\left(1^{2}\right)$, so that $\gamma^{\prime}=\sqrt{2}$, where $I=\left|\begin{array}{ll}1 & 0 \\ 0 & 1\end{array}\right| \rightarrow 1^{2}(\overline{0})$ and $\operatorname{Tr}(I)=2$ and $\operatorname{det}(I)=1^{2}$. If $\frac{\gamma^{\prime}}{\sqrt{2}}=1^{\prime}$ is established as the new "momentum" basis, then we can define $s=\frac{h}{\sqrt{2}}=\frac{\gamma^{\prime}}{\sqrt{2}}$ as

If $E_{0}=\gamma^{2}$ represents "energy" for a single "photon" and $P_{0}=\gamma$ represents "momentum", then the prescription for "spin" is represented by $s=\gamma^{2}+\gamma=\gamma(\gamma+1)$, For $\gamma=n$ ("momenta"), then $s=n(n+1)$ However, classical "momentum" is dimensionally inconsistent with classical energy.

For relativistic momentum, however $P=\left(m_{0} \gamma\right) \beta c=\left(m_{0} \gamma\right)\left(\frac{v}{c}\right) c=m_{0} c(\beta \gamma)=m_{0} c, \beta \gamma=1$ $\mathrm{P}_{(\text {rel })}=m_{0} c, \beta \gamma=1$ (Note that for $m_{0}=c t_{c}, \mathrm{P}_{(\text {rel })}=c^{2} t_{c}=c^{2}, t_{c}=1$ From Maxwell's result, $c^{2}=\frac{1}{\varepsilon_{0} \mu_{0}}$ where the permittivity $\varepsilon_{0}$ and the permeability $\mu_{0}$ are force coupling constants from Coulombs' and Ampere's laws, respectively. This means that $\mathrm{P}_{(\text {rel) }}$ is a relativistic energy, not a classical momentum.

Therefore, if $E_{0}$ is orthogonal to $P_{\text {rel }}$ in the relation $\left(E_{0}, P_{\text {rel }}\right)$, so that $\left(E^{\prime}\right)^{2}=\left(P_{\text {classical }}\right)^{2}+\left(E_{0}\right)^{2}$, the relation can be made dimensionally consistent by including $c=1$ to satisfy the relativistic momentumenergy relationship so that $\left(E^{\prime}\right)^{2}=\left(P_{0} c\right)^{2}+\left(E_{0}\right)^{2}$

Then for $\mathrm{c}=1$, we can set $\gamma=\gamma c$ so that $s=\gamma^{2}+\gamma=\gamma(\gamma+1)$ and for $\gamma c=n, s=n^{2}+n=n(n+1)$
Then $h=E_{0} t_{c}, E_{0}=\frac{h}{t_{c}}=\frac{h c}{\lambda_{c}}, \lambda_{c}=c t_{c}$ so that if $c t_{c}=1, \lambda_{c}=1$

Then the "spin" is an additional energy to the relativistic unit circle (which is a model of two independent particles) and characterizes the interaction between them in the new basis $\left(1^{\prime}\right)=\frac{\gamma^{\prime}}{\sqrt{2}}$ where $\gamma^{\prime} c=h$ and $\gamma^{\prime}=h, c=1$
$\frac{\gamma^{\prime}}{\sqrt{2}}=1^{\prime}$, where Planck's constant can be identified with $\gamma^{\prime}=\sqrt{h}$, so that $\left(1^{\prime}\right) 2=\frac{h}{\sqrt{2}}$.

Spin
$1_{0}{ }^{2}=\left(\gamma_{0}{ }^{2}+\beta_{0}{ }^{2}\right), \gamma<1_{0}, \beta<1_{0}$
$2=\left(\gamma_{1}^{2}+\beta_{1}^{2}\right), \gamma_{1}=\beta_{1}=1_{1}$
$\sqrt{\left(\gamma_{1}^{2}+\beta_{1}^{2}\right)}=\sqrt{2}$
$\gamma_{1} \beta_{1}=1_{1}{ }^{2}$
$s=\frac{\gamma_{1} \beta_{1}}{\sqrt{\left(\gamma_{1}{ }^{2}+\beta_{1}{ }^{2}\right)}}=\frac{1_{1}{ }^{2}}{\sqrt{2}}$
$h=1_{1}{ }^{2}$
$s=\frac{h}{\sqrt{2}}$
$(\gamma+\beta)^{2}=(\gamma)^{2}+(\beta)^{2}+2 \gamma \beta$
$h=2 \gamma \beta$
$\gamma=1$
$(\gamma+\beta)^{2}=(1)^{2}+(\beta)^{2}+2 \beta(1)$
$\beta=1$
$(\gamma+\beta)^{2}=2\left(1^{2}\right)+2\left(1^{2}\right)=4\left(1^{2}\right)=4 \gamma \beta$
$\sqrt{(\gamma+\beta)^{2}}(1)=(\gamma+\beta)(1)=\sqrt{2} \gamma \beta(1)=\sqrt{2}(1)$
$(\gamma+\beta)^{2}=(\gamma)^{2}+(\beta)^{2}+2 \gamma \beta$
$h^{2}=2(\gamma \beta)^{2}, h=\sqrt{2} \gamma \beta$
$s=\gamma \beta=\frac{h}{\sqrt{2}}$
$h^{2}=2 s$
$(\gamma+\beta)^{2}=1^{2}+2 \gamma \beta=1^{2}+2\left(\frac{h}{\sqrt{2}}\right)=1^{2}+2 s$

$$
\begin{aligned}
& \left|\begin{array}{cc}
\sqrt{s} & 0 \\
0 & \sqrt{s}
\end{array}\right| \rightarrow s(\overline{0}) \\
& \left|\begin{array}{cc}
-\sqrt{s} & 0 \\
0 & -\sqrt{s}
\end{array}\right| \rightarrow s(\overline{0}) \\
& \left|\begin{array}{cc}
\sqrt{s} & 0 \\
0 & \sqrt{s}
\end{array}\right|+\left|\begin{array}{cc}
-\sqrt{s} & 0 \\
0 & -\sqrt{s}
\end{array}\right| \rightarrow 2 s(\overline{0}) \\
& (\gamma+\beta)^{2}(\overline{0})=\left[1^{2}+2 \gamma \beta\right](\overline{0})=1\left[{ }^{2}+2\left(\frac{h}{\sqrt{2}}\right)\right](\overline{0})=\left[1^{2}+2 s\right](\overline{0}) \\
& (\gamma+\beta)^{2}=1^{2}, s=0
\end{aligned}
$$

For equal and opposite interactions where $\gamma_{1}=\beta_{1}=1$ we have $\gamma_{1}{ }^{2}+\beta_{1}{ }^{2}=2\left(1_{0}{ }^{2}\right)=\left(1_{1}\right)^{2}$ so that $\sqrt{2}(1)=\left(\sqrt{\gamma^{2}+\beta^{2}}\right)(1)$ which represents an increase in the radius of the relativistic unit circle from the case $1_{0}{ }^{2}=\left(\gamma_{0}{ }^{2}+\beta_{0}{ }^{2}\right), \gamma<1_{0}, \beta<1_{0}$. However, $\gamma \beta=\left(1_{1}\right)^{2}$, which now represents an increase in the radius of the relativistic unit circle to a final condition with increased energy due to the interaction (equal and opposite "rotation" in a quadrant), so that $E_{\gamma \beta}=\sqrt{2} \gamma \beta=\sqrt{2}\left(1^{2}\right)$. If we identify Planck's constant with the (equal and opposite) "rotation" energy $E_{\gamma \beta}$ so that $h=E_{\gamma \beta}$, then the quantized "spin" interaction energy can be represented as $s=1^{2}=\gamma \beta=\frac{h}{\sqrt{2}}$

## Division by zero

Addition (groups) only require one dimension; $\vec{c}=c \vec{i}=(a \pm b) \vec{i}$, where the sum on the right refers to the single integer on the left side of the equality, but multiplication and its inverse (division (rings)) require two dimensions. Consider the equation $z_{(x, y)}=\frac{x}{y}, x=y z_{(x, y)}$ If $y=0, x=y z_{(x, y)}$ means that x and y can still exist, but their relation is undefined, unless $y \vec{i}=x \vec{i}$ in one dimension, so that $x \vec{i}=0 \vec{i}$, and therefore $x=0$, which is a point (the midpoint of all possible metrics (lengths) in one dimension).

If $y \neq 0$, then $A=\frac{x}{y}$ is the slope of the line $y=A x+b$ in two (Cartesian) dimensions.
$\left|\begin{array}{ll}x & 0 \\ 0 & \frac{1}{y}\end{array}\right| \rightarrow\left(\frac{x}{y}\right) \overline{0}=A(\overline{0})$
Likewise, in the equation for radial coordinates $s=r \theta$ then $r=\frac{s}{\theta}$; for $\theta=0$ the arc length is undefined unless $s \vec{i}=r \vec{i}$ in one dimension. Also for $\theta=\frac{s}{r}$, for $r=0$, then $\theta \bar{i}=s \vec{i}$ in one dimension. $\left|\begin{array}{ll}r & 0 \\ 0 & \theta\end{array}\right| \rightarrow(r \theta)(\overline{0})=s(\overline{0})$

Using the Einstein summation convention, we have:
$S^{m}{ }_{i}=\left(\gamma_{i}+\beta_{i}\right)^{m}=\gamma_{i}^{m}+\beta_{i}^{m}+\operatorname{Re} m\left(\gamma_{i}, \beta_{i}\right)$, so that
$E_{0}(0)=\left(S^{m}{ }_{i}\right)^{2}(0)=\left[\left(\gamma_{i}+\beta_{i}\right)^{m}=\gamma_{i}^{m}+\beta_{i}^{m}+\operatorname{Re} m\left(\gamma_{i}, \beta_{i}\right)\right]^{2}(0)$
$E_{0}=\left[\gamma_{i}^{m}+\beta_{i}^{m}+\operatorname{Re} m\left(\gamma_{i}, \beta_{i}\right)\right]^{2}=\gamma_{i}^{2 m}+\beta_{i}^{2 m}+\operatorname{Re} m\left(\gamma_{i}, \beta_{i}, m, 2\right)$, where $\operatorname{Re} m\left(\gamma_{i}, \beta_{i}, m, 2\right.$ consists of all elements in the expansion that are not either $\gamma_{i}^{2 m}$ or $\beta_{i}^{2 m}$

## Relation to "space-time"

If we define $x_{c}=c t_{c}$ and $x_{v}=v t_{v}$ in the (Cartesian) "space-time" domain, then we have the relation $\frac{x_{v}}{x_{c}}=\frac{v t_{v}}{c t_{c}}=\beta_{(x, t)} \frac{t_{v}}{t_{c}}$. If $\beta_{(x, t)}$ is to be identified with $\beta_{(v, c)}$ in the energy- momentum domain then we must have $\frac{t_{v}}{t_{c}}=\gamma$, so $\frac{x_{v}}{x_{c}}=\frac{v t_{v}}{c t_{c}}=\beta_{(x, t)} \gamma$. But $\frac{t_{v}}{t_{c}}=\frac{1}{\sqrt{1-\beta^{2}}}$, which contradicts the linear relation in Cartesian space-time for $\frac{x_{v}}{x_{c}}=\frac{v t_{v}}{c t_{c}}=\beta_{(x, t)} \frac{t_{v}}{t_{c}}=\beta_{(x, t) \gamma} \gamma$.

This can be remedied in one of two ways. If we set $x_{v}=x_{c}$, we have the relation $1=\beta_{(x, t)} \gamma=\frac{v}{c} \gamma$ so that $v=\frac{c}{\gamma}$ Then $c=v \gamma$

## The "Wow" and "Ow" of observation

For $v<c$ consider the end result $1^{2}$ as the result of a single observations at an instant of time (or all possible observations (anywhere, anywhen), since $v$ and $c$ are independent of any particular position or time interval $(x, t)$, as are the scaling factors $t$ and $t^{\prime}$

One can then identify $\gamma$ as the "Wow" of physics - the observation at the zero-point level (i.e., at the lowest possible observable temperature in the parking lot), and $\beta$ with the "Ow" of physics - any sense perception not involving light (i.e. sunburn, gravity, ponderable matter, etc.) so that the total of a single observation consists of $1^{2}=\beta^{2}+\gamma^{2}=(\text { "Ow" })^{2}+(\text { "Wow" })^{2}$

Consider the expansion $(\gamma+\beta)^{2}=\left(\gamma^{2}+\beta^{2}\right)+2 \gamma \beta$ Then the "Interaction" term $2 \gamma \beta$ must be something that was missed in the observation (a "virtual" thought (virtual "wow") imagining an "Felt" event (a virtual "Ow") .

This term is then responsible for "Dark Energy", "Dark Matter", or "god", depending on your religious inclination - that is, it is an effect with an unseen cause that can be accounted for by either the "Wow" or "Ow" of physics. (This is the distinction between the local constant " g " of gravity that one feels when missing a step on a stairway vs. the imagined global constant $G$ (e.g. from the Solar System observations of the planets, or the concepts of "parallel Universes" in science (fantasy).

For "red shift" is a change in energy in an unobserved source (or a loss in energy by an unseen intervening density) from a far-away galaxy (or an act of god). For "Einstein rings", it is the bending of light due to an unseen central mass (or individual photons along an imagined geodesic - or an act of god).

In either case, it amounts to an imagined change in h in quantum mechanics, or an imagined change in the zero-point level (i.e., temperature) in the parking lot.
(Einstein Rings):
If we "imagine" we know the "distance" (the "energy of unperturbed observation)) to the center the ring as $r=\gamma, r^{2}=\gamma^{2}$ and we know the "force" that moves the galaxy off center as $\beta$, then $2 \gamma \beta$ is the imaginary energy required to move the galaxy as observed on our focal plane:

$$
(\gamma+\beta)^{2}=\gamma^{2}+\beta^{2}+2 \gamma \beta
$$

Note that the galaxies are distorted around the circle because of a real energy associate with a Lorentz rotation.

(Red Shift)
In this case, $\gamma=h$ in an imaginary parking lot far away and $\beta=i h^{\prime}$ is the energy lost to the imaginary cosmological background noise that interacts with the actual incoming photon as detected at our present observation site in our own parking lot as an observation.

Then the imagined energy of the source and the interaction will be given by the area of the triangle $E=\frac{1}{2} \gamma \beta$, but since all four quadrants of the relativistic circle must be taken into account, then the total energy of observation must be $E=4\left(\frac{1}{2} \gamma \beta\right)=2 \gamma \beta$, so is "virtual".

$1^{2}=h^{2}+\left(i h^{\prime}\right)^{2}+2 h\left(i h^{\prime}\right)=h^{2}-\left(h^{\prime}\right)^{2}+2 h\left(i h^{\prime}\right)$

## The Derivative

The analysis above (where a spin perturbation is characterizes as $s_{n}=n(n+1)$ suggests a new perspective on the derivative, since the precise limit of
$f^{\prime}\left(\gamma^{2}\right)=\left[\lim _{\beta \rightarrow 0} \frac{f(\gamma+\beta)-f(\gamma)}{\beta}\right]^{2}(\overrightarrow{0})=\left[\lim _{h \rightarrow 0} \frac{f(\gamma+h)-f(\gamma)}{h}\right]^{2}=f^{\prime}\left(1^{2}\right)$
And
$\left[\lim _{n(n+1) \rightarrow n^{2}} \frac{f(n(n+1))-f\left(n^{2}\right)}{n^{2}}\right]^{2}=\left(1^{2}\right)$

## The Interaction of Light with Matter

The metric of $s^{2}=(\overrightarrow{0})$ applies to all possible perturbations of the rest mass defined by $m_{\beta}=\left(v t^{\prime}\right)_{\beta}$ which is the "rest mass" of the perturbation to a given rest mass $m_{0}=c t$ which was identified as that of an unperturbed photon.

If we introduce a second "rest" photon (as either a photon with a different rest mass, or a ponderable mass, like a locomotive), we can use the same analysis to derive the "Quantized Angular Momentum"
$\overleftrightarrow{L_{m}}=\left|L_{m}\right|(\ddot{0})=\left|s_{m}\right|(\ddot{0})=\frac{h_{m}}{\sqrt{2}}(\ddot{0})$, where $h_{m}=c_{m} t_{m}$ can be identified with the DeBroglie "period" so that $m c_{m}{ }^{2}=E_{m}=h_{m} v=\frac{h_{m} c_{m}}{\lambda_{m}}=\frac{h_{m}}{\tau_{m}}$

If the ponderable mass is independent of spin (characterized as equal and opposite photon polarization for a single photon), then $\vec{L}$ and $\vec{s}$ (matter and light) independently exist in the relation ( $L, s$ ), but can be related as before by the scaling factors $\tau_{m}$ and $\tau_{m}$ ' in the vector space $\left(L_{m}, s\right)$, where $\overrightarrow{J_{(m+s)}}=J_{(m+s)} \vec{r}=\overrightarrow{L_{m}}+\vec{s}=L_{m} \vec{i}+s \vec{j}$, and $\tau_{m}=\tau_{s} \gamma_{L s}, \gamma_{L s}=\frac{1}{\sqrt{1-\beta_{L s}}}, \beta_{L S}=\frac{s}{L_{m}}$, so if $s=0$, there is no interaction between the photon and other forms of matter (including other photons).

If $L$ represents the energy of an atom in a unperturbed single "state", then $s$ represents a photon that is either absorbed ( $n_{L_{m}}{ }^{\prime}=(n+1)$ or emitted $n_{L_{m}}{ }^{\prime}=(n-1)$, where $n=0$ represents the case when no photon is either emitted or absorbed.
(If $\gamma_{L}=0$, we don't even get a sunburn.... ©

## Gravity

For single values of $\vec{L}$ (e.g., me) and interaction $\vec{s}$, the interaction (as felt in Einstein's elevator by me) is then characterized by $\Delta s^{2} \overrightarrow{0}=\left(\frac{d \beta^{\prime}}{d \beta}\right)^{2} \overrightarrow{0}$ for a single photon or $\left(\Delta E_{g r a v i t y}\right)^{2}(\overrightarrow{0})=\left(\frac{\overleftrightarrow{\Delta \beta_{L}}+\overleftrightarrow{\Delta \beta_{s}}}{\overleftrightarrow{\Delta \gamma_{J}}}\right)^{2}(\overrightarrow{0})$ which implies there is "something else out there" *e.g., the earth).

## Quantum Mechanics

If the wave function is defined by $\psi_{s}=\exp \left(\frac{2 \pi i}{h_{s}}[P x-v t]_{s}\right)=\exp \left(\frac{2 \pi i}{h_{s}}\left[P_{s} x_{s}-v_{s} t_{s}\right]\right)$ then the probability is characterized by only the perturbation $\beta_{s}=h_{s}=v t_{s}, v=0 \Leftrightarrow \beta_{s}=0$

