

# The Schrödinger Equation

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## [The Relativistic Unit Circle \(RUC\)](#)

This is an attempt to clarify the role of the LaGrangian with respect to the Schrodinger equation and the RUC.

## [The LaGrangian](#) (Wiki)

**“Interacting particles**[\[edit\]](#) (Wiki)

For a given system, if two subsystems  $A$  and  $B$  are non-interacting, the Lagrangian  $L$  of the overall system is the sum of the Lagrangians  $L_A$  and  $L_B$  for the subsystems:<sup>[36]</sup>

$$L = L_A + L_B$$

If they do interact this is not possible. In some situations, it may be possible to separate the Lagrangian of the system  $L$  into the sum of non-interacting Lagrangians, plus another Lagrangian  $L_{AB}$  containing information about the interaction,

$$L = L_A + L_B + L_{AB}$$

This may be physically motivated by taking the non-interacting Lagrangians to be kinetic energies only, while the interaction Lagrangian is the system's total potential energy. Also, in the limiting case of negligible interaction,  $L_{AB}$  tends to zero reducing to the non-interacting case above.

The extension to more than two non-interacting subsystems is straightforward – the overall Lagrangian is the sum of the separate Lagrangians for each subsystem. If there are interactions, then interaction Lagrangians may be added.”

Consider the expressions

$$\varphi = (c\tau) + (v\tau') = (m_0c) + (m_v v) \text{ where } m_0 \triangleq \tau \text{ and } m_v \triangleq \tau'$$

If non-interacting, the system  $\varphi$  can be represented by the matrix

$$|\varphi_\eta| = \begin{vmatrix} (m_0c) & 0 \\ 0 & (m_v v) \end{vmatrix} \text{ so that}$$

$$|\varphi_\eta|^2 = \begin{vmatrix} (m_0c) & 0 \\ 0 & (m_v v) \end{vmatrix}^2 = \begin{vmatrix} (m_0c)^2 & 0 \\ 0 & (m_v v)^2 \end{vmatrix}$$

$$Tr(|\varphi_\eta|^2) = (m_0c)^2 + (m_v v)^2$$

Corresponding to  $(L)^2 = (L_A)^2 + (L_B)^2$  where the squared quantities represent equal and opposite LaGrangians interacting with themselves, thus satisfying Newton's Third Law.

However, if they do interact, then the equation becomes

$$\begin{aligned} \varphi^2 &= [(c\tau) + (v\tau')]^2 = (c\tau)^2 + (v\tau')^2 + 2(c\tau)(v\tau') \\ &= (m_0c)^2 + (m_v v)^2 + 2(m_0c)(m_v v) \end{aligned}$$

Where the form is similar to that of the LaGrangian except for the factor of 2 in the interaction expression  $2(m_0c)(m_v v) \neq \det|\varphi_\eta|^2 = (m_0c)^2 (m_v v)^2$

Setting  $\varphi^2 = (c\tau')^2$  expresses the total interaction in terms of a scalar multiple from each of the interacting particles/fields, so that

$$(c\tau')^2 = (c\tau)^2 + (v\tau')^2 + 2(c\tau)(v\tau')$$

and

$$(m_v c)^2 = (m_0c)^2 + (m_v v)^2 + 2(m_0c)(m_v v)$$

**(Note that the interaction term  $2(m_0c)(m_v v) \neq \det|\varphi_\eta|^2 = (m_0c)^2 (m_v v)^2$**

**and  $2(m_0c)(m_v v) \neq \det|\varphi_\eta| = (m_0c)(m_v v)$  so that matrices cannot be used to characterize the interaction.**

Dividing the equations by the l.h.s.  $((c\tau)^2$  or  $(m_v c)^2$ ) gives the radiation final state from the initial states, and by the initial states  $(c\tau)^2$  or  $(m_0 c)^2$  gives the absorption final state. In terms of masses (the first equation is covered in the RUC):

### Radiation

$$(1)^2 = \frac{1^2}{(m_v c)^2} \left[ (m_0 c)^2 + (m_v v)^2 + 2(m_0 c)(m_v v) \right] = \left( \frac{m_0}{m_v} \right)^2 + \beta^2 + 2 \frac{(m_0 c)(m_v v)}{(m_v c)^2}$$

$$2 \frac{(m_0 c)(m_v v)}{(m_v c)^2} = 2 \frac{(m_0)(v)}{(m_v)(c)} = 2 \frac{\beta}{\gamma}, \beta = \frac{v}{c}, \gamma = \left( \frac{m_v}{m_0} \right), m_v < m_0, \beta < 1, \frac{1}{\gamma} < 1, \frac{\beta}{\gamma} < (1^2)$$

$$(1)^2 = \left( \frac{1}{\gamma} \right)^2 + \beta^2 + 2 \frac{\beta}{\gamma}$$

$$\frac{1}{\gamma} \triangleq \cos \theta, \beta \triangleq \sin \theta$$

$$(1)^2 = \cos^2 \theta + \sin^2 \theta + 2 \sin \theta \cos \theta$$

### Absorption

$$\frac{(m_v c)^2}{(m_0 c)^2} = \left[ 1^2 + \frac{(m_v v)^2}{(m_0 c)^2} + 2 \frac{(m_0 c)(m_v v)}{(m_0 c)^2} \right] = 1^2 + \left( \frac{m_v}{m_0} \right)^2 \beta^2 + 2 \frac{(m_0 c)(m_v v)}{(m_0 c)^2}$$

$$\frac{(m_0 c)(m_v v)}{(m_0 c)^2} = \frac{(m_v)(v)}{(m_0)(c)} = \beta \gamma, \beta = \frac{v}{c}, \gamma = \left( \frac{m_v}{m_0} \right), m_v > m_0, \beta > 1, \gamma > 1, \beta \gamma > (1^2)$$

$$(\gamma)^2 = 1^2 + (\beta \gamma)^2 + 2 \beta \gamma$$

$$(\gamma) \triangleq \cosh \theta, \beta \gamma \triangleq \sinh \theta$$

$$\cosh^2 \theta = 1^2 + \sinh^2 \theta + 2 \cosh \theta \sinh \theta$$

Note that if

$$\sqrt{L} = \sqrt{L_A} + \sqrt{L_B} \text{ then}$$

$$(\sqrt{L})^2 = (\sqrt{L_A} + \sqrt{L_B})^2$$

$$L = L_A + L_B + 2L_A L_B$$

And for  $L_A = L_B = 1$  then

$$L = L_A + L_B + 2L_A L_B = 4$$

But if the condition

$$(\sqrt{L})^2 = (\sqrt{L_A})^2 + (\sqrt{L_B})^2$$

$$L = L_A + L_B$$

Can only be met by

$$\sqrt{L} = \sqrt{L_A} + i\sqrt{L_B}$$

$$"(\sqrt{L})^2" = (\sqrt{L})(\sqrt{L})^* = [\sqrt{L_A} + i\sqrt{L_B}][\sqrt{L_A} - i\sqrt{L_B}] = L_A + L_B$$

$$L = L_A + L_B$$

Note that:

$$(1, i) \Leftrightarrow (1, i)^2 = (1^2, i^2) = (1^2, -1)$$

$$\log(1^2) = \log(1)^2 = 2 \neq \log(-1) = 1$$

$$-1 \neq -1$$

$$\begin{vmatrix} 1 & 0 \\ 0 & i \end{vmatrix}^2 = \begin{vmatrix} 1^2 & 0 \\ 0 & -1 \end{vmatrix} \neq \begin{vmatrix} 1^2 & 0 \\ 0 & -1^2 \end{vmatrix}$$

## The Schrodinger Equation

$$i\hbar\left(\frac{\partial}{\partial t}\right)|\psi(t)\rangle = \widehat{H}|\psi(t)\rangle$$

$$i\hbar\frac{\partial}{\partial t}\psi(\vec{r},t) = \left[\frac{-\hbar^2}{2m}\nabla^2 + V(\vec{r},t)\right]\psi(\vec{r},t)$$

Note that  $V(\vec{r},t)$  is not multiplied by  $\hbar$  which implies that only  $Px$  (mass) is part of the interaction, but not  $Et$  (electromagnetism, charge). That is, mass is only associated with space, and Energy only associated with time.

### **From the RUC**

$$h^2 = 2(ct)(v\tau') = 2S^2$$

$$(\hbar)^2 = \left(\frac{h}{2\pi}\right)^2$$

Note that  $\frac{d}{dh}(h)^2 = 2h$  so that  $2h$  characterizes the “gradient” of the “scalar field”  $h^2$  and is thus the “gauge” (entropy) of the system:

$$(c\tau')^2 = (c\tau)^2 + (v\tau')^2 + 2(c\tau)(v\tau') \text{ where } h^2 = 2(c\tau)(v\tau')$$

### **The LaGrangian**

$$\varphi = L = T + V$$

$$\varphi^2 = L^2 = (T + V)^2 = T^2 + V^2 + 2TV$$

$$\psi = L = T + iV$$

$$\psi\psi^* = (T + iV)(T - iV) = T^2 + V^2 \triangleq \text{Tr} \left( \begin{pmatrix} T^2 & 0 \\ 0 & V^2 \end{pmatrix} \right) \Leftrightarrow \text{Tr} \left( \begin{pmatrix} T & 0 \\ 0 & V \end{pmatrix} \right) = \text{Tr} \left( \sqrt{\begin{pmatrix} T^2 & 0 \\ 0 & V^2 \end{pmatrix}} \right) = T + V$$

### Unpacking the Schrodinger equation

$$i\hbar \frac{\partial}{\partial t} \psi(\vec{r}, t) = \left[ \frac{-\hbar^2}{2m} \nabla^2 + V(\vec{r}, t) \right] \psi(\vec{r}, t)$$

$$\psi(\vec{r}, t) = \sin \left[ \left( \frac{2\pi i}{h} \right) (Px - Et) \right] = \sin \left[ \left( \frac{i}{\hbar} \right) (Px - Et) \right]$$

$$\theta = \left( \frac{i}{\hbar} \right) (Px - Et)$$

$$\frac{d\theta}{dx} = \left( \frac{i}{\hbar} \right) P$$

$$E = h\nu = \frac{hc}{\lambda} = \frac{h}{t}$$

$$Et = h$$

### Kinetic Energy

$$\nabla \psi_x = \frac{Pi}{\hbar} \cos \left[ \left( \frac{i}{\hbar} \right) (Px - Et) \right]$$

$$\nabla^2 (\psi_x) = - \left[ \left( \frac{i}{\hbar} \right) P \right]^2 \sin \left[ \left( \frac{i}{\hbar} \right) (Px - Et) \right] = - \left( \frac{P}{\hbar} \right)^2 \sin \left[ \left( \frac{i}{\hbar} \right) (Px - Et) \right]$$

$$\left( \frac{-\hbar^2}{2m} \right) \nabla^2 (\psi_x) = \left( \frac{P^2}{2m} \right) \sin \left[ \left( \frac{i}{\hbar} \right) (Px - Et) \right] = \left( \frac{(mv)^2}{2m} \right) \sin \left[ \left( \frac{i}{\hbar} \right) (Px - Et) \right] = \left( \frac{m(v)^2}{2} \right) \sin \left[ \left( \frac{i}{\hbar} \right) (Px - Et) \right]$$

$$\left( \frac{-\hbar^2}{2m} \right) \nabla^2 (\psi_x) = T \sin \left[ \left( \frac{i}{\hbar} \right) (Px - Et) \right], \quad T = \frac{1}{2} mv^2$$

Then

$$\left[ \left( \frac{-\hbar^2}{2m} \right) \nabla^2 (\psi_x) \right]^2 = (T)^2 \sin^2 \left[ \left( \frac{i}{\hbar} \right) (Px - Et) \right]$$

### Potential Energy

$$\hbar \frac{\partial}{\partial t} \psi(\vec{r}, t) = \left[ \frac{-\hbar^2}{2m} \nabla^2 + V(\vec{r}, t) \right] \psi(\vec{r}, t)$$

Note that in one dimension,  $r = x$  and  $V(\vec{r}, t) = V(x, t) = E_x t$

$$\frac{d\theta}{dt} = \frac{d}{dt} \left[ \left( \frac{i}{\hbar} \right) (Px - Et) \right] = - \left( \frac{i}{\hbar} \right) E = - \left( \frac{i}{\hbar} \right) \frac{h}{t} = - \frac{1}{t} (2\pi i)$$

$$\frac{d}{dt} \psi(\vec{r}, t) = - \frac{1}{t} (2\pi i) \cos \left[ \left( \frac{i}{\hbar} \right) (Px - Et) \right]$$

$$\hbar \frac{d}{dt} \psi(\vec{r}, t) = - \left( \frac{h}{2\pi} \right) \left( \frac{E}{\hbar} \right) (2\pi i) \cos \left[ \left( \frac{i}{\hbar} \right) (Px - Et) \right] = - (Ei) \cos \left[ \left( \frac{i}{\hbar} \right) (Px - Et) \right]$$

$$\left[ \hbar \frac{d}{dt} \psi(\vec{r}, t) \right]^2 = (E)^2 \cos^2 \left[ \left( \frac{i}{\hbar} \right) (Px - Et) \right]$$

$$\frac{\partial}{\partial t} \psi(\vec{x}, t) = \frac{\partial}{\partial t} \sin \left[ \left( \frac{i}{\hbar} \right) (Px - Et) \right] = - \left( \frac{i}{\hbar} \right) E \cos \left[ \left( \frac{i}{\hbar} \right) (Px - Et) \right]$$

$$\hbar \frac{\partial}{\partial t} \psi(\vec{x}, t) = -\hbar \left( \frac{i}{\hbar} \right) E \cos \left[ \left( \frac{i}{\hbar} \right) (Px - Et) \right]$$

$$\frac{\partial}{\partial t} \psi(\vec{x}, t) = iE \cos \left[ \left( \frac{i}{\hbar} \right) (Px - Et) \right]$$

$$\psi(\vec{r}, t) = \sin \left[ \frac{i}{\hbar} (Px - Et) \right]$$

$$V(\vec{r}, t) = \sin \left[ \frac{i}{\hbar} (Px - Et) \right]$$

$$V(\vec{r}, t) \psi(\vec{r}, t) = \sin^2 \left[ \frac{i}{\hbar} (Px - Et) \right]$$

## The LaGrangian

$$\hbar \frac{\partial}{\partial t} \psi(\vec{x}, t) = \left[ \frac{-\hbar^2}{2m} \nabla^2 + E_x t \right] \psi(\vec{x}, t) = (T + V) (\psi(\vec{x}, t)) = (T + E_x) (\psi(\vec{x}, t))$$

Then:

(Kinetic Energy)

$$\left( \frac{-\hbar^2}{2m} \right) \nabla^2 (\psi_x) = T \sin \left[ \left( \frac{i}{\hbar} \right) (Px - Et) \right], \quad T = \frac{1}{2} mv^2$$

$$\left[ \left( \frac{-\hbar^2}{2m} \right) \nabla^2 (\psi_x) \right]^2 = (T)^2 \sin^2 \left[ \left( \frac{i}{\hbar} \right) (Px - Et) \right]$$

(Potential Energy)

$$\left[ \hbar \frac{d}{dt} \psi(\vec{r}, t) \right]^2 = (E)^2 \cos^2 \left[ \left( \frac{i}{\hbar} \right) (Px - Et) \right] = (V)^2 \cos^2 \left[ \left( \frac{i}{\hbar} \right) (Px - Et) \right]$$

$$\left[ \left( \frac{-\hbar^2}{2m} \right) \nabla^2 (\psi_x) \right]^2 + \left[ \hbar \frac{d}{dt} \psi(\vec{r}, t) \right]^2 = (T)^2 \sin^2 \left[ \left( \frac{i}{\hbar} \right) (Px - Et) \right] + (E)^2 \cos^2 \left[ \left( \frac{i}{\hbar} \right) (Px - Et) \right]$$

$$V = E, \quad T^2 = V^2, \quad T = V$$

$$\left[ \left( \frac{-\hbar^2}{2m} \right) \nabla^2 (\psi_x) \right]^2 + \left[ \hbar \frac{d}{dt} \psi(\vec{r}, t) \right]^2 = (T)^2 + (V)^2 = 2(T)^2 \left\{ \sin^2 \left[ \left( \frac{i}{\hbar} \right) (Px - Et) \right] + \cos^2 \left[ \left( \frac{i}{\hbar} \right) (Px - Et) \right] \right\} = (2(V))^2$$

$$\left[ \left( \frac{-\hbar^2}{2m} \right) \nabla^2 (\psi_x) \right]^2 + \left[ \hbar \frac{d}{dt} \psi(\vec{r}, t) \right]^2 = (2(V)^2) = (T + V)^2 = (L)^2$$

(In Progress)

$$\hbar \frac{\partial}{\partial t} \psi(\vec{x}, t) = \left[ \frac{-\hbar^2}{2m} \nabla^2 + E_x t \right] \psi(\vec{x}, t) = (V)(\psi(\vec{x}, t)) = (E_x)(\psi(\vec{x}, t))$$

$$h = (E_x)(\psi(\vec{x}, t))$$

**The Wave Equation and Complementary Variables**  $(P, x)$  and  $(E, t)$

$$\varphi = \cos(Px \pm Et) + \sin(Px \pm Et)$$

$$\varphi^2 = \cos^2(Px \pm Et) + \sin^2(Px \pm Et) + 2 \cos(Px \pm Et) \sin(Px \pm Et)$$

$$\psi = \cos(Px \pm Et) + i \sin(Px \pm Et)$$

$$\psi^* = \cos(Px \pm Et) - i \sin(Px \pm Et)$$

$$\psi \psi^* = 1^2 = \cos^2(Px \pm Et) - (i)^2 \sin^2(Px \pm Et) = \cos^2(Px \pm Et) + \sin^2(Px \pm Et)$$

**Complimentary Variables**

Consider a locomotive traveling through empty space with a momentum  $P = mv = \tau'v = v\tau'$  where  $\tau'$  is the time it takes to create the locomotive with a mass creation rate  $v$  where  $\tau'$  then represents the mass of the locomotive that characterizes its momentum. Then  $L = v\tau'$  also represents the length the locomotive will travel at velocity  $v$  during time  $\tau'$  for a locomotive of length  $L$

While it encounters nothing in its path, these values will remain invariant

Consider also a light patch of length  $x_c = c\tau$  which also has a mass  $m_0 = c\tau$  where  $c$  is the mass creation rate during the time  $\tau$  and where  $\tau$  can also characterize the mass  $m_0$  for a photon of momentum  $P_c = m_0c = m_0\tau$

If the photon encounters nothing in its path, its values will remain invariant as well.

If the locomotive and the photon do not interact, they can be represented by the matrix  $|P| = \begin{vmatrix} c\tau & 0 \\ 0 & v\tau' \end{vmatrix}$

where  $|P|^2 = \begin{vmatrix} c\tau & 0 \\ 0 & v\tau' \end{vmatrix}^2 = \begin{vmatrix} (c\tau)^2 & 0 \\ 0 & (v\tau')^2 \end{vmatrix}$  where  $(c\tau)^2$  represent the energy of two photons

encountering each other with equal opposite momentum, and similarly  $(v\tau')^2$  represents the energy of two locomotives encountering each other with equal but opposite momentum.

If the locomotive and the photon do interact, then the initial state of the mass of the photon can be related to the change of state of the locomotive by the expression:

$c\tau' = c\tau + v\tau'$  and the energy of the interaction (for equal and opposite momentum) can then be expressed as

$(c\tau')^2 = (c\tau)^2 + (v\tau')^2 + 2(c\tau)(v\tau')$  which also represents the change of state of the photon.

$$(c\tau')^2 = (c\tau)^2 + (v\tau')^2 + 2(c\tau)(v\tau')$$

$$(P')^2 = (P_c)^2 + (P)^2 + 2(P_c)(P)$$

$$(m_0v)^2 = (m_0c)^2 + (P)^2 + 2(m_0c)(P)$$

And  $2(m_0c)(P)$  represents the interaction between the locomotive and the light path during the time it takes for the length of the locomotive ( $L$ ) to pass through a light patch of length  $L$

$$\varphi = \cos \theta + \sin \theta$$

$$\varphi^2 = \cos^2 \theta + \sin^2 \theta + 2 \cos \theta \sin \theta = \cos^2 \theta + \sin^2 \theta + (h_\theta)^2$$

$$(\varphi_{Px}) = P + x$$

$$(\varphi_{Px})^2 = P^2 + x^2 + 2Px = P^2 + x^2 + (h_{Px})^2$$

$$(\varphi_{Et}) = E + t$$

$$(\varphi_{Et})^2 = E^2 + t^2 + 2Et = E^2 + t^2 + (h_{Et})^2$$