Proof of Fermat's Theorem

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02/13/2017 4:11 PM PST

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Updates: Latest revision: **03./05/2017 1:14 PM PST Case n=2 dot product**

Fermat's Theorem: $c^n \neq a^n + b^n$ for a, b, c and n positive integers.

The Binomial Theorem

The Binomial Expansion is given by the equation $(x+y)^n = \sum_{k=0}^{n} \binom{n}{k} x^{n-k} y^k$, $\binom{n}{k}$ is a positive integer taken to be the binomial coefficient. I use the expression $Rem(x,y,n) = \sum_{k=0}^{n} \binom{n}{k} x^{n-k} y^k - (x^n + y^n)$ to indicate those terms in the Binomial Expansion not equal to either x^n or y^n.

The Binomial Expansion applies to integers (in fact, it was first formulated this way), so using a and b , the expression is given by:

$\sum_{k=0}^{n} \binom{n}{k} a^{n-k} b^k$ Setting this expression equal to an arbitrary integer c^n so that

$c^n = \sum_{k=0}^{n} \binom{n}{k} a^{n-k} b^k = (a+b)^n = a^n + b^n + \mathrm{Re}\, m(a,b,n)$, where

$Rem(a,b,n) = \sum_{k=0}^{n} \binom{n}{k} a^{n-k} b^k - (a^n + b^n) = m$ where $m > 0$ (is a positive integer) proves Fermat's Theorem by inspection, since $c^n \neq a^n + b^n$

03./05/2017 07:05 AM PST Case n=2 dot product

It is important to understand that the Pythagorean Theorem means that the two legs of the right triangle a and b are orthogonal, so that $\vec{a} \cdot \vec{b} = 0$ For the Binomial Expansion, this means that $c^2 = a^2 + b^2 + 2\left(\vec{a} \cdot \vec{b}\right) = a^2 + b^2$, $a \perp b$ in the pair (a,b). If they are not orthogonal, then there will be a cross product which characterizes an multiplicative "interaction" between the two; $rem(a,b,2)=2ab$, so the expression is a vector equation where $c^2 r = a^2\vec{i}+b^2\vec{j}+2\left(\vec{a}\otimes\vec{b}\right)=a^2\vec{i}+b^2\vec{j}+(2ab)\vec{k}$, where 2ab is in a distinct set, forming the three dimensional vector space $(a,b,rem(a,b,2))=(a,b,2ab)$, where 2ab is an integer. However, this means that $\sqrt{2ab}$ is not an integer, so the expressio $c\vec{r}=a\vec{i}+b\vec{j}+\left(\sqrt{2ab}\right)\vec{k}$ means that c cannot be an integer for (a,b,c) if they do not form a Pythagorean triple. This is easily expanded to the case $n>2$ by the Binomial Expansion

$c^n=(a+b)^2(a+b)^{n-2}=a^n+b^n+rem(a,b,n)$ where $rem(a,b,n)\neq 0$ unless $a=0$ or $b=0$

so Fermat's expression $c^n=a^n+b^n$ cannot be complete, since it doesn't include the cases where $2ab\neq 0$ (the Pythagorean Triples) in the expansion above.

Therefore, the expression $c^2=a^2+b^2$ cannot be complete for three arbitrary integers a,b and c , and it can't be consistent if the arithmetic properties of integer addition, multiplication, and powers obtain, and by extension $c^n\neq a^n+b^n$ for all integers in the set (a,b,c) since there also exists the set $x=f(a,b,rem(a,b,n)$ where f is the Binomial Theorem and x cannot be an integer; Fermat's Theorem is proved, (as is Gödel's).

Note that one can always divide $c^n=(a+b)^2(a+b)^{n-2}=a^n+b^n+rem(a,b,n)$ by $(a+b)$ successively to retrieve the case $c^2\vec{r}=a^2\vec{i}+b^2\vec{j}+2ab\vec{k}$, but if $2ab$ exists, one cannot divide it further to obtain $\vec{c}=\vec{a}+\vec{b}=a\vec{i}+b\vec{j}$ for c an integer if (a,b,c) are not a Pythagorean Triple.

The Relativistic Unit Circle establishes the basis states for the Pythagorean Theorem when the vectors (a,b) are orthogonal. If they are orthogonal, the basis is $\left(1_a{}^2,1_b{}^2\right)$ That means the for one of the circles $\theta_a=0$ and the other $\theta_a=\frac{\pi}{2}$. However, if both these conditions do not hold, 1_a will not be orthogonal to 1_b so $\vec{a}$ is not orthogonal to $\vec{b}$ in the pair (a,b) accounting for the relation $2\gamma_a\beta_b=2\cos\theta_a\sin\theta_b\neq 0$ with γ_a and β_b referring to different circles, so the circles are not independent.

This factor is the effect of relativistic spin polarization in QFT.

However, to deeply understand why this is so requires understanding the basic relation of Special Theory of Relativity as applied to Quantum Field Theory to understand why this is so, particularly to understand why the "constructivist" approach in the Foundations of Mathematics relies on a subjective understanding of "counting" in terms of naïve realism…., since the philosophical position of STR/QFT is essentially solipsistic (a conclusion now recognized as Quantum Triviality, and the gist of the fundamental disagreement between Albert Einstein and Neils Bohr at in the early part of the last century.

In the context of a single relativistic circle $(ct, vt') = (a, b) = \left(a\gamma, b\beta\right)$, if $\vec{\gamma}$ and $\vec{\beta}$ are not orthogonal as in the RUC; there will be an additional vector product $\sqrt{2ab}\vec{k} = \sqrt{2\gamma\beta}\vec{k}$ corresponding to spin of a single particle (e.g., and electron), so that again, $\vec{c}$ cannot be an integer in the relation $\vec{c} = \vec{a} + \vec{b}$

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For a single relativistic circle, $1^2 = \cos^2\theta + \sin^2\theta$, but if not a circle, the unit basis doesn't exist because $\vec{\gamma}$ is not perpendicular to $\vec{\beta}$; that is the ratio $\beta \neq \frac{v}{c}$ (This is the case for the General Theory of Relativity or relativistic spin of a single particle) .

For two relativistic circles, note that $\cos(\theta_a + \theta_b) = \gamma_a\gamma_b - \beta_a\beta_b \neq 2(1^2)$ unless $\theta_a = 0$ and $\theta_b = 0$.

In only one dimension one dimension, $\gamma = 1$ is an integer only if $\beta = 0$ so $\overrightarrow{a\gamma_a} = a$ Otherwise, $x_{a,\beta} = \gamma_a\beta_a = \cos\theta_\gamma \cos\theta_\beta - \sin\theta_\gamma \sin\theta_\beta = \cos(\theta_\gamma + \theta_\beta)$ where the angles are those which the relativistic unit circles makes with the γ and β axes, respectively.

The relation between the Binomial Theorem and Special Relativity

The proof resides on being able to establish a connection between Fermat's Theorem and the Binomial Theorem. This cannot be accomplished by simply declaring $c = a + b$ and then using $c^n = (a+b)^n$ to establish the equation $c^n = (a+b)^n = a^n + b^n + rem(a,b,n)$ from elementary algebra, because the symbols on each side of the equality refer to the same unique integer. Rather, the two integers must be distinguished in the two-dimensional vector space (a,b)

This involves a relativistic comparison of the metric (at, bt') with a and b invariants as integers, with the relativistic unit circle the integer generator, rather than simple mental counting, as is the case in constructivist mathematics (Frege, Russell, et. Al.) Instead, a, b, and c must "exist" as integers defined by the relativistic unit circle.

If the Theory of Relativity is correct in defining the common metric for a and b, it establishes the foundation of the Binomial Expansion as a linear acceleration resulting from the interaction of two conserved particles for $n = 2$ (two dimensions) and non-linear for $n > 2$, and not only for integers. It therefore has profound implications for GTR and the metric tensor. The issue about distinguishing between a and b as relativistic is important to the concept of distinguishable particles in physics, addressed especially by Dirac with a foundation from Pauli). It involves an understanding of the "zero-point" energy (the "vacuum", and as a result an analysis of the null vector and its role as characterizing equal and opposite contact force.

(In the case of the Pythagorean triple in two dimensions, there is no linear relativistic acceleration).

O2/28/2017 10:44 AM PST

Consider a single relativistic unit circle in its final state $(ct)_a = (ct')_a$, with a unit radius $\gamma_a = 1_a$ and an area ("energy") $(E_a)^2 = (1_a)^2$ and a second relativistic unit circle $(ct)_b = (ct')_b$ with a unit radius $\gamma_b = 1_b$ ("energy") $(E_b)^2 = (1_b)^2$.

Each circle can be multiplied by an arbitrary integer, designated by its subscript:

$(E'_a)^2 = (a(1_a))^2 = a^2$ and $(E'_b)^2 = (b(1_b))^2 = b^2$, where the metric relation is the same for each circle in its final state, even though $(ct')_a \neq (ct')_b$, but the metrics (γ_a, γ_b) are independent, i.e. can be varied independently by varying the initial rest energy and perturbing states (relativistic momentum) for each one. The two metrics are exhaustive for all possible final states of the two relativistic systems.

That is, $(E_{total})^2 \vec{r} = c^2 \vec{r} = a^2 \vec{i} + b^2 \vec{j}$, where $a^2 = (a\vec{i}) \cdot (a\vec{i}) = a^2(\vec{i} \cdot \vec{i})$ and $b^2 = (b\vec{j}) \cdot (b\vec{j}) = b^2(\vec{j} \cdot \vec{j})$

So that $c^2 = a^2 + b^2$ for the two independent systems, where the relationship is characterized by the orthogonal vector space (a^2, b^2) .

If c^2 is an integer, then the relation is that of a Pythagorean Triple, true for all such metrics of non-interacting number lines composed of integers (i.e., orthogonal coordinates).

If the relation is not that of a Pythagorean triple, we can form the relation from $z^2 = a^2\left(\vec{i}\cdot\vec{i}\right) + b^2\left(\vec{j}\cdot\vec{j}\right) + 2ab\left(\overleftrightarrow{0}\right)$ where z is not an integer, and the interaction coefficient can be characterized as arising from equal and opposite "cross products" when all combinations are included for both number lines, so that

$$\overleftrightarrow{a}\otimes\overleftrightarrow{b} = 2ab(\overleftrightarrow{0}) = 2ba(\overleftrightarrow{0}) = 2ab\left(\vec{0}\cdot\overleftarrow{0}\right)$$

in the first order, so that a and b commute. Otherwise, $\left(\vec{a}\cdot\vec{b}\right) = 0$ which obtains in the case of Pythagorean Triples.

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in the first order, so that a and b commute. Then $c^2 = a^2 + b^2 + 2ab = (a+b)^2$ so for the case $n = 2$ either the Binomial Expansion or the Pythagorean Triple obtains. This case is easily extended to $n > 2$ by $c^n = (a+b)^n (a+b)^{n-2} = a^n + b^n + rem(a,b,n)$ where the Pythagorean triple no longer applies, and $rem(a,b,n) > 0$ so that Fermat's equation $c^n = a^n + b^n$ cannot be valid, and Fermat's Theorem is proven.

03/01/2017 09:02 AM PST Relation to Binomial Theorem

Case n=2

The reason why both the Pythagorean triple and the Binomial Expansion are valid for $n = 2$ is that for independent variables (a,b), they lie in the same plane. For any given pair, one can always draw a vector with length c from the origin to the point in the plane.

If the legs of the triangle $\{a,b,c\}$ from that point to each of the axes is orthogonal, then the areas in second order to each axis are described by the area of the rectangle (or circle) by the independent values (a^2,b^2) where $c^2=a^2+b^2$. This is the minimum value such an area can have, and the relation is true for all possible values of the lengths a and b.

If the legs of the triangle to the point are outside the rectangle (or circle), then an additional area must be added because of the additional triangle thus created, so the lines are no longer perpendicular to the axes. This is the case of the Binomial Expansion for $n=2$, where $2ab$ is the additional area of the triangle $\left(A=\left(1/2\right)ab\right)$, and accounts for the factor of 24 in the triple (not Pythagorean) (3,4,7) where $7^2=25+24=49$. If the "outer" legs are rotated back so they are perpendicular to the axes again, the "24" disappears, and the area is once again 25.

These relations is true for all integer triples $\{a,b,c\}$ for $n=2$. The additional "interaction" area (an integer constant) can be characterized by the vector $\vec{k}$ in three dimensions so that $c^2\vec{r}=a^2\vec{i}+b^2\vec{j}+(2ab)\vec{k}$. The fact that the actual (total)l additional area is actually $2ab$ instead of $\left(1/2\right)ab$ arises from the fact that all 4 areas must be added from the quadrants of the rectangle (or square) relative to the center (the origin $\left(0,0\right)$), so the total area is $4\left(1/2\right)ab=2ab$.

Case n>2

The case $n=3$ is actually a case of 3 integers characterized by the Binomial expansion. In this case, the vector $\vec{c}$ is no longer in the two dimensional $\left(a,b\right)$ and hence cannot be described by the minimum additional area =0 (the Pythagorean Triple) in two dimensions. However, the Binomial Expansion is still valid since the factors ab^2 and ba^2 characterize the additional volumes when moved of the spheres (a^3,b^3) in three dimensions, where $z^3\vec{r}=a^3\vec{i}+b^3\vec{j}+rem(a,b,3)\vec{k}$. However, z can no longer be an integer, since it is no longer in the integer plane $\left(a,b\right)$ which characterizes all independent integers.

This case is then easily extended to $n>3$ for all such expansions $z^n=a^n\vec{i}+b^n\vec{j}+rem(a,b,n)\vec{k}=(a+b)^3(a+b)^{n-3}$ where $rem\left(a,b,n\right) > 0$ where $n>2$ and Fermat's Theorem is proved.

The analysis above is true for all real numbers as well as integers.

That is, to prove Fermat's Theorem to be true all that is required is to show:

$c^n = (a+b)^n \neq a^n + b^n$ for $n > 2 \quad a,b,c,n$ positive integers, a trivial result by the Binomial Theorem.

If there are no interactions, then the Pythagorean triangle applies.

03/02/2017 12:07 PM PST Energy-Area Equivalence (n=2)

$E_{ab\perp} = (s_a)^2 + (s_b)^2 + \sqrt{(2ab)^2}$ s is the side of a square

$E_{ab\bigcirc} = (r_a)^2 + (r_b)^2$ r is the radius of a circle

Equivalent Area

$$\pi E_{ab\bigcirc} = \pi(r_a)^2 + \pi(r_b)^2 + \pi\sqrt{(2ab)^2} = \pi\left[(r_a)^2 + (r_b)^2 + \sqrt{(2ab)^2}\right] = \pi\left[(r_a)^2 + (r_b)^2 + (r_{ab})^2\right],$$

where $r_{ab} = 2ab$ is the area of an equivalent circle.

r is the radius of a circle

Non-Interacting Integers

$$s_a = (\gamma_a)^2 a = (1_a{}^2)a$$
$$s_b = (\gamma_b)^2 b = (1_a{}^2)b$$
$$E_a = (s_a)^2 a^2 = (1_a{}^4)a^2 = a^2$$
$$E_b = (s_b)^2 b^2 = (1_a{}^4)b^2 = b^2$$
$$E_{a+b} = a^2 + b^2$$

Interacting Integers

$$E_{ab} = \sqrt{(2a'b')^2} = 2a'b' = \Delta$$
$$E_{total} = E_{a+b} + E_{ab} = a'^2 + b'^2 + \Delta$$
$$\pi E_{total} = \pi\left(E_{a+b} + E_{ab}\right) = \pi\left(a'^2 + b'^2 + \Delta\right)$$

$$E' = (a')^2 + (b')^2 + \Delta$$

Completeness

$$\psi_{\circ}^{\,2}\overrightarrow{r_0} = a^2\cos^2\theta\vec{i} + b^2\sin^2\theta\vec{j}$$

$$\psi^2\overrightarrow{r_0} = c^2\vec{r} = a^2\vec{i}, \quad \theta = 0 \Rightarrow c = a$$

$$\psi^2\overrightarrow{r_0} = c^2\vec{r} = b^2\vec{i}, \quad \theta = \frac{\pi}{2} \Rightarrow c = b$$

Pythagorean Triples (Pythagorean Integers)

$c^2 = a^2 + b^2$, where c is the total number (count) of non-interacting integers (a, b)

Non Pythagorean Integers (Interacting Integers)

$(c')^2 = (a')^2\vec{i} + (b')^2\vec{j} + \Delta\vec{0},\ \Delta = \sqrt{(2a'b')^2}$, $\sqrt{\Delta} = 2a'b'$ is a positive integer

c' is the total number (count) of interacting integers in the triple (a', b', Δ)

The sum (count) $c + c'$ consists of the total number of interacting and non-interacting integers, and therefore exhausts the set of all integers.

The set $c^2 = a^2 + b^2$ consists of all non-interacting integers in the sets $\{a\}$ and $\{b\}$, $(\{a\}, \{b\})$, and so $(\{a^2\}, \{b^2\})$. In a similar manner, the set $c'^2 = a'^2 + b'^2 + \Delta$, where $\Delta = \sqrt{(2a'b'^2)} = 2a'b' = rem(a', b', n)$ consists of all interacting integers in the sets a', b', and Δ .

The sum (count) of integers in each set is c and c', respectively

$\{S\} = \{c\} \oplus \{c'\}$ is the count of both interacting an non-interacting integers, and exhausts the set of integers for n=2.

Then $S^2 = (c + c')^2 = c^2 + c'^2 + 2cc'$

Then (the sum of all integers) cannot consist of only non-interacting integers c unless c' = 0.

Therefore $S^2 \neq c^2 = a^2 + b^2$ unless $2cc' = 0$
]
But $2cc'$ is not an empty set, since there will always be a product of interacting integers which is itself an integer.

Therefore $S = \sqrt{a^2 + b^2} + \sqrt{(a')^2 + (b')^2 + \Delta} \neq c^2 = \sqrt{a^2 + b^2}$ which includes non-interacting and interacting integers cannot be an integer, since $c = \sqrt{a^2 + b^2}$ exhausts all the non-interacting integers as Pythagorean Triples. The Binomial Theorem is then proven for all integers for $n = 2$, and includes all possible positive integers (it is complete).

S can only be an integer if $\sqrt{(a')^2 + (b')^2 + \Delta} = 0$.

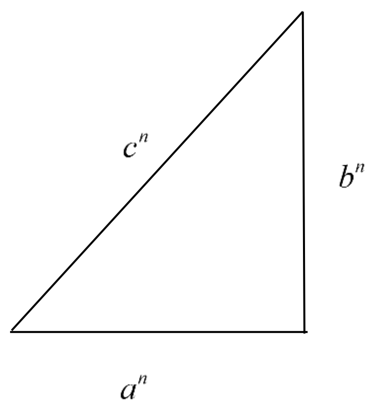
Fermat's Theorem is then proved for $n = 2$, since S must include all integers.

That is, $r^2 = x^2 + y^2$ (areas) does NOT mean $r = (x + y)$ (lengths) in Cartesian coordinates; rather, $r = \sqrt{(x^2 + y^2)}$

That is, $r^2 = x^2 + y^2$ (areas) does NOT mean $r = (x + y)$ (lengths) in Cartesian coordinates; rather, $r = \sqrt{(x^2 + y^2)}$, In the same way, $(c^n)^2 \neq (a^n)^2 + (b^n)^2$ does not mean $c^n = a^n + b^n$, unless $(a^n \perp b^n)$ (i.e., independent; no interaction; i.e., a Pythagorean Triple where $c^n = \sqrt{(a^n)^2 + (b^n)^2}$); otherwise the Binomial Expansion applies: $c^n = a^n + b^n + rem(a^n, b^n, n)$

If Fermat's expression were true, $c^n = a^n + b^n$ would be a Pythagorean Triple.

But the Binomial Expansion always applies for $n > 2$, so $c^n \neq \sqrt{(a^n)^2 + (b^n)^2}$, $n > 2$.



(The vector analysis is shown in the PDF document)

Fermat's expression is not a Pythagorean Triple.

QED

I'm still waiting for an intelligent counter-argument. I think the idea the Fermat's theorem has not been proven many times over is an urban legend, designed to get unwary young students excited about mathematics. Quite possibly a scam by math departments the world over …

Or a Jedi mind trick….

Comment:

The diagram above together with the Binomial Theorem establishes the inequality for Fermat's Theorem. However, it still remains to prove that c^n is not an integer.

This is accomplished by the fact that the relativistic unit circle is an "integer generator" which compares metrics between two real numbers via the scaling factors t and t' in the pair (ct, vt') where a unit integer is generated only for $t = t'$. The variable "v" acts as a perturbation; when $v = 0$ there is no change, and when $v = c$ the next integer is created.

This is clear from the relativistic energy equation"

$$(E')^2 = (Pc)^2 + (E_0)^2$$

which is positive definite, but periodic in the circle where the effect of adding integers is cumulative (where P oscillates from 0 to E_0, each time creating an integer so that $n(E')^2 = n\left[(P_n c)^2 + (E_0)^2\right]$ This means that γ increments by $\cos n\theta$ when $\sin n\theta = 0$ The "perturbation" effect ($\sin n\theta$) for a linear system can be removed by the process $e^{i\theta} = \cos\theta + i\sin\theta$ so that $n(ct)^2 = n\big(x + i(vt)\big)\big(x - i(vt)\big) = n\left[x^2 + (vt)^2\right] = n\left(x^2\right)$ with $x = ct$ at $v = 0$ and "rest mass" interpreted as $m_0 = ct$

For Fermat's Theorem, the pair (ct, vt') is replaced by (at, bt') where a and b are independent positive integers, and new integers are created from the basis $(1,1)$ at each $n+1$ rotation of the unit circle where $t' = t$.

The relativistic increments must be carried out in two dimensions for independent (a,b), so the vectors are orthogonal for Pythagorean Triples, or if not the Binomial Expansion for $n \geq 2$ applies (either $a = 0$ or $b = 0$).

(See The Relativistic Unit Circle for further discussion on this issue.)

Axiom of Choice: It is always possible to arrange any three given integers a, b, c in the equation $c^2 = a^2 + b^2$ where $c \geq a$ and $c \geq b$.

Consider three positive integers a, b, and c

Choose $c \geq a \geq b$

Let $c^2 = a^2 + b^2$ (e.g. $25 = 4^2 + 3^2 = 16 + 9$)

Note that $\frac{b}{a} \le 1$ ($\frac{3}{4} \le 1$)

, so if $b < a$, $\frac{b}{a}$ is not an integer.

Divide both sides by a so that $\frac{c^2}{a^2} = 1^2 + \frac{b^2}{a^2}$

Assume that $\frac{c^2}{a^2} = d^2$ is an integer. (e.g., $d^2 = 1^2 + \left(\frac{3}{4}\right)^2$)

Then $d^2 = \frac{c^2}{a^2} = 1^2 + \frac{b^2}{a^2}$ cannot be an integer, since $b \le a$ so $\frac{b^2}{a^2} \le 1^2$

If $b = a$ then $d^2 = \frac{c^2}{a^2} = 1^2 + 1^2$ so $d = \sqrt{2}$, so $\frac{c}{a} = \sqrt{2}$, an irrational.

Therefore, $c = a\sqrt{2}$ cannot be an integer.

If $b \ne a$, then $b < a$ so d cannot be an integer $d = \sqrt{1^2 + \left(\frac{3}{4}\right)^2}$, so $\frac{c}{a}$ cannot be an integer so $c = a\sqrt{1^2 + \left(\frac{3}{4}\right)^2}$ cannot be an integer, so the equation $c^2 = a^2 + b^2$ where a,b and c are integers cannot be valid (the common denominator a where $a \ge b$ cannot be an integer).

Cross Products

Consider a set of three arbitrary positive integers (a,b,c) . The integers can be selected so that $c \ge a$ and $c \ge b$ (the Axiom of Choice) to form the relation $c^2 = a^2 + b^2$. For Pythagorean Triples, the pair (a,b) must be orthogonal ($a \perp b$) so that their vector relation is

$a \cdot b = ab\cos\theta = 0,\ \theta = \left(\frac{\pi}{2}\right)$ and $a \otimes b = ab\sin\theta = ab,\ \theta = \frac{\pi}{2}$, with no other metric relationship.

The cross product is characterized by the matrix representations

$$\begin{vmatrix} a\vec{i} & 0 & 0 \\ 0 & b\vec{j} & 0 \\ 0 & 0 & 0 \end{vmatrix} \xrightarrow{a\vec{i}\otimes b\vec{j}} \begin{vmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & (ab)\vec{k} \end{vmatrix} = (ab)\vec{k}$$

$$\begin{vmatrix} 0 & a\vec{i} & 0 \\ b\vec{j} & 0 & 0 \\ 0 & 0 & 0 \end{vmatrix} \xrightarrow{b\vec{j}\otimes a\vec{i}} \begin{vmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & (ab)\overrightarrow{-k} \end{vmatrix} = (ab)\left(\overrightarrow{-k}\right)$$

So that $\left(a\vec{i}\otimes b\vec{j}\right)+\left(b\vec{j}\otimes a\vec{i}\right)=(ab)\vec{k}+(ab)\left(\overrightarrow{-k}\right)=(ab)\left(\vec{0}\right)$

We also must include the fact that the scalar product $ab=ba$ commutes, so that

$$\begin{vmatrix} b\overleftarrow{i} & 0 & 0 \\ 0 & a\overleftarrow{j} & 0 \\ 0 & 0 & 0 \end{vmatrix} \xrightarrow{b\overleftarrow{i}\otimes a\overleftarrow{j}} \begin{vmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & (ba)\overleftarrow{k} \end{vmatrix} = (ba)\overleftarrow{k}$$

$$\begin{vmatrix} 0 & a\overleftarrow{i} & 0 \\ b\overleftarrow{j} & 0 & 0 \\ 0 & 0 & 0 \end{vmatrix} \xrightarrow{b\overleftarrow{j}\otimes a\overleftarrow{i}} \begin{vmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & (ba)\overleftarrow{-k} \end{vmatrix} = (ba)\left(\overleftarrow{-k}\right)$$

So that $\left(a\overleftarrow{i}\otimes b\overleftarrow{j}\right)+\left(b\overleftarrow{j}\otimes a\overleftarrow{i}\right)=(ab)\overleftarrow{k}+(ab)\left(\overleftarrow{-k}\right)=(ab)\left(\overleftarrow{0}\right)$.

Adding the two results gives an equal and opposite reaction at the null vector connection between a and b (the juncture of the independent number lines:

$(ab)\left(\vec{0}\right)+(ab)\left(\overleftarrow{0}\right)=2ab\left(\vec{0}\right)$, where the interaction is represented by $2ab$.

In order to eliminate the interaction product, the prescription $c^2=a^2+b^2=(a+ib)(a-ib)$ must be used, where ai and bi are now complex scalars $i=\sqrt{-1}$, $i^2=-1$.

Note that products such as $a^p b^q$ do not commute on different number lines for $p\neq q$, so these terms cannot be canceled in rem(a,b,n), $n>2$.

Emphasizing the obvious, note that if either a is not an integer or b is not an integer then $2ab$ is not an integer (the triple is not Pythagorean)

This corresponds to the Binomial Theorem for the case $n=2$, where $c^2 = a^2 + b^2 + 2ab$; if there is no interaction, the vectors are said to be “affine”, with no common origin (the number lines are parallel, and do not intersect).

If a and b are on the same number line, then the left and right sides of the equality $c\vec{i} = (a+b)\vec{i} = d\vec{i}$ refer to the same unique integer. Nevertheless, $c^2\vec{i} = (a+b)^2\vec{i} = \left[a^2 + b^2 + 2ab\right]\vec{i} = \left[a^2 + b^2 + m\right]\vec{i}$ as scalars, so that $c^2 = a^2 + b^2$ means that $m = 2ab = 0$ (i.e., $a = 0$ or $b = 0$)

Proof of Fermat's Theorem

Fermat's Theorem says that $c^n \neq a^n + b^n$ for a,b,c and n positive integers for $n > 2$.

The above results for $n = 2$ for triples that are not Pythagorean can be easily extended by the Binomial Expansion:

$$c^n = \left(a+b\right)^n = \left(a+b\right)^2 (a+b)^{n-2} =$$
$$\left(a^2 + b^2 + 2ab\right)(a+b)^{n-2} =$$
$$a^2(a+b)^{n-2} + b^2(a+b)^{n-2} + 2ab(a+b)^{n-2} =$$
$$a^2\left[a^{n-2} + b^{n-2} + rem(a,b,n-2)\right] + b^2\left[a^{n-2} + b^{n-2} + rem(a,b,n-2)\right] + \left(2ab\right)rem(a,b,n-2)$$

$k = rem(a,b,n-2) > 0$, $k > 0$, a positive integer, $k = 0$ only if $a = 0$ or $b = 0$.

Then

$$c^n = a^2\left[a^{n-2} + b^{n-2} + k\right] + b^2\left[a^{n-2} + b^{n-2} + k\right] + \left(2ab\right)k =$$
$$a^n + a^n b^{n-2} + a^2 k + b^2 a^{n-2} + b^n + b^2 k + (2ab)k =$$
$$a^n + b^n + a^n b^{n-2} + b^2 a^{n-2} + a^2 k + b^2 k + (2ab)k$$

Let $m = a^n b^{n-2} + b^2 a^{n-2} + k\left[a^2 + b^2 + (2ab)\right]$, a positive integer, $m = 0$ only if $a = 0$ or $b = 0$

$c^n = a^n + b^n + m$, $c^n = a^n + b^n$ only if $m = 0$

Therefore,

$c^n = a^n + b^n + m \neq a^n + b^n$, since $m > 0$, $m = 0$ if $a = 0$ or $b = 0$.

Q.E.D.

If there is no interaction (the equivalent of a Pythagorean triple for $n > 2$), then the equation for independent $\overrightarrow{a^n}$ and $\overrightarrow{b^n}$

$$c^n \vec{r} = a^n \bar{i} + b^n \bar{j}$$

$$\left(c^n\right)^2 = \left(a^n\right)^2 + \left(b^n\right)^2$$

$$c^n = \sqrt{\left(a^n\right)^2 + \left(b^n\right)^2} \neq a^n + b^n$$

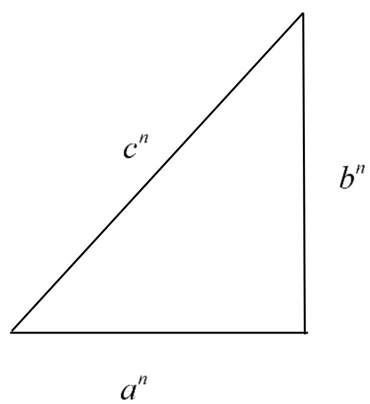
Q.E.D.

If there are no interactions, then the Pythagorean triangle applies.

That is, $r^2 = x^2 + y^2$ (areas) does NOT mean $r = (x + y)$ (lengths) in Cartesian coordinates; rather, $r = \sqrt{(x^2 + y^2)}$

In the same way, $(c^n)^2 = (a^n)^2 + (b^n)^2$, where $(a^n \perp b^n)$ (i.e., independent; no interaction)

$c^n = \sqrt{(a^n)^2 + (b^n)^2} \neq a^n + b^n$



Discussion

The Binomial Theorem is generated from the equation: $c^n = (a+b)^n$ which establishes the additive, multiplicative, and power relations between all integers a,b,c,n where $c \geq a$ and $c \geq b$ which is always possible by rearranging the integers so that $c \geq (a+b)$

If this cannot be done so that $c \neq (a+b)$, then $c^n \neq (a+b)^n = a^n + b^n + rem(a,b,n)$ so the inequality exists (in particular for $n > 2$).

The Binomial Theorem is true for all positive integers for the case $n = 2$. The reason that Pythagorean Triples exist is because in those cases, $\vec{a} \cdot \bar{b} = 0$. That is, a and b are the legs of a right triangle, so the equation $c^2 = a^2 + b^2 + 2ab = a^2 + b^2 + 2(\vec{a} \cdot \vec{b}) = a^2 + b^2$

If the three integers are NOT Pythagorean triples, then the interaction term $rem(a,b,n)$ exists because the vectors to the integer pairs (from the origin, in a connected vector space, so the vectors are not affine) are not at right angles in the two dimensional integer plane, so the cross product exists, and thus the interaction product.

Consider the product $a^p b^q$ where $p \neq q$, which are characteristic of terms within Rem(a,b,n) in the Binomial Expansion, with a and b independent (on different number lines).

When the number lines are compared in the expansion, they do not commute for n>2 as was the case for the product 2ab for n=2, and so cannot be eliminated in those cases. That means that the vectors are not equal and opposite (do not satisfy the null vector), so "energy" is not conserved (which is why GTR and QFT are ultimately incompatible) for multiple positive particles (bosons) at the origin for n>2, and cannot be factored out by using complex numbers.

In particular, for $n=3$ the terms a^2b and b^2a , a and b do not commute on different number lines, so that

$$m'c^2 = (m_0\gamma)c^2 = \frac{1}{\sqrt{\varepsilon_0\mu_0}}(ct_c\gamma)^2 \text{, } m' = m_0\gamma, = c(t_c\gamma), m_0 = ct_c,\ c = \frac{1}{\sqrt{\varepsilon_0\mu_0}},\ t_c = 1 \text{ where}$$

$a = \mathrm{m}', b = c$ does not commute with $\mathrm{b} = \mathrm{m}', a = c$ in the Binomial Expansion, and m′ is the result of photon (“mass”)- on photon spin interaction (polarization) in the Special Theory of Relativity (Lorentz Transform), where $c' \neq c + v$ as scalars (as opposed to Newton and Galileo).

(Compare this with the expression for total angular momentum $\vec{J} = \vec{L} + \vec{S}$) where $\vec{S}$ refers to photon spin polarization and $\vec{L}$ is orbital angular momentum in the Standard Model.

This is true for all integers of all powers in the two dimensional integer plane (a,b)..
(and certainly for $n > 2$)

It is obvious in a single dimension if the integers are invariant; the Binomial Theorem still holds for positive integers that retain their identity under addition and multiplication.

The Binomial Theorem, Relativity, and Fermat's Theorem

Euler's Formula (02/26/2017 12:21 AM)

Euler's Formula (Math is Fun)

Euler's Formula for Solids (n>2)

(The vectors that make up the edges of polyhedral (Platonic Solids) are affine; that is, they have no common origin. Collapsing any two opposite points of a polyhedron to its origin results in $n = 1$ (contraction of a tensor to the unit metric in the (planar) relativistic unit circle in one (positive) dimension for each distinct integer (a,b) where $\sqrt{(-1^2)} = +1$; otherwise $n = 2$ in Euler's Equation F+V-E = n.

In one dimension, a and b are on the same number line (n=1).

In two dimensions (n=2), either the Pythagorean theorem (triples, triangle) or the Binomial expansion applies.

For n>2, only the Binomial Expansion applies.

Since a and b are arbitrary integers, each product term in $rem(a,b,n)$ in the Binomial Expansion will have the form $\begin{pmatrix} n \\ k \end{pmatrix} a^p b^q$ where $\begin{pmatrix} n \\ k \end{pmatrix}$ is the binary coefficient (a positive integer) for the term and $p < n,\ q < n$. The analysis for the case $n = 2$ can then be applied to each term by applying the interaction matrix

$\begin{vmatrix} a^p \vec{i} & 0 & 0 \\ 0 & b^q \vec{j} & 0 \\ 0 & 0 & 0 \end{vmatrix}$ which results in $\begin{pmatrix} n \\ k \end{pmatrix} a^p b^q (\vec{\vec{0}})$ for each term where the coefficients a^p and b^q commute. The final form of the Binomial Expansion is then

$c^n\left(\vec{\vec{0}}\right) = a^n\left(\vec{\vec{0}}\right) + b^n\left(\vec{\vec{0}}\right) + rem(a,b,n)\left(\vec{\vec{0}}\right) = \left[a^n + b^n + rem(a,b,n)\right]\left(\vec{\vec{0}}\right)$,

where $c^n \neq a^n + b^n$ and in fact $c^2 \neq a^2 + b^2$

Note that no distinction is made between Cartesian and Polar Coordinates, since the two equations $c^n = a^n + b^h + rem(a,b,n)$ and $\pi c^n = \pi\left[a^n + b^h + rem(a,b,n)\right]$ and $\sqrt{2}\left(c^n\right) = \sqrt{2}\left[a^n + b^h + rem(a,b,n)\right]$ (or in fact any multiple of the equation) is valid.

the only stipulation is that a and b are independent and less than or equal to c. The analysis also applies to real numbers.

(as an exercise, set $a = b = 1$ as the basis set for a two-dimensional vector space).

Pythagorean Triples

The proof above proves Fermat's Theorem for independent pairs (a,b) where a common origin exists, so the resultant can be defined, where there is always the interaction term $rem(a,b,2) = 2ab$ for the case $n = 2$, where the arithmetic operation of multiplication between integers applies; an apparent problem remains that there are Pythagorean triples for the case $n = 2$ which apparently contradicts the generality of the Binomial Theorem.

The point is that the arithmetic operation of multiplication (interaction) of terms within the coefficients of each vector (e.g. ab) does not apply to Pythagorean Triples, even though the it appears that way in a triangle diagram; the equation $c^2 = a^2 + b^2$ for Pythagorean Triples, because a, $b$and $c$because the products with exponents only are calculated independently for each vector.

Pythagorean triples then exist because all arithmetic operations are carried out within the coefficients of their respective vectors, with no interactive operations between coefficients of the vectors as specific instances of counting.

For example, $\left(25\right)(1^2)\vec{i} = (23+2)(1^2)\vec{i} = \left(3^2+4^2\right)(1^2)\vec{i} = (8+1)(1^2)\vec{i} + (13+3)(1^2)\vec{i} = ...$, where the factor of $1^2 = \cos^2\theta$ is a result of the metric of the relativistic unit circle being one dimension $\theta = 0$ for positive integers.

In other words

The Binomial Expansion $c^n = (a+b)^n = a^n + b^n + rem(a,b,n)$ provides an expression that is true for all values of a,b,c and n positive integers for $n > 2$. For complete generality, a and b must be independent $\vec{a} \perp \vec{b}$ in the vector space (a,b)

In order to provide an expression that is true for all values of $c^n = a^n + b^n$, we simply equate them so that $c^n = (a+b)^n = a^n + b^n$ for all values of c, and ask "what condition needs to be true so that the expression

$c^n = a^n + b^n$ is valid (for all values of c)?

That condition is $rem\left(a,b,n\right) = 0$, which can only be true if $a = 0$ or $b = 0$. Therefore, a or b cannot be a positive integer.

QED

In other words, suppose there was an integer d such that $d^n = a^n + b^n$

Then $c^n = d^n + rem(a,b,n)$

But we already know that $c^n = (a+b)^n$ is valid for all values of a , b and n .
Therefore d^n is valid only if $rem\left(a,b,n\right) = 0$ so that $d^n = c^n$.

$rem\left(a,b,n\right) = 0$ only iff $a = 0$ or $b = 0$, so a or b cannot be a positive integer. QED

Addendum

The context of the proof is that for two generalized variables, two dimensions characterized by independent variables (a,b) or (x,y) are required for mathematical (i.e., arithmetical) operations on real numbers as well as integers, in particular multiplication and division. This is because the dimensions have to be related by a common metric (i.e., must touch, cannot be parallel in two dimensions), so there is a common connection (the "zero" point, which defines the origin of each dimension as the midpoint of all possible lengths for that dimension). Thus, two dimensions must have a common origin in order to compare metrics.

This is true for real numbers, as well as integers, which is why the Binomial Theorem is applicable - Pythagorean Triplets begin with orthogonal dimensions, and the case for powers is expanded via the Binomial Expansion.

If there is no such connection, it can be removed by "imaginary" (no pun intended); i.e. complex numbers $(a+ib)(a-ib)=a^2+b^2$ which is an operation fundamental to the Pauli characterization of "spin" via the σ_1 and σ_2 matrices which treats number lines on either side of the origin as independent (1,-1), expanded by Dirac to the complete description of the circle (1,-1,1,-1) for two dimensions.

The Relativistic Unit Circle shows that the first integer for positive definite integers is created as $1^2=(\frac{1}{\gamma})^2+\beta^2$ when an arbitrary metric is applied via the so-called "time dilation" equation.

Quantum Field Theory then applies the result to two dimensions (1,1) where $1^2+1^2=2(1^2)$ for positive definite quantities equally and oppositely directed in terms of the null vector, like the analysis above, where the interaction in terms of n and h can be identified with $2ab$ from the Binomial Expansion for $n=2$.

The derivative then becomes a function of first dividing out the successive interactions given by $(a+b)$, and then taking the limit as $b->0$, where $f'(\gamma+\beta)=\lim_{\beta\to 0}\left[\frac{f(\gamma+\beta)-f(\gamma)}{\beta}\right]$ where $f(\gamma+\beta)$ is the Binomial Expansion, and the result is $c=\lim_{\beta\to 0}(\gamma+\beta)=\gamma=\sqrt{1^2}$ by the relativistic unit circle, so that $c^n=a^n$ for $\gamma=a$, and $c^n(\vec{0})=a^n(\vec{0})$. (if $a=\gamma=0$, there is no integer to begin with). When this is accomplished, the constant a can be identified with the slope of a straight line in the coordinate relation $y(\vec{0}) = (Ax)(\vec{0}),\ a = A$, where the derivatives can then be identified with the terms in the Jacobian of the metric tensor in GTR.

The constant $a = ct = m_0(ct)$ can be identified as the "mass" of a particular photon relative to the "zero point" energy at which the photon has no perceived mass (e.g., the parking lot on earth, unless you get a sunburn). The "c" is identified as a specific value (t=1) from the "measured" displacement current from Maxwell's equation, via the force constants from Ampere's and Coulomb's laws.

The process of setting h and n equal to zero in the QFT characterization removes interactions from the theory to the two dimensional characterization of Dirac for two photons (photon -equivalent particles by deBroglie), and finally two a single photon by setting $\beta = \frac{v}{c} = 0$ so the energy of the single photon is given by $1^2 = (\frac{1}{\gamma})^2$ where $m_0 = ct$ has a different value for each (unperturbed) photon depending on its relation to the zero point energy.

Probabilistically, that energy is usually given by a Gaussian spread around some reference black-body temperature in the parking lot. The context can be expanded to the solar system, the galaxy, and beyond if one uses one's fertile imagination. May the Force be with you. (it ain't with me, as far as I know... 😊

This result can be extended to GTR where the non-linear terms in rem(a,b,n) can be shown to correspond to the Christoffel curvature symbols, which relates STR, GTR, The Binomial Theorem and Fermat's Theorem to show that the classical Lagrangian for a two-particle sysem cannot conserve energy for n>2 (and not even for n=2 if spin is included).

I believe this can be extended through the Euler Conjecture for many-body particles (integers), since if Fermat's Theorem is true for two particles, multiple particles can be arranged by the Axiom of Choice in a similar fashion (computer simulation of Mathematics notwithstanding), etc. etc. as well)

(There is much more to be said about this, but I don’t have space to write it in this document…

☺