The Pauli/Dirac Matrices

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This document shows the relationship of the Pauli/Dirac matrices to the relativistic unit circle

(I’m currently writing it, but this will give you a taste so far).

The Relativistic Unit Circle (under revision)

There are three Pauli Matrices for the case of two identical particles $\gamma = 1, \beta = 1$:

$$\sigma_1 = \begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix} = \begin{vmatrix} 0 & \gamma \\ \gamma & 0 \end{vmatrix}, \gamma = 1$$

$$\sigma_2 = \begin{vmatrix} 0 & i \\ -i & 0 \end{vmatrix} = \begin{vmatrix} 0 & i\beta \\ -i\beta & 0 \end{vmatrix}, \beta = 1$$

$$\sigma_3 = \begin{vmatrix} 1^2 & 0 \\ 0 & i^2 \end{vmatrix} = \begin{vmatrix} \gamma & 0 \\ 0 & -\gamma^2 \end{vmatrix}, \gamma = 1$$

$$\sigma_3 = \begin{vmatrix} 1 & 0 \\ 0 & -1 \end{vmatrix} = \begin{vmatrix} \gamma & 0 \\ 0 & -\gamma \end{vmatrix}, \gamma = 1$$

$$\begin{vmatrix}\gamma\\0\end{vmatrix}\vec{i}\otimes\begin{vmatrix}0\\-\gamma\end{vmatrix}\vec{j}=-\gamma^2\vec{k}=\gamma^2\left(\overrightarrow{-k}\right)$$

$$\begin{vmatrix}0\\-\gamma\end{vmatrix}\vec{j}\otimes\begin{vmatrix}\gamma\\0\end{vmatrix}\vec{i}=\gamma^2(\vec{k})$$

$$E_3=(i\otimes j)+(j\otimes i)=(\gamma^2+\gamma^2)\left[\vec{k}+\left(\overrightarrow{-k}\right)\right]=2\left(\gamma^2\right)(\bar{0})=2(1^2)(\bar{0})$$

Stern=Gerlach Experiment

Although the energy represented as null is equal and opposite, it is directed along an axis perpendicular to the direction of motion classically; globally, we call the direction of motion $\vec{v}\cdot\vec{I}$, and indicate the equal and opposite attractive forces by $(\bar{0}\cdot\vec{J})$

$$\sigma_1+\sigma_2=\begin{vmatrix}0&\gamma\\\gamma&0\end{vmatrix}+\begin{vmatrix}0&i\beta\\-i\beta&0\end{vmatrix}=\begin{vmatrix}0&\gamma+i\beta\\\gamma-i\beta&0\end{vmatrix}$$

$$E_{\sigma_1+\sigma_2}=\vec{i}\otimes\vec{j}=\begin{vmatrix}0\\\gamma-i\beta\end{vmatrix}\vec{i}\otimes\begin{vmatrix}\gamma+i\beta\\0\end{vmatrix}\vec{j}=-(\gamma+i\beta)(\gamma-i\beta)\vec{k}=\left(\gamma^2+\beta^2\right)\left(-\vec{k}\right)$$

$$\vec{j}\otimes\vec{i}=\begin{vmatrix}\gamma+i\beta\\0\end{vmatrix}\vec{j}\otimes\begin{vmatrix}0\\\gamma-i\beta\end{vmatrix}\vec{i}=(\gamma+i\beta)(\gamma-i\beta)\vec{k}=\left(\gamma^2+\beta^2\right)\vec{k}$$

$$\left(\vec{i}\otimes\vec{j}\right)+\left(\vec{j}\otimes\vec{i}\right)=2\left(\gamma^2+\beta^2\right)\left(\vec{k}+\left(-\vec{k}\right)\right)=2\left(\gamma^2+\beta^2\right)(\bar{0})=\left(2(\gamma^2)+2\beta^2\right)(\bar{0})$$

Here we see there is an extra term $2\beta^2$ that is different from the initial condition of σ_3

This term represents the energy carried to the system from the B field that was turned on to separate the “spins”

We can add this term to the σ_3 initial condition by assuming it is not interacting; thus the total system is represented by the interaction charges and a separate interacting B field:

$$\sigma_4=\begin{vmatrix}\beta&0\\0&-\beta\end{vmatrix}$$

$$\vec{i}\otimes\vec{j}=\begin{vmatrix}\beta\\0\end{vmatrix}\vec{i}\otimes\begin{vmatrix}0\\-\beta\end{vmatrix}\vec{j}=(\beta)(-\beta)\vec{k}=\left(\beta^2\right)\overrightarrow{-k}$$

$$\vec{j}\otimes\vec{i}=\begin{vmatrix}0\\-\beta\end{vmatrix}\vec{j}\otimes\begin{vmatrix}\beta\\0\end{vmatrix}\vec{i}=(-\beta)(\beta)\overrightarrow{-k}=\left(\beta^2\right)\vec{k}$$

$$\left(\vec{i}\otimes\vec{j}\right)+\left(\vec{j}\otimes\vec{i}\right)=2\beta^2\left(\vec{0}\right)$$

The total of the interactions from and σ_3 σ_4 as initial states is then

$E_3 + E_4 = 2\gamma^2(\bar{0}) + 2\beta^2\left(\vec{0}\right) = 2\left(\gamma^2 + \beta^2\right)$, and is equal to that of the final states:

$$E_{\sigma_1+\sigma_2} = E_{\sigma_3+\sigma_4} = 2\left(\gamma^2 + \beta^2\right)(\bar{0})$$

The Dirac γ^0 Matrix

The Dirac γ^0 matrix can then be represented from the relativistic unit circle as a ccw Lorentz rotation corresponding to separate rotations of $\theta = \frac{\pi}{2}$ starting with $\theta = 0$:

$$\gamma^0 = \begin{vmatrix} 1^2 & 0 & 0 & 0 \\ 0 & 1^2 & 0 & 0 \\ 0 & 0 & -1^2 & 0 \\ 0 & 0 & 0 & -1^2 \end{vmatrix} = \begin{vmatrix} \gamma^2 & 0 & 0 & 0 \\ 0 & \beta^2 & 0 & 0 \\ 0 & 0 & (i\gamma)^2 & 0 \\ 0 & 0 & 0 & (i\beta)^2 \end{vmatrix} = \begin{vmatrix} \cos^2\theta & 0 & 0 & 0 \\ 0 & \sin^2\theta & 0 & 0 \\ 0 & 0 & -\cos^2\theta & 0 \\ 0 & 0 & 0 & -\sin^2\theta \end{vmatrix},$$

where the trace of the upper left quadrant together with the trace of the lower right quadrant yields the Pauli σ_3 matrix :

$$\sigma_3 = \begin{vmatrix} 1^2 & 0 \\ 0 & -1^2 \end{vmatrix} = \begin{vmatrix} Tr(A) & 0 \\ 0 & Tr(B) \end{vmatrix}, \ |A| = \begin{vmatrix} \cos^2\theta & 0 \\ 0 & \sin^2\theta \end{vmatrix}, \ |B| = \begin{vmatrix} -\cos^2\theta & 0 \\ 0 & -\sin^2\theta \end{vmatrix}$$

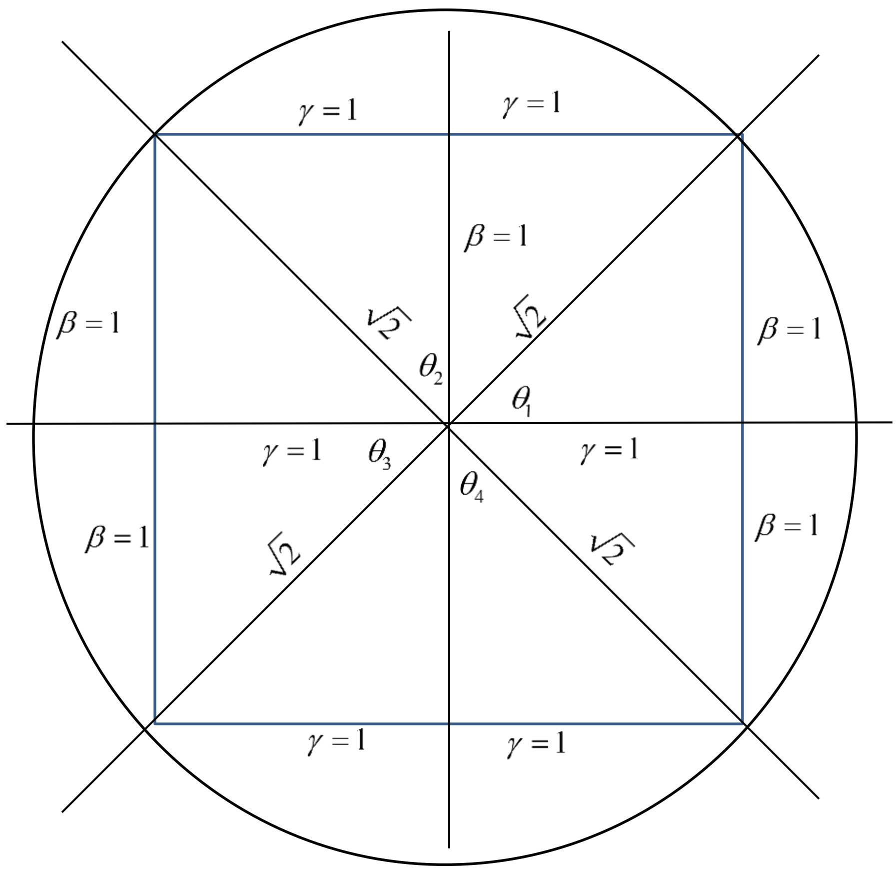
$$\gamma^0 = \begin{vmatrix} |A| & 0 \\ 0 & |B| \end{vmatrix}$$

This matrix can be reconfigured to represent polarity reversal:

$$\gamma_0 = \begin{vmatrix} 1^2 & 0 & 0 & 0 \\ 0 & 1^2 & 0 & 0 \\ 0 & 0 & i^2 & 0 \\ 0 & 0 & 0 & i^2 \end{vmatrix} = \begin{vmatrix} \gamma & 0 & 0 & 0 \\ 0 & -\gamma & 0 & 0 \\ 0 & 0 & \beta & 0 \\ 0 & 0 & 0 & -\beta \end{vmatrix} = \begin{vmatrix} \cos\theta & 0 & 0 & 0 \\ 0 & -\cos\theta & 0 & 0 \\ 0 & 0 & \sin\theta & 0 \\ 0 & 0 & 0 & -\sin\theta \end{vmatrix}$$

Spin and Angular Momentum quantization is represented by Lorentz rotation to the diagonals, where $\gamma = \beta$ introduces a factor of $\frac{1}{\sqrt{2}}$, so that an initial Lorentz rotation of $\frac{\pi}{4}$ corresponds to:

$$\gamma_0 = \begin{vmatrix} \frac{1}{\sqrt{2}}\gamma & 0 & 0 & 0 \\ 0 & \frac{1}{\sqrt{2}}\beta & 0 & 0 \\ 0 & 0 & -\frac{1}{\sqrt{2}}\gamma & 0 \\ 0 & 0 & 0 & -\frac{1}{\sqrt{2}}\beta \end{vmatrix} = \begin{vmatrix} \frac{1}{\sqrt{2}}\cos\theta & 0 & 0 & 0 \\ 0 & \frac{1}{\sqrt{2}}\sin\theta & 0 & 0 \\ 0 & 0 & -\frac{1}{\sqrt{2}}\cos\theta & 0 \\ 0 & 0 & 0 & -\frac{1}{\sqrt{2}}\sin\theta \end{vmatrix}$$



CCW Rotation

Positive Definite $\left(\gamma^0\right)^2$

$$\left(\gamma^0\right)^2 = I^4 = \begin{vmatrix} 1^4 & 0 & 0 & 0 \\ 0 & 1^4 & 0 & 0 \\ 0 & 0 & 1^4 & 0 \\ 0 & 0 & 0 & 1^4 \end{vmatrix} = \begin{vmatrix} \gamma^4 & 0 & 0 & 0 \\ 0 & \beta^4 & 0 & 0 \\ 0 & 0 & (i\gamma)^4 & 0 \\ 0 & 0 & 0 & (i\beta)^4 \end{vmatrix} = \begin{vmatrix} \cos^4\theta & 0 & 0 & 0 \\ 0 & \sin^4\theta & 0 & 0 \\ 0 & 0 & \cos^4\theta & 0 \\ 0 & 0 & 0 & \sin^4\theta \end{vmatrix}$$

Note that the trace of both matrices in the upper left and lower right quadrants is now:

$$Tr(A) = Tr(B) = \cos^4\theta + \sin^4\theta$$

Compare this result with the equation:

$$\left(\cos^2\theta + \sin^2\theta\right)^2 = \cos^4\theta + 2\sin^2\theta\cos^2\theta + \sin^4\theta = \gamma^4 + 2\gamma^2\beta^2 + \beta^4$$

Note that there is now an “interaction” term $2\gamma^2\beta^2$ that now must be accounted for in the expansion for the positive definite representation of γ^0 This is done by the reamaining Dirac gamma matrices, γ^1, γ^2 and γ^3.

$$\gamma^0 = \left(\sqrt{\gamma^0}\right)^2 = \begin{vmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & i & 0 \\ 0 & 0 & 0 & i \end{vmatrix} \begin{vmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & i & 0 \\ 0 & 0 & 0 & i \end{vmatrix} = \begin{vmatrix} 1^2 & 0 & 0 & 0 \\ 0 & 1^2 & 0 & 0 \\ 0 & 0 & i^2 & 0 \\ 0 & 0 & 0 & i^2 \end{vmatrix} = \begin{vmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{vmatrix}$$

$$\left(\gamma^0\right)^2 = \begin{vmatrix} 1^2 & 0 & 0 & 0 \\ 0 & 1^2 & 0 & 0 \\ 0 & 0 & i^2 & 0 \\ 0 & 0 & 0 & i^2 \end{vmatrix}^2 = \begin{vmatrix} 1^4 & 0 & 0 & 0 \\ 0 & 1^4 & 0 & 0 \\ 0 & 0 & 1^4 & 0 \\ 0 & 0 & 0 & 1^4 \end{vmatrix}$$

$$\gamma^0 = \left(\sqrt{\gamma^0}\right)^2 = \begin{vmatrix} \gamma & 0 & 0 & 0 \\ 0 & \gamma & 0 & 0 \\ 0 & 0 & i\beta & 0 \\ 0 & 0 & 0 & i\beta \end{vmatrix} \begin{vmatrix} \gamma & 0 & 0 & 0 \\ 0 & \gamma & 0 & 0 \\ 0 & 0 & i\beta & 0 \\ 0 & 0 & 0 & i\beta \end{vmatrix} = \begin{vmatrix} \gamma^2 & 0 & 0 & 0 \\ 0 & \gamma^2 & 0 & 0 \\ 0 & 0 & -\beta^2 & 0 \\ 0 & 0 & 0 & -\beta^2 \end{vmatrix}$$

$$\gamma^1 = \begin{vmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & i & 0 \\ 0 & 0 & 0 & i \end{vmatrix} \begin{vmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & i & 0 & 0 \\ i & 0 & 0 & \end{vmatrix} = \begin{vmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{vmatrix}$$

$$= \left[\left(\sqrt{\gamma_0}\right)\left(\sqrt{\gamma_0}\right)^T \right]$$

$$\gamma^2 = \begin{vmatrix} 0 & 0 & 0 & -i \\ 0 & 0 & i & 0 \\ 0 & i & 0 & 0 \\ -i & 0 & 0 & 0 \end{vmatrix}$$

$$\gamma^3 = \begin{vmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \\ -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{vmatrix} = \begin{vmatrix} 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \\ -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{vmatrix} \begin{vmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{vmatrix}$$

$$\left(\gamma^0\right)^2 = \begin{vmatrix} 1^2 & 0 & 0 & 0 \\ 0 & 1^2 & 0 & 0 \\ 0 & 0 & i^2 & 0 \\ 0 & 0 & 0 & i^2 \end{vmatrix}^2 = \begin{vmatrix} 1^4 & 0 & 0 & 0 \\ 0 & 1^4 & 0 & 0 \\ 0 & 0 & 1^4 & 0 \\ 0 & 0 & 0 & 1^4 \end{vmatrix}$$

$$\left(\gamma^1\right)^2 = \begin{vmatrix} 0 & 0 & 0 & 1^2 \\ 0 & 0 & 1^2 & 0 \\ 0 & i^2 & 0 & 0 \\ i^2 & 0 & 0 & 0 \end{vmatrix}^2 = \begin{vmatrix} -\left(1^4\right) & 0 & 0 & 0 \\ 0 & -\left(1^4\right) & 0 & 0 \\ 0 & 0 & -\left(1^4\right) & 0 \\ 0 & 0 & 0 & -\left(1^4\right) \end{vmatrix}$$

$$\left(\gamma^2\right)^2 = \begin{vmatrix} 0 & 0 & 0 & -i \\ 0 & 0 & i & 0 \\ 0 & i & 0 & 0 \\ -i & 0 & 0 & 0 \end{vmatrix}^2 = \begin{vmatrix} -(1)^2 & 0 & 0 & 0 \\ 0 & -(1)^2 & 0 & 0 \\ 0 & 0 & -(1)^2 & 0 \\ 0 & 0 & 0 & -(1)^2 \end{vmatrix} = \begin{vmatrix} i^2 & 0 & 0 & 0 \\ 0 & i^2 & 0 & 0 \\ 0 & 0 & i^2 & 0 \\ 0 & 0 & 0 & i^2 \end{vmatrix}$$

$$\left(\gamma^3\right)^2 = \begin{vmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \\ -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{vmatrix}^2 = \begin{vmatrix} 0 & 0 & \gamma & 0 \\ 0 & 0 & 0 & (i\gamma) \\ (i\gamma) & 0 & 0 & 0 \\ 0 & \gamma & 0 & 0 \end{vmatrix}^2 = \begin{vmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{vmatrix} = \begin{vmatrix} i^2 & 0 & 0 & 0 \\ 0 & i^2 & 0 & 0 \\ 0 & 0 & i^2 & 0 \\ 0 & 0 & 0 & i^2 \end{vmatrix}$$

$$\gamma^5 = \begin{vmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{vmatrix}$$

$$\gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3$$

The interaction product

The Interaction Product is the area ("energy") of the interaction triangle $A_{\gamma\beta} = \frac{1}{2}\gamma\beta = \frac{1}{2}1^2$ in each quadrant at the null vector at the vector connection (or at the circumference only if equal and opposite as the tangent, since independent on the plane (γ, β) For $\gamma = \beta = 1$, there is a phase difference of $\Delta\theta = \frac{\pi}{2}$ in each quadrant, and of $\Delta\theta = \pi$ for each axis considered separately.

For each axis separately, with equal and opposite interaction at the origin:

$(+\gamma, -\gamma)$, $(+\beta, -\beta)$ phase difference π

$$\begin{vmatrix} \beta & 0 \\ 0 & -\beta \end{vmatrix} \rightarrow (1^2)\beta^2(\vec{\vec{0}})$$

$$\begin{vmatrix} \gamma & 0 \\ 0 & -\gamma \end{vmatrix} \rightarrow (1^2)\gamma^2(\vec{\vec{0}})$$

For $\gamma = \beta = 1$, the sum is equal to $2\gamma\beta(\vec{\vec{0}}) = 2(1^2)(\vec{\vec{0}})$

Energy as sum of quadrants

For the energy in each quadrant, Lorentz rotating ccw in the relativistic unit circle with phase difference $\frac{\pi}{2}$:

$(+\gamma, +\beta)$ 1st Quadrant

$$\begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} \rightarrow (\vec{i} \otimes \vec{j}) = 1^2(\vec{k})$$

$$\begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix} \rightarrow (\vec{j} \otimes \vec{i}) = 1^2(\overrightarrow{-k}) .$$

$$(\vec{i} \otimes \vec{j}) + (\vec{j} \otimes \vec{i}) = 1^2(\vec{\vec{0}})$$

Interaction Product:

$$\begin{vmatrix} \gamma & 0 \\ 0 & \beta \end{vmatrix} \rightarrow \left(\frac{1}{2}\right)\beta\gamma(\vec{\vec{0}})$$

$(+\beta, -\gamma)$ 2nd quadrant

$$\begin{vmatrix} 1 & 0 \\ 0 & -1 \end{vmatrix} \rightarrow \left(\vec{i} \otimes \vec{j}\right) = 1^2\left(\overrightarrow{-k}\right)$$

$$\begin{vmatrix} 0 & 1 \\ -1 & 0 \end{vmatrix} \rightarrow \left(\vec{j} \otimes \vec{i}\right) = 1^2\left(\vec{k}\right)$$

$$\left(\vec{i} \otimes \vec{j}\right) + \left(\vec{j} \otimes \vec{i}\right) = 1^2\left(\vec{0}\right)$$

Interaction Product:

$$\begin{vmatrix} \beta & 0 \\ 0 & -\gamma \end{vmatrix} \rightarrow \left(\frac{1}{2}\right)\beta\gamma\left(\vec{0}\right)$$

$(-\gamma, -\beta)$ 3rd quadrant

$$\begin{vmatrix} -1 & 0 \\ 0 & -1 \end{vmatrix} \rightarrow \left(\vec{i} \otimes \vec{j}\right) = 1^2\left(\vec{k}\right)$$

$$\begin{vmatrix} 0 & -1 \\ -1 & 0 \end{vmatrix} \rightarrow \left(\vec{j} \otimes \vec{i}\right) = 1^2\left(\overrightarrow{-k}\right)$$

$$\left(\vec{i} \otimes \vec{j}\right) + \left(\vec{j} \otimes \vec{i}\right) = 1^2\left(\vec{0}\right)$$

Interaction Product

$$\left(\frac{1}{2}\right)\begin{vmatrix} -\beta & 0 \\ 0 & -\gamma \end{vmatrix} \rightarrow \left(\frac{1}{2}\right)\beta\gamma\left(\vec{0}\right)$$

$(-\beta, +\gamma)$ 4th quadrant

$$\begin{vmatrix} -1 & 0 \\ 0 & 1 \end{vmatrix} \rightarrow \left(\vec{i} \otimes \vec{j}\right) = 1^2\left(\overrightarrow{-k}\right)$$

$$\begin{vmatrix} 0 & -1 \\ 1 & 0 \end{vmatrix} \rightarrow \left(\vec{j} \otimes \vec{i}\right) = 1^2\left(\vec{k}\right)$$

$$\left(\vec{i} \otimes \vec{j}\right) + \left(\vec{j} \otimes \vec{i}\right) = 1^2\left(\vec{0}\right)$$

Interaction Product:

$$\left(\frac{1}{2}\right)\begin{vmatrix} -\beta & 0 \\ 0 & \gamma \end{vmatrix} \rightarrow \left(\frac{1}{2}\right)\beta\gamma\left(\vec{0}\right)$$

$$E\left(\vec{0}\right) = 4\left(\frac{1}{2}\gamma\beta\right) = 2\gamma\beta$$

The interaction energy can also be calculated for $\gamma = \beta$ in each quadrant as the matrix (e.g. in the first quadrant) as

$$\begin{vmatrix} \frac{\gamma}{\sqrt{2}} & 0 \\ 0 & \frac{\beta}{\sqrt{2}} \end{vmatrix} \rightarrow \frac{1}{2}(\gamma\beta)$$, consistent with the "argument by geometry" above.

This interaction relation is actually spin polarization energy of photon on photon interaction (with β representing a change (or perturbation) to the initial "rest energy of the photon γ. However, if β is characterized as the "spin" of a photon-equivalent electron (i.e., a massive heavier photon), then the "spin" is independent of the electron mass as defined by the relativistic unit circle; if there is no "spin" interaction with an external field, then the initial state of the electron is the same as the final state $ct = ct', \gamma^2 = 1^2, \beta^2 = 0$ and its "rest energy" or "rest mass" is invariant.

The spin can be quantized; for the case $\gamma = \beta = 1 = h$ describing two photon - equivalent electrons (or photons) interaction, the interaction energy is then defined as $E_s = s^2 = \frac{1}{2}h^2$ for each quadrant of the relativistic circle, so that

$s = \frac{1}{\sqrt{2}}h = \frac{h}{\sqrt{2}}$. For n such interactions, then $s_n = \frac{nh}{\sqrt{2}} = ns$

Consider the case where $\gamma\beta > 1$ (i.e., the initial state is invariant). Then $(\gamma')^2 = 1^2 + (\gamma\beta)^2$ Since $\gamma = 1$ in the case of the relativistic unit circle, this becomes

$(\gamma')^2 = \gamma^2 + (\gamma\beta)^2$, where γ can be thought of as the photo-equivalent electron.

Then for n such electrons, the equation becomes $(\gamma')^2 = (n\gamma)^2 + (n\gamma)^2(\beta)^2 = n^2 + n(\beta)^3$ However, since $\beta^3 = (1)^3$, $\beta > 0$ the equation can be written as $(\gamma')^2 = n^2 + n(1)^3 = n(n + (1)^3)$, (where $(1)^3$ usually appears as "1", so that $(\gamma')^2 = n^2 + n$), bearing in mind that β is always positive if the interaction is between bases in the first quadrant. Note that if $\beta = 0$, $(\gamma')^2 = n^2$ (i.e., there is no interaction). In particular, $\lim_{n\to\infty}(\gamma')^2 = \lim_{n\to\infty} n^2 + n = \lim_{n\to\infty} n(n+1) = n^2$; i.e., n^2 is analogous to a "continuum" in the state model of atomic physics (note that β^3 suggests a three dimensional “volume”, but the precise discussion is outside the scope of this document)..

From the Binomial Expansion, $\psi^n = (\gamma + \beta)^n = \gamma^n + \beta^n + rem(\gamma, \beta, n)$ For the case $n = 2$, $rem(\gamma, \beta, 2) = 2\gamma\beta$ which is always positive in the first quadrant (and the third), so that it only vanishes if $\beta = 0,\ \gamma = 1$ (there is only one non-interacting photon or photo-equivalent electron).