(Charles Keyser, 1/15/2023)

## Planck's constant, Neutrinos and Spin

Consider the following:
Interactions require the existence of two positive definite particles, where the count is given by:
$(\#)^{2}=(a+b)^{2}=\left[a^{2}+b^{2}\right]+2 a b$ where the interaction ("entanglement") term is $2 a b$.

For $b=(a+\delta)>a$ and $a:=(c t), b:=\left(v t^{\prime}\right), a \geq 1_{(c t)}$; the difference $(\Delta)^{2}$ between the affine sum $\left(a^{2}+a^{2}\right)$ and the interacting sum $\left(a^{2}+b^{2}\right)$ is characterized by the relation:

$$
\left(\#_{\Delta}\right)^{2}:=(a-b)^{2}=\left(a^{2}+b^{2}\right)-2 a b=\left(\left[(c t)^{2}+\left(v t^{\prime}\right)^{2}\right]-2(c t)\left(v t^{\prime}\right)\right.
$$

The particle $b=\left(v t^{\prime}\right)=a+\delta$ can be characterized as the integer difference $\delta$ that must be subtracted ("radiated") from $b$ so $b-\delta=a$ resulting in identical (non-interacting, prime, affine) particles. $\delta$ is sometimes referred to as an "affine" connection.

Note that $a-b=0 \equiv b=a ; \delta=0$
Both $2 a b$ and $\delta$ are absent from the relation:

$$
\begin{aligned}
& \psi:=a+i b \\
& \psi^{*}:=a-i b \\
& \psi \psi^{*}=(a+i b)(a-i b)=a^{2}+[i b a-i a b]-(i b)^{2}
\end{aligned}
$$

where $\psi \psi^{*}=a^{2}+b^{2}$ only if $i^{2}=-1$
(Einstein was attempting to declare an affine connection where one doesn't exist in the context of complex numbers).

## Example

Here is an example using a basic $(3,4,5)$ Pythagorean Triad
$a=4$
$b=3$
$\left(\#_{\Delta}\right)^{2}=(a-b)^{2}=\left[a^{2}+b^{2}\right]-2 a b=(\sqrt{25})^{2}-(\sqrt{24})^{2}=1^{2}$
$\left(\#_{\Delta}\right)^{2}=1^{2}$
In general, for two particles $a$ and $b$, where $a>b$ and $a=b+c$,
$\left(\#_{\Delta}\right)^{2}:=c^{2}=[a+(a+c)]^{2}-2(a)(a+c)$, where $\left(\#_{\Delta}\right)^{2}:=c^{2}$ is the term that must be set to zero so the two particles will be affine (identical, prime, non-interacting). That is,
$c^{2}=0 \leftrightarrow\left[a^{2}+a^{2}\right]-2 a^{2}=0$ so that $\left[a^{2}+a^{2}\right]=2 a^{2}$
In the example of the triad above,

$$
\left(\#_{\Delta}\right)^{2}:=1^{2}=[a+(a+1)]^{2}-2(a)(a+1)
$$

Hint: try to calculate the expressions where $a=1, b=2, \delta=1$
$\left(\#_{\Delta}\right)^{2}=c^{2}=[1+(1+1)]^{2}-2(1)(1+1)=5(1)^{2}-4\left(1^{2}\right)=1^{2}$
(note there is no interaction between 1 and $(1+1)^{2}=1^{2}+1^{2}$ since $4(1)^{2}=(1+1)^{2} \neq 1^{2}+1^{2}=1^{2}-(i)^{2}$

## The Pauli Matrices

The latter is the foundation of the $\operatorname{SU}(2)$ group, in particular the three Pauli matrices:
$\left|\sigma_{1}\right|:=\left|\begin{array}{cc}0 & \sqrt{1} \\ \sqrt{1} & 0\end{array}\right|,\left|\sigma_{1}\right|^{2}=\left|\begin{array}{ll}1 & 0 \\ 0 & 1\end{array}\right|, \operatorname{Tr}\left|\sigma_{1}\right|^{2}=2, \operatorname{Det}\left|\sigma_{1}\right|^{2}=1^{2}$
Note that the trace of is zero (existence is omitted, only "interaction" is included)
$\left|\sigma_{2}\right|:=\left|\begin{array}{cc}0 & i \\ -i & 0\end{array}\right|,\left|\sigma_{2}\right|^{2}=\left|\begin{array}{ll}1 & 0 \\ 0 & 1\end{array}\right|=\left|\begin{array}{cc}-i^{2} & 0 \\ 0 & -i^{2}\end{array}\right|$
$\operatorname{Tr}\left|\sigma_{2}\right|^{2}=2(1)=2\left(-i^{2}\right), \operatorname{Det}\left|\sigma_{2}\right|^{2}=-\left(i^{2}\right)\left(i^{2}\right)=2 i^{2}=2(1)$

As in $\left|\sigma_{1}\right|$, the trace of $\left|\sigma_{2}\right|$ is zero, so only imaginary particles are represented in first ordser.

$$
\begin{aligned}
& \left|\sigma_{3}\right|:=\left|\begin{array}{cc}
\sqrt{1} & 0 \\
0 & \sqrt{-1}
\end{array}\right|=\left|\begin{array}{cc}
\sqrt{1} & 0 \\
0 & i
\end{array}\right| \\
& \operatorname{Tr}\left(\left|\sigma_{3}\right|\right)=\sqrt{1}-\sqrt{1} \neq 0 \rightarrow 1 \neq 1 \text { since } \\
& \operatorname{Tr}\left(\left|\begin{array}{cc}
\sqrt{1} & 0 \\
0 & \sqrt{1}
\end{array}\right|\right)=\sqrt{1}+\sqrt{1}=2(\sqrt{1}) \leftrightarrow(\sqrt{1}-\sqrt{1})=0 \rightarrow 1^{2}-1^{2}=0 \leftrightarrow 1=1 \rightarrow 1-1=0 \text { for two particles. } \\
& \left|\sigma_{1}\right|+\left|\sigma_{2}\right|=\left|\begin{array}{cc}
0 & 1+i \\
1-i & 0
\end{array}\right|=\left|\begin{array}{cc}
0 & \psi \\
\psi^{*} & 0
\end{array}\right| \\
& \psi \psi^{*}:=-\operatorname{Det}\left|\begin{array}{cc}
0 & \psi \\
\psi^{*} & 0
\end{array}\right|=(1+i)(1-i)=1^{2}+1 \neq 2
\end{aligned}
$$

This relation will work for any Pythagorean triad where the hypotenuse of the right triangle is defined by $\psi \psi^{*}$, but note that

$$
\begin{aligned}
& \psi:=(4+3 i) \\
& \psi^{*}:=(4-3 i) \\
& \psi \psi^{*}=(4+3 i)(4-3 i)=5^{2}=25=4^{2}+3^{2}=4^{2}+4[(3 i)-(3 i)]-(3 i)^{2}
\end{aligned}
$$

provided that $i^{2}=-1$. However, $i^{2}$ lies on the imaginary axis, and $-1<0$ lies on the (nonexistent) negative axis, (since in the complete case, all values are positive definite), so this prescription is invalid.

Note that since $a^{2}+a^{2}=2 a^{2}$ for $\left(\#_{\Delta}\right)^{2}=0, a^{2}$ must be odd prime $1_{a^{2}}:=\frac{a^{2}}{a^{2}} ; a^{2}\left(1_{a^{2}}\right)=a^{2}$ and therefore $2 a^{2}$ is even. Then $a+a=2 a$ is also even, true for any number $a$, which is a statement of Goldbach's Conjecture (every even number is the sum of two primes). The term $(a-a)=0$ for two particles expresses the tautology that $a=a$ for $a \neq 0$.

Note that $25=\psi \psi^{*}=(3+i 4)(3-i 4)$ (Pythagorean Triple) has the structure odd $=$ odd + even, as does $(\#)^{2}=(7)^{2}=(3+4)^{2}=\left[\psi \psi^{*}\right]+2 a b=25+24=49$ so that the term $\left[\psi \psi^{*}\right]$ is real only in the context where count is preserved.

## Pauli and the Stern-Gerlach experiment

For Pauli's analysis of the Stern-Gerlach experiment (modeling "Spin") in terms of (invariant) "mass"

$$
\begin{aligned}
& \left|\sigma_{1}\right|+\left|\sigma_{2}\right|=\left|\begin{array}{cc}
0 & E+i B \\
E-i B & 0
\end{array}\right| \\
& \psi:=\operatorname{det}\left(-\left[\left|\sigma_{1}\right|+\left|\sigma_{2}\right|\right]\right)=\left(E^{2}-(i B)^{2}\right)=\left(E^{2}+B\right) \neq 2
\end{aligned}
$$

This is equivalent to saying that the charge to mass ratio of a particle modeled by $E^{2}$ was unchanged by an imaginary $B$ so the mass each of the particles at the end of the experiment each having imaginary spin of $S=\frac{1}{2}$ somehow directed them in opposite directions due to the imaginary excitation of the $B$ field at the application point.

## Spin

The difference between $(\#)^{2}$ and $\left(\#_{\Delta}\right)^{2}$ is the "glue" that holds two different particles together (e.g. a proton and electron: i.e., a neutron).

Compare the two expressions for the sums $(\#)^{2}$ and $\left(\#_{\Delta}\right)^{2}$ where $b>a$ :
$(\#)^{2}=(a+b)^{2}=\left[a^{2}+b^{2}\right]+2(a b)_{\#}$
$\left(\#_{\Delta}\right)^{2}=(b-a)^{2}=\left[a^{2}+b^{2}\right]-2(a b)_{\Delta}$
$\left(h_{\#}\right)^{2}:=2(a b)_{\#}=2\left(S_{\#}\right)^{2}$
$\left(h_{\Delta}\right)^{2}:=2(a b)_{\Delta}=2\left(S_{\Delta}\right)^{2}$
$\left(S_{\#}\right)=\frac{h_{\#}}{\sqrt{2}}$
$\left(S_{\Delta}\right)=\frac{h_{\Delta}}{\sqrt{2}}$
Note that the difference $-2 S>0$ because $\left(\#_{\Delta}\right)^{2}>0 \rightarrow\left[a^{2}+b^{2}\right]>(-2 a b)$ so the negative sign refers to a difference between positive values. This gives two different values for Planck's constant in relation to the different spins. One can only assign a common spin

$$
\begin{aligned}
& a=b=1 \\
& S:=\frac{1}{\sqrt{2}}+\frac{i}{\sqrt{2}} \\
& S^{*}:=\frac{1}{\sqrt{2}}-\frac{i}{\sqrt{2}} \\
& S S^{*}=\frac{1}{2}+i^{2}\left(\frac{1}{2}\right) \rightarrow\left[\frac{1}{2}+\left(-\frac{1}{2}\right)\right] \neq 1
\end{aligned}
$$

if $S$ is first order and imaginary, which is inconsistent with positive definite existing values.
The difference $(\#)^{2}-\left(\#_{\Delta}\right)^{2}=\left(h_{\#}\right)^{2}-\left(h_{\Delta}\right)^{2}:=\delta^{2}$ is the energy of the particle that needs to be radiated in order for the particles to be identical, in which case $\left(h_{\#}\right)^{2}=\left(h_{\Delta}\right)^{2}=0$
$\delta^{2}$ then represents the change of state where the particle emitted changes the state of the system from interacting to non-interacting (affine) ( $b=a$ ).

## Planck's "constant"

Consider the Interactive equation for two identical particles $a$
\#: $=a+a$
$(\#)^{2}=(a+a)^{2}=a^{2}+a^{2}+2 a^{2}$
$S:=a$
$h^{2}:=2 a^{2}=2 S^{2}$
$S=\frac{h}{\sqrt{2}}$
Note that if there is only one particle, $(\#)^{2}=a^{2}$, so that $S=0$ (a Boson) and if the two particles are identical there is no affine connection (as above)

Therefore, for interactions there are two values of $h$ that distinguish the counts between positive and negative particles. Note that for two identical (non-interacting, prime) particles

$$
\begin{aligned}
& (\#)^{2}=(a+a)^{2}=\left[a^{2}+a^{2}\right]+2 a^{2} \\
& (\#)^{2}=(a-a)^{2}=\left[a^{2}+a^{2}\right]-2 a^{2}=0 \\
& {\left[a^{2}+a^{2}\right]=2 a^{2}}
\end{aligned}
$$

That is, the difference term $\left(\#_{\Delta}\right)^{2}$ doesn't exist.

Note:
$\varphi:=(a+1)-a$
$\varphi^{2}=\left[(a+1)^{2}+a^{2}\right]-2 a(a+1)=1^{2}$
$\psi:=(a+1)+i a$
$\psi^{*}:=(a+1)-i a$
$\psi \psi^{*}=\left[(a+1)^{2}+a^{2}\right]$

$$
\begin{aligned}
& b=(a+k) \\
& \varphi:=c=a+b \\
& \varphi^{2}=c^{2}=\left[a^{2}+(a+k)^{2}\right]-2 a(a+k) \\
& b^{2}=(a+k)^{2} \\
& c^{2}=\left[a^{2}+b^{2}\right]-2 a b=k^{2} \\
& c=k
\end{aligned}
$$

$$
\begin{aligned}
& \psi=a+i b=a+i(a+k) \\
& \psi^{*}=a-i b=a-i(a+k) \\
& \psi \psi^{*}=a^{2}+b^{2}+i a b-i a b=a^{2}+b^{2} \\
& c^{2}=\left[a^{2}+b^{2}\right]-2 a b=\left[\psi \psi^{*}\right]-2 a b \\
& \psi \psi^{*} \neq c^{2}
\end{aligned}
$$

## The Process

1. Initially there are two non-interacting identical particles of energy
$(5)^{2}=25$ so total energy is $50=25<+>25$, noting that both sides of the equation (and those that follow) are multiplied implicitly by $1_{(c t)^{2}}=\frac{(c t)^{2}}{(c t)^{2}}$ or $1_{\left.(c t)^{\prime}\right)^{2}}=\frac{\left(c t^{\prime}\right)^{2}}{\left(c t^{\prime}\right)^{2}}$
2. When they interact, they share one particle of interaction, so the total energy is $(49)=(7)^{2}=(3+4)^{2}+24+1=\left[p p^{*}\right]+24+1$
3. If they separate, either a particle of interaction is absorbed so the total is again 50 , or a second particle of interaction is radiated (to the cosmic background radiation), so the total is
$[48]=[24<->24]<->[1<->1]$ (i.e., 4 non-interacting particles.
Note that [48] is even, and so is the sum of two primes and that 24 is also even so it also is the sum of two primes.

The process of radiation continues until the final result $2=1<+>1$ is the final contribution to the cosmic background radiation.

