

Proof of Fermat's Theorem and Euler's Conjecture

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The Relativistic Unit Circle

Natural Logarithms (and Fermat's Theorem)

$$z^n = x^n + y^n$$

$$n \log z = n \log x + n \log y$$

$$\log z = \log x + \log y$$

$$z^n = \gamma^n + \beta^n$$

$$n \log z = n \log \gamma + n \log \beta$$

$$\log z = \log \gamma + \log \beta$$

Therefore, no unique solution for any n except $n=0$ which is a contradiction, since that would imply $z^0 = x^0 + y^0$ (i.e., $1+1=2$)

That is, z cannot be an algebraic (i.e., countable) unless $\log y=0$ for some integer algebraic number y , which can never be true (i.e., unless $\beta=0$).

For Fermat's Theorem, replace (z, x, y) by integers (c, a, b)

Euler's Conjecture

This can be extended to m variables;

$$z^n = \sum_m (x_m)^n = n \log x_0 + n \log x_1, \dots, + n \log x_m = n \log z$$

$$\log z = \sum_m (\log x_m)$$

For $n=0$, $\gamma_m = \sqrt{(\gamma_m)^2} = |\log x_m|$, this is the trace of the unit metric tensor:

$$I_m = \begin{vmatrix} |\log x_0| & 0 & 0 & 0 & 0 \\ 0 & |\log x_1| & 0 & 0 & 0 \\ 0 & 0 & \dots & 0 & 0 \\ 0 & 0 & 0 & \dots & 0 \\ 0 & 0 & 0 & 0 & |\log x_m| \end{vmatrix} = \begin{vmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & \dots & 0 & 0 \\ 0 & 0 & 0 & \dots & 0 \\ 0 & 0 & 0 & 0 & 1 \end{vmatrix} = \begin{vmatrix} \frac{x_0}{x_0} & 0 & 0 & 0 & 0 \\ 0 & \frac{x_1}{x_1} & 0 & 0 & 0 \\ 0 & 0 & \dots & 0 & 0 \\ 0 & 0 & 0 & \dots & 0 \\ 0 & 0 & 0 & 0 & \frac{x_m}{x_m} \end{vmatrix} =$$

$Tr(I_m) = \log z = 1+1+1+\dots,+1$ which cannot be an integer for $m > 0, \beta_m \neq 0,$

Expanding to m metrics defined by l initial conditions, gives the relation

$(c_1\tau^1, c_2\tau^2, \dots, c_p\tau^q)$ where the indices do not indicate powers, but rather distinct initial, final, or perturbations, in terms of which the other two components are to be interpreted.

For the relativistic unit circle, if the final state is invariant, then there will be a relativistic unit circle for each initial condition, and the changes of state will have the form

$(\gamma_1\beta^1, \gamma_2\beta^2, \dots, \gamma_p\beta^q)$, where $(c_p\tau^q)^2 = (v_q\tau^q)^2 + (c_p\tau^q)^2$ for each p and q taken individually (i.e., Einstein summing convention does NOT apply).

Bottom Line: Fermat is correct, and Einstein, Wittgenstein, and Goedel were all right and wrong for exactly the same reasons... ☺

From the Relativistic Unit Circle

1. Case $\gamma < 1, \beta < 1$

$$1^2 = \gamma^2 + \beta^2, c^2 1^2 = c^2 = (c\gamma)^2 + (c\beta)^2$$

$a = c\gamma < c, \gamma < 1$, γ is not an integer, and similarly, $b = c\beta < c, \beta < 1$ β is not an integer, so c^2 is not an integer, so c is not an integer.

2. Case $\beta = 1, \gamma = 0 \Rightarrow 1^2 = \beta^2, c^2 = \beta^2$

3. Case $\beta = 0, \gamma = 1 \Rightarrow 1^2 = \gamma^2, c^2 = \gamma^2$

4. Case $\beta = 1, \gamma = 1 \Rightarrow c^2 = 1^2 + 1^2 = 2(1)^2$. $c = \sqrt{2}$ is not an integer

5. Case $\gamma = 1, \beta > 1, (\gamma')^2 = \gamma^2 + \gamma\beta$

$$c = \gamma', a = \gamma, \beta = b$$

$$c^2 = a^2 + ab = a^2 + n^2$$

a. $n^2 = ab$ (Pythagorean Triple)

b. $n = 0 \Rightarrow c^2 = a^2$ (One Dimension)

c. $c^2 = (a+b)^2 = a^2 + b^2 + 2ab = a^2 + b^2 + 2n$

$$n = 0 \Rightarrow c^2 = (a+b)^2 \text{ (Pythagorean Triple)}$$

$$n = ab \Rightarrow c^2 = (a+b)^2 = a^2 + b^2 + 2ab \text{ (Binomial Theorem)}$$

c is not an integer in the vector space (a, b) If c is an integer, then either the vector space is one dimensional or (a, b, c) are a Pythagorean Triple, where a, b , and c are unrelated metrically (they are coefficients of affine vectors, and have no common metric origin).

$$(ct', vt''),$$

$$v < c$$

$$t'' = t' \gamma', \gamma' = \frac{1}{\sqrt{1 - (\beta)^2}}, \beta = \frac{v}{c}$$

$$v > c$$

$$\gamma'' = (1)^2 + \gamma\beta = ct''$$

$$ct'' = (1)^2 + \gamma\beta = (1)^2 + \frac{v}{c} \gamma = (1)^2 + \frac{vt}{ct} \gamma = (1)^2 + \frac{vt}{\sqrt{1}} \gamma$$

Case $\beta > 1$

$$(c')^2 = \gamma^2 + \gamma\beta = a^2 + a\beta, \beta > 1, \gamma = 1, \beta > 1 \text{ Suppose } n \text{ is an integer.}$$

$$(\gamma') = c, a = 1, b = \beta\gamma = \beta, \gamma = 1$$

Condition that b is an integer is that $\beta = \frac{v}{c}$ is an integer, so $\beta = \frac{bc}{c} = b$

$$(\gamma')^2 = 1^2 + b$$

Condition that $(\gamma')^2$ is an integer is that $\gamma' = \sqrt{(ct')^2 + b}$ is an integer

$$\gamma' = ct'' = c't$$

$$(c't'')^2 = (ct')^2 + b$$

$$a^2 = (ct')^2 \text{ is an integer if } ct' \text{ is an integer and } a = ct'$$

If both c and t are integers, then a is an integer, $(ct'')^2 = (a)^2 + b = (ct')^2 + b$.

Then $(ct'')^2$ is an integer if $\sqrt{(ct')^2 + b} = \sqrt{a^2 + b^2} = \sqrt{a^2 + \beta^2} = \sqrt{\gamma^2 + \beta^2}$ is an integer, so

that $k(ct'')^2 = m\sqrt{\gamma^2 + \beta^2} = m(\sqrt{1^2})$ so that $(r'')^2 = (ct'')^2 = \frac{m}{k} = \frac{\sqrt{m^2}}{\sqrt{k^2}}$ is an integer.

$$\text{For } b > a, (r'')^2 = (ct'')^2 = \frac{b}{a} = \frac{\sqrt{b^2}}{\sqrt{a^2}},$$

Then $n(\gamma')^2 = n(1^2) + n(\gamma\beta)$ $n(\gamma')^2 = c^2$ can only be an integer if $n(\gamma\beta) = b^2$ forms a Pythagorean triple so that $c^2 = 1^2 + b^2$; i.e. $1^2 = c^2 - b^2$ if (a,b,c) do not form a Pythagorean triple, then either c or b is not an integer, so that $1^2 \neq c^2 - b^2$. Then $n(1)^2 = n(c^2 - b^2) = nc^2 - nb^2$ cannot be an integer, so either c^2 or b^2 is not an integer.

Fermat's Theorem is true...

Christoffel Symbols

$$[(c(x)t(x), v(x)t'(x))] = [(c(x)t(x), c'(x)t'(x))]$$

$$(c(x)t(x), (c'(x)t'(x)))$$

$$m_0(x) = (c(x)t(x))$$

$$m'(x) = (c'(x)t'(x))$$

$$t'(x) = t(x)\gamma(x), \gamma(x) = \frac{1}{1 - [\beta(x)]^2}, \beta(x) = \frac{v(x)}{c(x)} = \frac{c'(x)}{c(x)}$$

$$c'(x) = c(x), \gamma(x) = 1, v(x) = 0$$

Tangent vectors (local coordinate basis) $e_i = \frac{\partial}{\partial x^i} = \partial_i, i = 1, 2, 3, \dots, n$

$$\bar{e}^i = \bar{e}^1 + \bar{e}^2 + \dots + \bar{e}^n$$

$$e^0 = 1$$

$$(e^i)^2 = (e^1 + e^2 + \dots + e^n)^2 = (e^1)^2 + (e^2)^2 \dots (e^n)^2 + \text{rem}(e^1, e^2, \dots, e^n, n)$$

$$\begin{vmatrix} \bar{e}^i & 0 \\ 0 & \bar{e}^i \end{vmatrix} \rightarrow (e^i)^2(\bar{0}) = [(e^1)^2 + (e^2)^2 \dots (e^n)^2 + \text{rem}(e^1, e^2, \dots, e^n, n)](\bar{0})$$

$$(e^i)^2(\bar{0}) = (e^1)^2 + (e^2)^2 \dots (e^n)^2 \Leftrightarrow \text{rem}(e^1, e^2, \dots, e^n, n) = 0$$

This is true if even one of the basis vectors $e^k = 0$, which "contracts" the tensor (reduces the dimension by one)...

$\frac{\partial \bar{x}}{\partial x} = \gamma_x, \partial \bar{x} = (\gamma_x) \partial x$; if $x = ct = m$, then $\gamma_x \geq 1$ is a density, the mass of the unit circle increases.

$\frac{\partial x}{\partial x} = \frac{1}{\gamma_x}, \partial x = \frac{1}{\gamma_x} \partial \bar{x}$ and $\gamma_x \geq 1$ then the radius of the relativistic unit circle decreases, and so does its mass.

If $\beta = \frac{v}{c}$ an increase in c will decrease the relative interaction β , and vice versa.

Conversely, and increase in v will increase the relative interaction β , and vice versa.

Note that $c \rightarrow 0$ implies that $v \rightarrow 0$, so $c=0 \Leftrightarrow v=0$

For ct' the final state of a single (possibly perturbed) system, $1^2 = \gamma^2 + \beta^2$, $\gamma \leq 1$, $0 \leq \beta < 1$

For $ct = ct'$, $\gamma = 1$, $\beta = 0$

For $ct = \gamma$ the initial state of the system, $(\gamma')^2 = \gamma^2 + \gamma\beta$. If $\gamma = n$, $\beta = 1$, then

$(\gamma')^2 = n^2 + n = n(n+1)$ so that $\lim_{n \rightarrow \infty} (\gamma')^2 = n^2$.

If $\beta \neq 0$, $(\beta')^2 = \beta^2 + \gamma\beta$ - If $\gamma = 1$, $\beta = n$ then $(\beta')^2 = n^2 + n = n(n+1)$ so that $\lim_{n \rightarrow \infty} (\beta')^2 = n^2$

, $\lim_{n \rightarrow \infty} \frac{1}{\gamma} = \frac{1}{m} = 0$

$\gamma_x = 1 \Leftrightarrow \gamma_j = \beta_j = 0, j \neq x$

$\vec{r}^{ij} = \vec{e}^i + \vec{e}^j$

$(\vec{r}^{ij})^2 = (\vec{e}^i)^2 \vec{i} + (\vec{e}^j)^2 \vec{j} + 2\vec{e}^i \vec{e}^j \vec{k}^{ij} = \left(\frac{\partial}{\partial x^i}\right)^2 \vec{i} + \left(\frac{\partial}{\partial x^j}\right)^2 \vec{j} + 2 \frac{\partial}{\partial x^i} \frac{\partial}{\partial x^j} \vec{k}^{ij}$

$f = g(dx^i \vec{e}^i, dx^j \vec{e}^j)$

Let $x = f(x)$ be a coordinate value and $h(x) = m(x) = \rho_x x = \gamma_x x$

$\frac{dx}{x} \vec{i} = f'(x) \vec{i} = \frac{df(x)}{dx} \vec{i} = \vec{1i}$

$$\text{Let } g(x)\vec{i} = (x + \gamma x)\vec{i} = x\vec{i} + (\gamma x)\vec{i}$$

$$\text{Then } \frac{d(g(x))\vec{i}}{dx} = (1 + \gamma)\vec{i} = (1)\vec{i} + \gamma\vec{i} \text{ and } \frac{d(g(x))\vec{i}}{d\gamma} = x\vec{i}$$

$$[g(x)]^n \vec{i} = (x + \gamma x)^n \vec{i} = x^n + (\gamma x)^n + \text{rem}(x, (\gamma x), n)$$

$$\frac{d[g(x)]^n \vec{i}}{dx} = (n-1)(x + \gamma x)^{n-1} \vec{i} = (n-1)x^{n-1} + (n-1)(\gamma x)^{n-1} + \frac{d[\text{rem}(x, (\gamma x), n)]}{dx}$$

$$\frac{d[g(x)]^n \vec{i}}{dn} = n \log(g(x))\vec{i}$$

$$g(x) = nx, \frac{d[g(x)]^n}{dn} = n^2$$

$$g(x) = x, \frac{d[g(x)]^n}{dn} = 1$$

$$g(x) = \frac{1}{x}, \frac{d[g(x)]^n}{dn} = -1$$

$$g(x) = \frac{1}{n}, \frac{d[g(x)]^n}{dn} = -n$$

Spin Quantization (ct Invariant Final Condition)

$$\begin{vmatrix} \gamma & 0 \\ 0 & \gamma \end{vmatrix} \rightarrow \gamma^2 (\vec{0})$$

$$\begin{vmatrix} \beta & 0 \\ 0 & \beta \end{vmatrix} \rightarrow \beta^2 (\vec{0})$$

$$1^2 (\vec{0}) = (\gamma^2 + \beta^2) (\vec{0}) = [\cos^2 \theta + \sin^2 \theta] (\vec{0})$$

$$s_1 = \begin{vmatrix} \gamma & 0 \\ 0 & \beta \end{vmatrix} = \gamma \vec{i} \otimes \beta \vec{j} = \gamma \beta (\vec{i} \otimes \vec{j}) = \gamma \beta \vec{k}$$

$$s_3 = \begin{vmatrix} -\gamma & 0 \\ 0 & -\beta \end{vmatrix} = (-\gamma) \vec{i} \otimes (-\beta) \vec{j} = \gamma \beta (\vec{i} \otimes \vec{j}) = \gamma \beta \vec{k}$$

$$s_2 = \begin{vmatrix} -\gamma & 0 \\ 0 & \beta \end{vmatrix} = (-\gamma) \vec{i} \otimes (\beta) \vec{j} = -\gamma \beta (\vec{i} \otimes \vec{j}) = \gamma \beta (-\vec{k}) = \gamma \beta (\vec{k})$$

$$s_4 = \begin{vmatrix} \gamma & 0 \\ 0 & -\beta \end{vmatrix} = (\gamma) \vec{i} \otimes (-\beta) \vec{j} = -\gamma \beta (\vec{i} \otimes \vec{j}) = \gamma \beta (-\vec{k}) = \gamma \beta (\vec{k})$$

$$s_1 + s_3 = 2\gamma\beta\vec{k}$$

$$s_2 + s_4 = 2\gamma\beta(\vec{k})$$

$$\gamma = h = h_\gamma, \quad \beta = h = h_\beta$$

$$(s_1 + s_3) + (s_2 + s_4) = 2\gamma\beta\vec{k} + 2\gamma\beta(\vec{k}) = 2\gamma\beta(\vec{0}) = 2h^2$$

$$s_1 = s_2 = s_3 = s_4 = s^2$$

$$4s^2 = 2h^2$$

$$s = \frac{h}{\sqrt{2}}$$

$$\text{Note that } 1^2 = (h_\gamma)^2 + (h_\beta)^2 \Leftrightarrow h_\gamma = 1, \quad h_\beta = 0.$$

Quantization by n (ct' invariant Initial Condition)

$$(\gamma')^2 = 1^2 + \gamma\beta = (h_\gamma)^2 = h_\gamma^2 + h_\gamma\beta$$

$$h_\gamma = n, \quad \beta = h_\beta = 1$$

$$E_n' = (\gamma')^2 = n(n+1)$$

Consequences of the Binomial Theorem

From the Binomial Theorem, $(h_\tau)^n = [(h_\gamma) + (h_\beta)]^n = (h_\gamma)^n + (h_\beta)^n + \text{rem}(h_\gamma, h_\beta, n)$.

If $h_\gamma = 1$, then $(h_\tau)^n = (h_\gamma^n) + (h_\beta^n) + \text{rem}(h_\gamma, h_\beta, n)$ cannot be an integer multiple of (h_γ^n) unless $(h_\beta^n) = 0$, so $\text{rem}(h_\gamma, h_\beta, n) = 0$ and

$$E_n = (nh_\tau v)^n = \left(n \frac{h_\gamma c}{\lambda}\right)^n = \left(\frac{nh_\gamma}{\tau}\right)^n = \left(\frac{n}{\tau}\right)^n = [(m_0)_{\gamma, \tau}]^n c^2, \text{ where } [(m_0)_{\gamma, \tau}] = \frac{1}{\sqrt{(\varepsilon_0)_{\gamma, \tau} (\mu_0)_{\gamma, \tau}}}$$

and $(\varepsilon_0)_{\gamma, \tau} \perp (\mu_0)_{\gamma, \tau}$ [i.e. $(m_0)_{\gamma, \tau} = \left(\frac{n}{\tau}\right)$, $h_\gamma = 1$

Impulse Response

(Final State invariant)

Note that the area (i.e., "energy") of the relativistic unit circle is independent of the "parity" of the "momenta", since

$$r^2 = 1^2 = (\gamma^2 + \beta^2) = (\gamma + i\beta)(\gamma - i\beta),$$

and

$$r^2 = (\pm\gamma)^2 + (\pm\beta)^2 = (\pm\gamma)^2 + (\mp\beta)^2 = (\mp\gamma)^2 + (\pm\beta)^2$$

(Initial State invariant)

This is not true of a system with "spin" however, since $(\gamma')^2 = 1^2 + 2\gamma\beta$ so that either γ or β can be negative.

If $\gamma\beta > 0$ That means that a system originally having energy of $E_0 = 1^2$ has been increased to $E' = 1^2 + 2\gamma\beta = E_0 + 2\gamma\beta$, corresponding to an absorption of $\Delta E = 2\gamma\beta$

If $\gamma\beta < 0$ That means that a system originally having energy of $E_0 = 1^2$ has decreased to $E' = (\gamma')^2 = 1^2 - 2\gamma\beta = E_0 - 2\gamma\beta$, corresponding to radiation of $\Delta E = 2\gamma\beta$

In both cases the sensor or source (absorbing or emitting ΔE , respectively, is not specified.

For $\Delta E = 2\gamma\beta$, $h' = \gamma\beta \Rightarrow \Delta E = 2(h')^2$, so $h' = \frac{\Delta E}{\sqrt{2}}$

Then for $E_0 = h = 1^2$, the total energy is given by $(E_{total})^2 = (E_0)^2 \pm (\Delta E)^2$

Lorentz Transform

For the Lorentz transform, this corresponds to

$$\gamma' = ct' = (ct)\gamma \pm (vt') = (ct)\gamma \pm (\beta ct')\gamma, \beta' = \pm \left(\frac{v'}{c}\right), \gamma = 1$$

$$t' = t \pm (\beta t') = t + (\pm\beta t')$$

$$(t')^2 = (t)^2 + (\pm\beta t')^2, |\beta t'| < |t|$$

Suppose a system is characterized by the Binomial Expansion for the basis $\vec{\gamma} = \gamma \vec{i}$, $\gamma = 1$

$$\begin{aligned} (\gamma + \beta)^n &= (\gamma + \beta)^2 (\gamma + \beta)^{n-2} = (\gamma^2 + \beta^2)(\gamma + \beta)^{n-2} + 2\gamma\beta(\gamma + \beta)^{n-2} \\ (\gamma + \beta)^n - 2\gamma\beta(\gamma + \beta)^{n-2} &= (\gamma^2 + \beta^2)(\gamma + \beta)^{n-2} \\ (\gamma + \beta)^{n-2}(\gamma^2 + \beta^2) &= (\gamma + \beta)^{n-2}(1^2) = (\gamma + \beta)^n - 2\gamma\beta(\gamma + \beta)^{n-2} \end{aligned}$$

Therefore, the system $(\gamma + \beta)^{n-2}$ multiplied by the impulse $1^2 = (\gamma^2 + \beta^2)$ yields the system response $(\gamma + \beta)^n - 2\gamma\beta(\gamma + \beta)^{n-2}$; i.e., $(\gamma + \beta)^{n-2}(1^2) = (\gamma + \beta)^n - 2\gamma\beta(\gamma + \beta)^{n-2}$. This is possible only if $\beta = 0$, so that $(\gamma + \beta)^{n-2}(1^2) = (\gamma^{n-2})(\gamma^2) = \gamma^n$ (lhs) and $(\gamma + \beta)^n - 2\gamma\beta(\gamma + \beta)^{n-2} = \gamma^n$ (rhs)

$$\begin{aligned} r^2 &= (\gamma + i\beta)(\gamma - i\beta) = (\pm\gamma)^2 + (\pm\beta)^2 = 1^2 \\ &= (\pm\gamma)^2 + (\mp\beta)^2 = (\mp\gamma)^2 + (\pm\beta)^2 \end{aligned}$$

Consider the equation

$$\begin{aligned} (\gamma + \beta)^n &= (\gamma + \beta)^2 (\gamma + \beta)^{n-2} = (\gamma^2 + \beta^2 + 2\gamma\beta)(\gamma + \beta)^{n-2} = 1^2 + 2\gamma\beta(\gamma + \beta)^{n-2} \quad \text{so that} \\ 1^2 &= (\gamma + \beta)^n - 2\gamma\beta(\gamma + \beta)^{n-2} \end{aligned}$$

Then 1^2 is the impulse for any linear system $\Psi(\gamma, \beta, n)$ where the system response is $\varphi(\gamma, \beta, n)$; that is $[\Psi(\gamma, \beta, n)][(\gamma + \beta)^n - 2\gamma\beta(\gamma + \beta)^{n-2}] = \varphi(\gamma, \beta, n)$

Suppose a system is given by the Binomial Expansion for the basis $\vec{\gamma} = \gamma \vec{i}$, $\gamma = 1$

$$\psi(\gamma, \beta, n) = \psi(1, \beta, n) = (\gamma + \beta)^n = (1 + \beta)^n$$

This can only be true if $[\Psi(\gamma, \beta, n)] = 1^{n-2}$ so that $\varphi(\gamma, \beta, n) = 1^n = (\gamma^2 + \beta^2)^n$,

that is, $2\gamma\beta=0$, which means that $\gamma=0$, $\beta=0$, or $\gamma\vec{i}\cdot\beta\vec{j}=\gamma\beta(\vec{i}\cdot\vec{j})=\pm\sin\theta=\pm 0$ and therefore $\theta=0$ or $\theta=\pi$ in the relativistic unit circle ($\beta=\pm 0$).

$$\begin{vmatrix} \beta & 0 \\ 0 & \beta \end{vmatrix} = \begin{vmatrix} \pm 0 & 0 \\ 0 & \pm 0 \end{vmatrix} \rightarrow (\pm 0)^2 (\vec{0}) \text{ so the } \vec{j} \text{ dimension does not exist (it has no metric length).}$$

Vector spaces

A vector space is formed for each Lorentz rotation of the relativistic unit circle where either $m_0=ct=1$ (no perturbation $v'=0$) or $m_0'=ct'=1$ is taken as the final state, and constitutes a dimension, where the dimension of the system is $n=n(ct)$ or $n=n(ct')$

That gives a set of positive integers $1^2, 2(1^2), 3(1^2), \dots, n(1^2)$; if no integers are created at all, then $ct=0$ or $ct'=0$. Within the relativistic unit circle, equal and opposite integer creation rates and times yield the null vector for that dimension, where

$$\begin{vmatrix} ct & 0 \\ 0 & ct \end{vmatrix} \rightarrow (\pm ct) \otimes (\pm ct) \rightarrow (ct)^2 (\vec{0}) \quad \text{or} \quad \begin{vmatrix} ct' & 0 \\ 0 & ct' \end{vmatrix} \rightarrow (\pm ct') \otimes (\pm ct') \rightarrow (ct')^2 (\vec{0})$$

This forms the basis set $I_n = \begin{vmatrix} 1^2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1^2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1^2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1^2 & 0 & 0 \\ 0 & 0 & 0 & 0 & \dots & 0 \\ 0 & 0 & 0 & 0 & 0 & 1^2 \end{vmatrix}$, where $Tr(I_n) = n(1^2)$ and

$det(I_n) = (1^2)^n$. Each dimension represented by a basis 1^2 can be multiplied by a metric length x , where the scalar $x=0$ is taken to be the midpoint of all possible metric lengths in that dimension. Each dimension is then represented by the symbols $\vec{i}, \vec{j}, \vec{k}, \dots$, so that a vector is represented by $\vec{x} = x\vec{i} = x=0(1^2)\vec{i}$ in that dimension. For $x=0$, there is no metric length associated with the dimension, so that if a common metric for all dimensions is assumed, the trace of the system is $n=0$. Setting $x=0$ for a single dimension removes that dimension from the system (a tensor "contraction"), so the dimension is then $n-1$.

This results when the "dot" product of two basis vectors are taken $\vec{i}\cdot\vec{j}=0$. Conversely, an additional dimension is added when the "cross" product of two basis vectors is taken: $\vec{i}\otimes\vec{j}=\vec{k}$, $\vec{j}\otimes\vec{i}=(\vec{-k})=(\vec{k})$, where the reverse arrow indicates an equal and opposite orientation of the $(\pm k)=(\vec{k})$ vectors. Adding these vectors yields the "null" vector representing an equal and opposite "force" in physics):

$(\vec{i} \otimes \vec{j}) + (\vec{j} \otimes \vec{i}) = \vec{k} + \vec{k} = (\vec{0})$, where a scalar multiple of the null vector gives the magnitude of the equal and opposite "force" $\vec{x} = x(\vec{0})$. If $x=0$, the magnitude of this force is equal to zero, so that $\vec{0} = 0(\vec{0})$. Which is a contraction of the system; if such a vector is retained in the system, then the determinant is zero for the whole system.

If interactions $\Delta vt' = (vt')\vec{s}$ are included in the system for the metrics represented by $(1)(\vec{ct}) = (1)(1^2)\vec{i}$, then if it is independent of the dimension, it is characterized by a second dimension \vec{s} so that $\vec{c} = c(\vec{r}) = c(\vec{i} \otimes \vec{s}) = c(\vec{k}_s)$ where $\vec{r} = \vec{k}_s$ is not a member of the original system, since it is not a basis vector unless explicitly included; it is therefore orthogonal to the hyperplane of the original system.

If the three vectors (\vec{ct}) , (\vec{ct}') , and (\vec{vt}') have no common metric, then they are "affine" – i.e., their individual metrics have no common origin, or unrelated (in the case of integers, these vectors form a Pythagorean triple $(\vec{a}, \vec{b}, \vec{c})$ with no common origin, and hence no common metric; in particular, there is no common null vector for the vector space.

If they have a common origin (so that the midpoints of all possible lengths in each dimension coincide, then the vector relation is given by $(ct')\vec{r} = (vt')\vec{s} + (ct)\vec{i}$. If it is understood that $\vec{r} = \vec{s}$ and \vec{k} represents vectors not in the hyper plane given by (\vec{i}_n, \vec{j}_n) the relation can then be characterized by the relation $(ct')\vec{r} = (vt')\vec{j} + (ct)\vec{i}$. This relation can also be characterized as an initial condition \vec{ct} , a "perturbation" (\vec{vt}') and a final condition (\vec{ct}') , where $(vt') = 0$ means that $\vec{ct}' = \vec{ct}$ so that $t' = t$ for the common "metric creation rate" c .

(ct') as Invariant Final Condition

The relativistic unit circle is then derived by considering the two "inputs" (ct) and (vt') in terms of the final "output" (ct') , so that dividing by the scalar (ct') yields the relation

$(1)\vec{r} = \left(\frac{v}{c}\right)\vec{j} + \left(\frac{t}{t'}\right)\vec{i} = \beta\vec{i} + \gamma\vec{j}$. The magnitude of the vector \vec{r} is given by the relation

$$1^2 = r^2 = \gamma^2 + \beta^2 \text{ so that } 1 = \sqrt{\gamma^2 + \beta^2}$$

For Fermat's Theorem that $n = n(\sqrt{\gamma^2 + \beta^2})$ cannot be an integer unless either $\gamma^2 = 0$ or $\beta^2 = 0$.

Since the metric relation given by the scaling factors t and t' , the relation $1^2 = r^2 = \gamma^2 + \beta^2$ includes all possible relations in the vector space defined by \vec{r} .

Therefore, consider the relation $(r')^2 = (\gamma + \beta)^2 = \gamma^2 + \beta^2 + 2\gamma\beta = 1^2 + 2\gamma\beta$. For the relativistic unit circle, $\gamma\beta$ must be equal to zero, so that $\gamma \cdot \beta = 0$, which means that $\beta \perp \gamma$ (γ and β are orthogonal, so γ is independent of any value of β , and vice versa. This also means that the interaction $\gamma\beta$ has no common metric, so its interaction "energy" is "virtual", or imaginary. It thus can be removed from $(\gamma + \beta)^2$ by the prescription

$$r^2 = (\gamma^2 + \beta^2) = (\gamma + i\beta)(\gamma - i\beta), \text{ where } i = \sqrt{-1}$$

(ct) as Invariant Initial Condition

If, however, the equation $(ct')\vec{r}' = (vt')\vec{s} + (ct)\vec{i}$ is characterized by the "initial" condition ct

, then $\left(\frac{t'}{t}\right)\vec{r} = \left(\frac{vt'}{ct}\right)\vec{s} + (1)\vec{i}$, so that $\left(\frac{t'}{t}\right)^2 = \left(\frac{vt'}{ct}\right)^2 + (1^2) = (\beta\gamma)^2 + (1^2)$. In terms of the

state as the final condition, this can only be valid if $v=0$ so that $t'=t$. However, if a second "interactive" metric is applied so that $(1') = (\gamma')^2 = (\gamma\beta) + 1^2$, then this characterizes

a new "basis" for the dimension characterized by \vec{r} . For any integer n , if $n^2 = \gamma\beta$ so that $\gamma=1, \beta=n$, then the new basis is given by $(1')^2 = (\gamma')^2 = (\gamma\beta) + 1^2$, so

$(1')^2 = (\gamma')^2 = (n\gamma) + 1^2 = (n\gamma) + \gamma^2 = \gamma(n+1)$. Finally, if $\gamma = n$, then $(1')^2 = (\gamma')^2 = S = n(n+1)$

. This new basis is then characterized as "spin", consisting of the energy of the relativistic unit circle plus the interaction energy $\gamma\beta$ for "momentum" $\beta=n$, so that $S^2 = [n(n+1)]^2$.

Note that for $\beta=1, S^2 = 2 = 2(\gamma^2)$, so $S = \sqrt{2}\gamma$. This corresponds to the equal and opposite

additional interaction between $\pm S$, characterized by $\begin{vmatrix} S & 0 \\ 0 & S \end{vmatrix} \rightarrow S^2(\vec{0})$

If γ has the quantized energy h , so that $\gamma = h = 1$ then for a single value of interaction

energy, $S = (1') = h\sqrt{2}$, so that $\gamma' = \frac{h}{\sqrt{2}}$. For n such spins, $nS = n\gamma' = \frac{nh}{\sqrt{2}}$ in the dimension

$\vec{r} = \vec{k}$, where $\vec{c}' = c^2\vec{r}' = (\gamma^2 + \beta^2) + \frac{n}{\sqrt{2}} = 1^2 + \frac{n}{\sqrt{2}}$

(For Fermat's Theorem, this means that c^2 cannot be an integer unless $n=0$ (i.e., $\beta=0$ for $\gamma=1$), so $c = \sqrt{c^2}$ cannot be an integer.

If c cannot be an integer for the case $n=2$ for either an invariant initial condition or invariant final condition (as an integer), then it can only be an integer if $vt'=0$; i.e. $\beta=0$

where ct and ct' have a common metric. (Again, if there is no common metric, then Pythagorean triples indicate that the vectors are affine, and the iondependent triples result from the base of the number systems as counted in either dimension – but not both).

Then $c^n = (\gamma + \beta)^n = \gamma^n + \beta^n + \text{Rem}(\gamma, \beta, n) = (\gamma + \beta)^2 (\gamma + \beta)^{(n-2)}$ (where $\text{Rem}(a, b, n)$ consists of all terms in the Binomial Expansion that are not γ^n or β^n) cannot be an integer unless either $\gamma = 0$ or $\beta = 0$, since all terms in $\text{Rem}(\gamma, \beta, n)$ must be positive for $\gamma > 0$ and $\beta > 0$

In addition, note that $(\gamma + \beta)^2 (\gamma + \beta)^{(n-2)} = \gamma^n + \beta^n + 2\gamma\beta \text{Rem}(\gamma, \beta, n-2)$, which shows the relation of Spin and the elements of relativistic (photon or photo-electron equivalent mass) interaction (in powers of n) to the Binomial Theorem...

Taking the cross product of two dimensions

If the scalar 0 is taken to be the midpoint of all possible metric lengths, then each of these basis vectors can then be

From the relativistic unit circle (in two dimensions), for ct' selected as the final state

((which is equivalent to the axiom of choice), the $t' = t\gamma$, $\gamma = \frac{1}{\sqrt{1-\beta^2}}$ yields $1^2 = \gamma^2 + \beta^2$, so

that $1 = \sqrt{\gamma^2 + \beta^2}$ where both $\gamma < 1$ and $\beta < 1$ for the circle, so that $c = c\sqrt{\gamma^2 + \beta^2}$ can only be an integer if either $\gamma = 0$ or $\beta = 0$ (i.e., the system is one dimensional).

(The analysis also applies to Cartesian coordinates for γ and β characterized as lengths (i.e., metrics of real number lines).

For two integers (a,b) , the relation for the Binomial Theorem for $n=2$ is given by (at, bt') , so that if a is considered the "reference" (the smallest integer (which amounts to the axiom of choice i.e., as the initial condition), the relation is

$$t' = t\gamma, \gamma = \frac{1}{\sqrt{1-\beta^2}}, \beta = \frac{b}{a} \quad \text{If } c \text{ is chosen as the final condition, then } (ct')^2 = (bt')^2 + (at)^2$$

Consider $\gamma = ct = ct' = 1$ to be the initial condition, so $t = t'$, so $\gamma = 1$ $ct = ct'$, $\gamma = 1$. Then

$$(\gamma')^2 = \gamma^2 + \gamma\beta = 1^2 + (1)\beta, \beta = \frac{v}{c} \quad (\text{A second dimension is being created for } 0 \leq \beta < 1, \text{ independent}$$

of γ) When v increases to $v=c$, $(\gamma')^2 = 1^2 + 1^2 = 2(1^2)$, so that $\gamma' = \sqrt{2}$, where

$$I = \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} \rightarrow 1^2(\bar{0}) \text{ and } Tr(I) = 2 \text{ and } \det(I) = 1^2.$$

If $\frac{\gamma'}{\sqrt{2}} = 1'$ is established as the new "momentum" basis, then we can define $s = \frac{h}{\sqrt{2}} = \frac{\gamma'}{\sqrt{2}}$ as

If $E_0 = \gamma^2$ represents "energy" for a single "photon" and $P_0 = \gamma$ represents "momentum", then the prescription for "spin" is represented by $s = \gamma^2 + \gamma = \gamma(\gamma+1)$, For $\gamma = n$ ("momenta"), then $s = n(n+1)$ However, classical "momentum" is dimensionally inconsistent with classical energy.

For relativistic momentum, however $P = (m_0\gamma)\beta c = (m_0\gamma)\left(\frac{v}{c}\right)c = m_0c(\beta\gamma) = m_0c, \beta\gamma = 1$

$P_{(rel)} = m_0c, \beta\gamma = 1$ (Note that for $m_0 = ct_c$, $P_{(rel)} = c^2 t_c = c^2, t_c = 1$ From Maxwell's result, $c^2 = \frac{1}{\epsilon_0\mu_0}$

where the permittivity ϵ_0 and the permeability μ_0 are force coupling constants from Coulombs' and Ampere's laws, respectively. This means that $P_{(rel)}$ is a relativistic energy, not a classical momentum.

Therefore, if E_0 is orthogonal to P_{rel} in the relation (E_0, P_{rel}) , so that $(E')^2 = (P_{classical})^2 + (E_0)^2$, the relation can be made dimensionally consistent by including $c = 1$ to satisfy the relativistic momentum-energy relationship so that $(E')^2 = (P_0c)^2 + (E_0)^2$

Then for $c = 1$, we can set $\gamma = \gamma c$ so that $s = \gamma^2 + \gamma = \gamma(\gamma+1)$ and for $\gamma c = n$, $s = n^2 + n = n(n+1)$

Then $h = E_0 t_c$, $E_0 = \frac{h}{t_c} = \frac{hc}{\lambda_c}$, $\lambda_c = ct_c$ so that if $ct_c = 1$, $\lambda_c = 1$

Then the "spin" is an additional energy to the relativistic unit circle (which is a model of two independent particles) and characterizes the interaction between them in the new basis $(1') = \frac{\gamma'}{\sqrt{2}}$ where $\gamma' c = h$ and $\gamma' = h$, $c = 1$

$\frac{\gamma'}{\sqrt{2}} = 1'$, where Planck's constant can be identified with $\gamma' = \sqrt{h}$, so that $(1')^2 = \frac{h}{\sqrt{2}}$.

Spin

$$1_0^2 = (\gamma_0^2 + \beta_0^2), \gamma < 1_0, \beta < 1_0$$

$$2 = (\gamma_1^2 + \beta_1^2), \gamma_1 = \beta_1 = 1_1$$

$$\sqrt{(\gamma_1^2 + \beta_1^2)} = \sqrt{2}$$

$$\gamma_1 \beta_1 = 1_1^2$$

$$s = \frac{\gamma_1 \beta_1}{\sqrt{(\gamma_1^2 + \beta_1^2)}} = \frac{1_1^2}{\sqrt{2}}$$

$$h = 1_1^2$$

$$s = \frac{h}{\sqrt{2}}$$

$$(\gamma + \beta)^2 = (\gamma)^2 + (\beta)^2 + 2\gamma\beta$$

$$h = 2\gamma\beta$$

$$\gamma = 1$$

$$(\gamma + \beta)^2 = (1)^2 + (\beta)^2 + 2\beta(1)$$

$$\beta = 1$$

$$(\gamma + \beta)^2 = 2(1^2) + 2(1^2) = 4(1^2) = 4\gamma\beta$$

$$\sqrt{(\gamma + \beta)^2} (1) = (\gamma + \beta)(1) = \sqrt{2}\gamma\beta (1) = \sqrt{2} (1)$$

$$(\gamma + \beta)^2 = (\gamma)^2 + (\beta)^2 + 2\gamma\beta$$

$$h^2 = 2(\gamma\beta)^2, h = \sqrt{2}\gamma\beta$$

$$s = \gamma\beta = \frac{h}{\sqrt{2}}$$

$$h^2 = 2s$$

$$(\gamma + \beta)^2 = 1^2 + 2\gamma\beta = 1^2 + 2\left(\frac{h}{\sqrt{2}}\right) = 1^2 + 2s$$

$$\begin{vmatrix} \sqrt{s} & 0 \\ 0 & \sqrt{s} \end{vmatrix} \rightarrow s(\bar{0})$$

$$\begin{vmatrix} -\sqrt{s} & 0 \\ 0 & -\sqrt{s} \end{vmatrix} \rightarrow s(\bar{0})$$

$$\begin{vmatrix} \sqrt{s} & 0 \\ 0 & \sqrt{s} \end{vmatrix} + \begin{vmatrix} -\sqrt{s} & 0 \\ 0 & -\sqrt{s} \end{vmatrix} \rightarrow 2s(\bar{0})$$

$$(\gamma + \beta)^2(\bar{0}) = [1^2 + 2\gamma\beta](\bar{0}) = 1 \left[1^2 + 2 \left(\frac{h}{\sqrt{2}} \right) \right] (\bar{0}) = [1^2 + 2s](\bar{0})$$

$$(\gamma + \beta)^2 = 1^2, s = 0$$

For equal and opposite interactions where $\gamma_1 = \beta_1 = 1$ we have $\gamma_1^2 + \beta_1^2 = 2(1_0^2) = (1_1)^2$ so that

$\sqrt{2}(1) = \left(\sqrt{\gamma^2 + \beta^2} \right) (1)$ which represents an increase in the radius of the relativistic unit circle from the

case $1_0^2 = (\gamma_0^2 + \beta_0^2)$, $\gamma < 1_0$, $\beta < 1_0$. However, $\gamma\beta = (1_1)^2$, which now represents an increase in the

radius of the relativistic unit circle to a final condition with increased energy due to the interaction

(equal and opposite “rotation” in a quadrant), so that $E_{\gamma\beta} = \sqrt{2}\gamma\beta = \sqrt{2}(1^2)$. If we identify Planck’s

constant with the (equal and opposite) “rotation” energy $E_{\gamma\beta}$ so that $h = E_{\gamma\beta}$, then the quantized

“spin” interaction energy can be represented as $s = 1^2 = \gamma\beta = \frac{h}{\sqrt{2}}$

Spin

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Division by zero

Addition (groups) only require one dimension; $\vec{c} = \vec{ci} = (a \pm b)\vec{i}$, where the sum on the right refers to the single integer on the left side of the equality, but multiplication and its inverse (division (rings))

require two dimensions. Consider the equation $z_{(x,y)} = \frac{x}{y}$, $x = yz_{(x,y)}$ If $y = 0$, $x = yz_{(x,y)}$ means

that x and y can still exist, but their relation is undefined, unless $y\vec{i} = x\vec{i}$ in one dimension, so that $x\vec{i} = 0\vec{i}$, and therefore $x = 0$, which is a point (the midpoint of all possible metrics (lengths) in one dimension).

If $y \neq 0$, then $A = \frac{x}{y}$ is the slope of the line $y = Ax + b$ in two (Cartesian) dimensions.

$$\begin{vmatrix} x & 0 \\ 0 & \frac{1}{y} \end{vmatrix} \rightarrow \left(\frac{x}{y}\right)\vec{0} = A(\vec{0})$$

Likewise, in the equation for radial coordinates $s = r\theta$ then $r = \frac{s}{\theta}$; for $\theta = 0$ the arc length is

undefined unless $s\vec{i} = r\vec{i}$ in one dimension. Also for $\theta = \frac{s}{r}$, for $r = 0$, then $\theta\vec{i} = s\vec{i}$ in one dimension.

$$\begin{vmatrix} r & 0 \\ 0 & \theta \end{vmatrix} \rightarrow (r\theta)(\vec{0}) = s(\vec{0})$$

Using the Einstein summation convention, we have:

$$S^m_i = (\gamma_i + \beta_i)^m = \gamma_i^m + \beta_i^m + \text{Rem}(\gamma_i, \beta_i), \text{ so that}$$

$$E_0(0) = (S^m_i)^2(0) = [(\gamma_i + \beta_i)^m = \gamma_i^m + \beta_i^m + \text{Rem}(\gamma_i, \beta_i)]^2(0)$$

$E_0 = [\gamma_i^m + \beta_i^m + \text{Rem}(\gamma_i, \beta_i)]^2 = \gamma_i^{2m} + \beta_i^{2m} + \text{Rem}(\gamma_i, \beta_i, m, 2)$, where $\text{Rem}(\gamma_i, \beta_i, m, 2)$ consists of all elements in the expansion that are not either γ_i^{2m} or β_i^{2m}

Relation to "space-time"

If we define $x_c = ct_c$ and $x_v = vt_v$ in the (Cartesian) "space-time" domain, then we have the relation

$$\frac{x_v}{x_c} = \frac{vt_v}{ct_c} = \beta_{(x,t)} \frac{t_v}{t_c} . \text{ If } \beta_{(x,t)} \text{ is to be identified with } \beta_{(v,c)} \text{ in the energy- momentum domain then we}$$

must have $\frac{t_v}{t_c} = \gamma$, so $\frac{x_v}{x_c} = \frac{vt_v}{ct_c} = \beta_{(x,t)} \gamma$. But $\frac{t_v}{t_c} = \frac{1}{\sqrt{1-\beta^2}}$, which contradicts the linear relation in

$$\text{Cartesian space-time for } \frac{x_v}{x_c} = \frac{vt_v}{ct_c} = \beta_{(x,t)} \frac{t_v}{t_c} = \beta_{(x,t)} \gamma .$$

This can be remedied in one of two ways. If we set $x_v = x_c$, we have the relation $1 = \beta_{(x,t)} \gamma = \frac{v}{c} \gamma$ so

$$\text{that } v = \frac{c}{\gamma} \text{ Then } c = v\gamma$$

The “Wow” and “Ow” of observation

For $v < c$ consider the end result 1^2 as the result of a single observations at an instant of time (or all possible observations (anywhere, anywhen), since v and c are independent of any particular position or time interval (x, t) , as are the scaling factors t and t'

One can then identify γ as the “Wow” of physics – the observation at the zero-point level (i.e., at the lowest possible observable temperature in the parking lot), and β with the “Ow” of physics – any sense perception not involving light (i.e. sunburn, gravity, ponderable matter, etc.) so that the total of a single observation consists of $1^2 = \beta^2 + \gamma^2 = (\text{"Ow"})^2 + (\text{"Wow"})^2$

Consider the expansion $(\gamma + \beta)^2 = (\gamma^2 + \beta^2) + 2\gamma\beta$ Then the “Interaction” term $2\gamma\beta$ must be something that was missed in the observation (a “virtual” thought (virtual “wow”) imagining an “Felt” event (a virtual “Ow”)).

This term is then responsible for “Dark Energy”, “Dark Matter”, or “god”, depending on your religious inclination – that is, it is an effect with an unseen cause that can be accounted for by either the “Wow” or “Ow” of physics. (This is the distinction between the local constant “g” of gravity that one feels when missing a step on a stairway vs. the imagined global constant G (e.g. from the Solar System observations of the planets, or the concepts of “parallel Universes” in science (fantasy)).

For “red shift” is a change in energy in an unobserved source (or a loss in energy by an unseen intervening density) from a far-away galaxy (or an act of god). For “Einstein rings”, it is the bending of light due to an unseen central mass (or individual photons along an imagined geodesic – or an act of god).

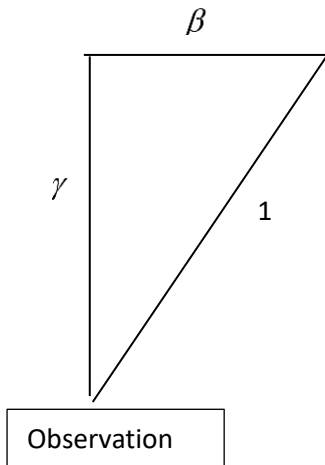
In either case, it amounts to an imagined change in h in quantum mechanics, or an imagined change in the zero-point level (i.e., temperature) in the parking lot.

(Einstein Rings):

If we “imagine” we know the “distance” (the “energy of unperturbed observation)) to the center the ring as $r = \gamma$, $r^2 = \gamma^2$ and we know the “force” that moves the galaxy off center as β , then $2\gamma\beta$ is the imaginary energy required to move the galaxy as observed on our focal plane:

$$(\gamma + \beta)^2 = \gamma^2 + \beta^2 + 2\gamma\beta$$

Note that the galaxies are distorted around the circle because of a real energy associate with a Lorentz rotation.



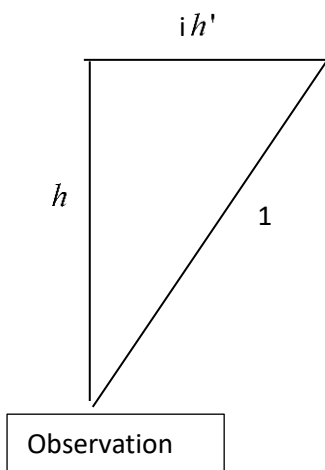
(Red Shift)

In this case, $\gamma = h$ in an imaginary parking lot far away and $\beta = ih'$ is the energy lost to the imaginary cosmological background noise that interacts with the actual incoming photon as detected at our present observation site in our own parking lot as an observation.

Then the imagined energy of the source and the interaction will be given by the area of the triangle

$E = \frac{1}{2}\gamma\beta$, but since all four quadrants of the relativistic circle must be taken into account, then the

total energy of observation must be $E = 4\left(\frac{1}{2}\gamma\beta\right) = 2\gamma\beta$, so is "virtual".



$$1^2 = h^2 + (ih')^2 + 2h(ih') = h^2 - (h')^2 + 2h(ih')$$