Mathematical Foundation

(for Fermat's Theorem)
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The Relativistic Unit Circle (under revision)

A single vector space in one dimension, it is characterized by a scalar and a unit vector $(\vec{1}) = (1)\vec{i}$ The unit vector for that dimension can be multiplied by any function of scalars $y\vec{i} = f(x)\vec{i}$ so that $y = f(x)$ represents a single real number (integer or otherwise) in that dimension (i.e., $f(x)$ represents a single valued scalar, no matter what the form of $f(x)$.

Thus, for integers, the complete set of individual integers in a single dimension is represented by the scalar variable a, in the form $\vec{a} = a\vec{i}$. The scalar (1) is said to form the basis for that set since $a*1=a$ (product of scalars in $(a)\vec{i}$

The set of scalar multiplicative products in a single dimension for two scalars in the set is given by $\vec{c}_{ab} = \overrightarrow{(ab)} = (ab)\vec{i}$ and the set of two scalar additives in the set is given by $\vec{c}_{(a+b)} = \overrightarrow{(a+b)} = (a+b)\vec{i}$, and the rules for multiplication and distribution apply.

The set of such scalars in a single dimension (a "vector Space $\langle A \rangle \vec{i}$) is called a **ring**, with a metric (length) defined by $a = 0 \pm a$, where 0 is the midpoint of all possible lengths defined by that metric. Note that 0 is not a length; if a does not exist, there is no dimension (or rather, the dimension is fictitious); therefore, the specification of a is a condition of the (mathematical) existence of at least one metric for that dimension. The expression $0*a=0$ symbolizes the concept that 0 is the midpoint in terms of scaling $0*a=0*(-a)=0$, while the expression $0 \pm a = a$ expresses the fact that the set $\{a\}\vec{i} = (a)\vec{i}$ has at least one element (i.e., the "axiom of choice")

If the inverse operation to multiplication is included (e.g., $\left(\frac{b}{a}\right)\left(\frac{a}{b}\right)\vec{i} = (1)\vec{i}$) , the set of scalars is called a "division ring". Note that division by zero is undefined (actually means the dimension does not "exist" in the sense that there are no metric scalars (lengths) within it (associated with it); (only its conceptual "midpoint") since $b\vec{i} = \frac{a}{0}\vec{i} \Leftrightarrow (b*0)\vec{i} = 0\vec{i}$ Note that 0 remains a scalar (i.e., it is not a basis vector) for that dimension – it is also called the "origin" for that dimension.

Note that if a division ring is specified, two dimensions are necessary, since the relation $A = \frac{a}{b}$ defines the slope of a straight line in the geometrical ("Cartesian" coordinate system) characterization of the two independent vectors (a,b) or (x,y)

A second dimension $\langle B \rangle \vec{j}$ can be created in exactly the same way, where an independent (non-interacting) sum of vector spaces can be defined in terms of "Cartesian coordinates" (a,b) (or in general terms (x,y) where the prescription of "non-interacting" can be graphed by orthogonal axes $a \perp b \Leftrightarrow b \perp a$ or $x \perp y \Leftrightarrow y \perp x$

The two dimensions can be characterized by their basis vectors $(a,b) = (a\vec{i}, b\vec{j})$ where $(1_a\vec{i}, 1_b\vec{j}) = (1,1)$ represent unit bases in each dimension, where the independence of the vector spaces represented by $(1,1)$ If there is a common midpoint to all possible metrics in both dimensions, it is represented by the "origin" $(0,0)$ where there is no metric in either dimension. If one of the dimensions is represented by the scalar $b*0 = 0 = 1*0$ then there is no common metric origin – in terms of $\vec{a} = a\vec{i}$, the dimension represented by $\vec{b} = b\vec{j}$ does not exist. Therefore, the vector space $(a,0) = (a)$

Division by zero

Addition (groups) only require one dimension; $\vec{c} = c\vec{i} = (a \pm b)\vec{i}$, where the sum on the right refers to the single integer on the left side of the equality, but multiplication and its inverse (division (rings)) require two dimensions. Consider the equation $z_{(x,y)} = \frac{x}{y}$, $x = yz_{(x,y)}$ If $y = 0$, $x = yz_{(x,y)}$ means that x and y can still exist, but their relation is undefined, unless $y\vec{i} = x\vec{i}$ in one dimension, so that $x\vec{i} = 0\vec{i}$, and therefore $x = 0$, which is a point (the midpoint of all possible metrics (lengths) in one dimension).

If $y \neq 0$, then $A = \frac{x}{y}$ is the slope of the line $y = Ax + b$ in two (Cartesian) dimensions.

$$\begin{vmatrix} x & 0 \\ 0 & \frac{1}{y} \end{vmatrix} \rightarrow \left(\frac{x}{y}\right)\bar{0} = A(\bar{0})$$

Likewise, in the equation for radial coordinates $s = r\theta$ then $r = \frac{s}{\theta}$; for $\theta = 0$ the arc length is undefined unless $s\vec{i} = r\vec{i}$ in one dimension. Also for $\theta = \frac{s}{r}$, for $r = 0$, then $\theta\bar{i} = s\vec{i}$ in one dimension.

$$\begin{vmatrix} r & 0 \\ 0 & \theta \end{vmatrix} \rightarrow (r\theta)(\bar{0}) = s(\bar{0})$$

If there are no lengths in either of the vector spaces, the common connection $(0,0)$ still represents a common "origin" for the two spaces in terms of which existing metrics can be related. If there is no common origin, then the vectors spaces are said to be "affine", and no such relation can be established.

If a relation between the metrics of the two vector spaces can be established, such a relation can be established in terms of the relative scaling factors τ_a and τ_b where the orthogonal vectors spaces not have the relation $(a\tau_a, b\tau_b)$, or by the relation in terms of the basis vectors for each dimension $((1_a)\tau_a,(1_b)\tau_b)$ which represents different metrics in the two dimensions.

If the spaces are orthogonal, then the resultant vector can be chosen so that $\vec{c} = c\vec{r} = \vec{a} + \vec{b}$, where the symbol "+" now represents the relation between the two dimensions $\vec{c} = c\vec{r} = a\tau_a\vec{i} + b\tau_a\vec{j}$ Note that if either of the vector spaces has no metric lengths associate with it (e.g. $\vec{b} = b\vec{j} = (0)\vec{j}$) , then that dimension does not "exist" $\vec{c} = c\vec{r} = a\tau_a\vec{i}$, so that $c = a\tau_a$ and $\vec{r} = \vec{i}$. That is, only the vector space represented by $\vec{a} = a\vec{i}$ "exists" (i.e., is relevant to the mathematical context under discussion).

In order to proceed further, a choice must be made as to which metric (dimension) is to serve as the foundation for comparison. The choice made is than called an "initial state", which means that by itself, it has no relation to any other metric in any other vector space.

If the state represented by $\vec{a}$ is chosen to be the initial state, then a related state $\vec{b}$ can be thought of as producing a change in the total system $\vec{c}$, so that $\vec{b} \neq (0)\vec{j}$ implies that $\vec{c} \neq \vec{a}$. $\vec{c} = c\vec{r} = a\tau_a\vec{i} + b\tau_b\vec{j}$

For clarity, from here on out we will use the notation $(at, bt') = (a\tau_a, bt_b)$ with the association understood.

The linear relation can be established:

$(ct')^2 = (bt')^2 + (at)^2$ which establishes a metric relationship between the integer spaces (a,b), where at is considered the initial metric ((or state; the foundation for comparison), and bt' is a second metric (or state) for comparison, with the result of the comparison given by the final metric (or state) ct' , which includes interactions from both metrics. Note that $d = (at)$ is an integer only if t is an integer and $e = bt'$ is an integer only if t' is an integer, so that $f = ct'$ is an integer only if c is an integer, where $f^2 = e^2 + d^2$. Dividing by f yields $1^2 = \left(\frac{e}{f}\right)^2 + \left(\frac{d}{f}\right)^2$ where $\left(\frac{e}{f}\right)^2 < 1^2 \Leftrightarrow \frac{e}{f} < 1$ and $\left(\frac{d}{f}\right)^2 < 1^2 \Leftrightarrow \frac{d}{f} < 1$ Multiplying by an arbitrary integer n^2 yields $n^2 = n^2(\frac{e}{f})^2 + n^2(\frac{d}{f})^2$, where n cannot be an integer since neither $\frac{e}{f}$ nor $\frac{d}{f}$ is not an integer (both are less than one) unless $e = 0$ or $d = 0$, in which case $n^2 = n^2(\frac{d}{f})^2$ or $n^2 = n^2(\frac{e}{f})^2$ respectively, so $\frac{e}{f} = \sqrt{1^2}$

Therefore, $c^2 = a^2 + b^2$ cannot be an integer unless $a = 0$ or $b = 0$, and if a is specified to exist as the foundation for comparison then $b = 0$

Therefore, the integers a, b, and c cannot have a metric relationship in two dimensions.

In one dimension, $c^2 = a^2 + b^2 = d^2$ all refer to the same unique integer $c = d$. The expression $a^2 + b^2$ is established by merely partitioning the same unique integer in tat set in different ways (e.g., $c = a + b = d, c^2 = d^2$). The same approach can be taken in a second dimension, but if there is no common origin, the dimensions are unrelated, or "affine"; that is they are parallel number lines, and there is no connection between the elements in one dimension with that in the other.

If the dimensions are not "affine", then there must be a common intersection point $(0,0)$; if the dimensions are perpendicular, their relation is given by the cross product, where the null vector (see below) indicates equal and opposite relationship. However, if a dimension has no metric (scalar) associated with it, then the origin of that dimension is merely a point, so there is no metric relation with that dimension and other existing dimensions in the null space.

In two dimensions where a metric relation, where the integer values of a and b are preserved in each dimension (i.e., can refer to different integers), the resultant is given by the Binomial Expansion for $n = 2$ where $c^2 = (a+b)^2 = a^2 + b^2 + 2ab$, where $2ab$ is an interaction term, which vanishes only if $a = 0$ or $b = 0$

Null Vectors

The orthogonal vectors (i, j) can be represented as unit vectors in the matrix $\begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix}$

, where the cross product of these vectors is given by $\begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{vmatrix} \to \begin{vmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1^2 \end{vmatrix}$, which

characterizes the operation $\vec{i} \otimes \vec{j} = \vec{k}$, where the new vector is represented in its own dimension as $\vec{k}$. Reversing the order of the cross product is accomplished by swapping the first two columns of the matrix so that $\begin{vmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{vmatrix} \to \begin{vmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1^2 \end{vmatrix}$, or

$\vec{j} \otimes \vec{i} = 1^2\left(\overrightarrow{-k}\right)$ The sum of these vectors represent an equal and opposite interaction at their connection; note that the negative sign applies to the vector, but not the scalar. Adding the two vectors yields:

$\left(1^2\right)\left[\left(\vec{i} \otimes \vec{j}\right) + \left(\vec{j} \otimes \vec{i}\right)\left(\overleftarrow{i} \otimes \vec{j}\right) + \left(\overleftarrow{i} \otimes \overleftarrow{j}\right)\right] = \left(1^2\right)\left[\vec{k} + \left(\overrightarrow{-k}\right)\right] = 1^2\left[\vec{k} + \left(\overleftarrow{k}\right)\right] = 1^2\overleftrightarrow{(0)}$, so the null vector characterizes an equal but opposite interaction, the magnitude of which is given by the product of the scalars.

This operation is indicated here by $\begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} \to 1^2\left(\overleftrightarrow{0}\right)$, where is assumed the cross product relation above has been taken.

For further discussion, see "The Pauli/Dirac Matrices"

The interactions are then defined by
$\begin{vmatrix} a & 0 \\ 0 & b \end{vmatrix} \rightarrow ab(\vec{k})$ and $\begin{vmatrix} 0 & a \\ b & 0 \end{vmatrix} \rightarrow ab(\overrightarrow{-k})$, (the scalars a and b commute) so that adding the interaction for multiplicative commuting variables is given by

$ab(\vec{k}) + ab(\overrightarrow{-k}) = ab(\overleftrightarrow{0})$

However, the cases

$\begin{vmatrix} b & 0 \\ 0 & a \end{vmatrix} \rightarrow ba(\vec{k})$ and $\begin{vmatrix} 0 & b \\ a & 0 \end{vmatrix} \rightarrow ba(\overrightarrow{-k})$ must also be included, so that

$ba(\vec{k}) + ab(\overrightarrow{-k}) = ba(\overleftrightarrow{0})$ and adding the cases for the final result gives
$ab(\overleftrightarrow{0}) + ba(\overleftrightarrow{0}) = 2ab$, which is the interaction term in the Binomial Expansion.

In the case where there is a common metric, then proof of Fermat's Theorem follows directly from the Binomial Expansion, where

$c^n = (a+b)^n = \left(a+b\right)^2 (a+b)^{n-2} = a^n + b^n + \left(2ab\right) rem(a,b,n-2)$. For Fermat's Theorem to be true, then $\left(2ab\right) rem(a,b,n-2)$ must vanish, which means that either $a=0$ or $b=0$, and from the argument above, if a is chosen as the foundation for metric comparison, then $b=0$ means that $c^n = a^n$ and there is no relation vis a vis the dimension in which b is defined ($\vec{a}$ and $\vec{b}$ are affine i.e., parallel in two dimensions).

The equation $c^n = a^n + b^n$ cannot therefore be valid for c, a, b, n positive integers.

Q.E.D