Proof of Fermat's Theorem

(Matrix version)

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Fermat's Theorem

The Binomial Theorem

(I'm not a set theorist, or a number theorist, but I know what I mean, and will defend it if you spring for the beer and pizza… ☺

Consider the Cartesian product of two independent sets $\{C\}=\{A\}\times\{B\}$ whose elements are positive integers $\{c\}=\{a\}\times\{b\}$. The sets are complete under the operations of arithmetic addition, multiplication, and positive powers.

A single vector with two components can be characterized by the column vector $\begin{vmatrix} a \\ b \end{vmatrix}\vec{i}$ where the vector components satisfy arithmetic operations; in particular, in a single dimension $\vec{i}$ the following relations are satisfied for $a,b \in A,\ a,b \notin B$:

$$\begin{aligned} a\vec{i}+b\vec{i} &= (a+b)\vec{i}=(b+a)\vec{i} \\ a\vec{i}\cdot b\vec{i} &= (ab)\vec{i}=(ba)\vec{i} \\ a^n\vec{i}+b^n\vec{i} &= (a^n+b^n)\vec{i}=(b^n+a^n)\vec{i} \end{aligned},$$

and in particular for the case $a=b$. These operations are also satisfied in an orthogonal dimension $\begin{vmatrix} a \\ b \end{vmatrix}\vec{j}$ in the two dimensional vector space represented by $(\vec{i},\vec{j})$ where all arithmetic operation are carried out independently in each dimension: $(f(a,b)\vec{i}, g(a,b)\vec{j})$ where $c_{ab}\vec{i}=f(a,b)\vec{i}$ represents the single-valued result of the operation $f(a,b)$, and so the equality represents the same unique number in the dimension $\vec{i}$, and similarly for $d_{ab}\vec{j} = g(a,b)\vec{j}$, including the case where $c_{ab} = f(a,b) = g(a,b) = d_{ab}$, and where $f(a,b) \in A, \; g(a,b) \in B$ and $A \cup B = \{0\}$.

In each dimension, if these operations are restricted to positive arithmetic in each dimension, the variables a and b commute: i.e.,

$f(a,b) = f(b,a)$ and $g(a,b) = g(b,a)$ so that $c_{ab} = c_{ba}$ and $d_{ab} = d_{ba}$

if $f(a,b) = g(a,b)$ then $c_{ab} = c_{ba} = d_{ab} = d_{ba}$

Let a and b be independent positive integers in the space $(a,b)\vec{i}$ and also in the space $(a,b)\vec{j}$, where $a\left(\vec{i}\cdot\vec{i}\right) = a$, $b\left(\vec{i}\cdot\vec{i}\right) = b$, and similarly $a\left(\vec{j}\cdot\vec{j}\right) = a$ and $b\left(\vec{j}\cdot\vec{j}\right) = b$. Note that the distinction between the orthogonal dimensions are removed when taking dot products in each dimension to produce the scalars a and b . If a and b are instantiated to integers (e.g. $a = 3, b = 4$, then they are invariants within each dimension, and cannot be changed without changing c_{ab} or d_{ab}; that is, such changes depend on the arithmetic operations within the dimension.

Two dimensional vectors with two component each can be represented by a 2×2 matrix with components $\begin{vmatrix} a & b \\ c & d \end{vmatrix}$, where the column vectors $\begin{vmatrix} a \\ c \end{vmatrix}$ and $\begin{vmatrix} b \\ d \end{vmatrix}$ represent variables with arithmetic operations in the dimensions $(\vec{i}, \vec{j})$; e.g. $(a+c)\vec{i} = a\vec{i} + c\vec{i}$ or $(b^n + d^n)\vec{j} = b^n\vec{j} + d^n\vec{j}$, etc.

Consider the matrix where $c = b$ and $d = a$.

$$C = \begin{vmatrix} a & b \\ b & a \end{vmatrix} = \begin{vmatrix} a & 0 \\ 0 & a \end{vmatrix} + \begin{vmatrix} 0 & b \\ b & 0 \end{vmatrix} = A + B$$

This represents the case where a and b are independent in each dimension, even though they have the same value as scalars. That is, changing the value in the top row of each column will not affect the value in the bottom row of each column, although the sum of the vector components will, of course, change.

$$C^2 = \left(A+B\right)^2 = A^2 + AB + BA + B^2 =$$

$$\begin{vmatrix} a & 0 \\ 0 & a \end{vmatrix}^2 + \begin{vmatrix} a & 0 \\ 0 & a \end{vmatrix}\begin{vmatrix} 0 & b \\ b & 0 \end{vmatrix} + \begin{vmatrix} 0 & b \\ b & 0 \end{vmatrix}\begin{vmatrix} a & 0 \\ 0 & a \end{vmatrix} + \begin{vmatrix} 0 & b \\ b & 0 \end{vmatrix}^2 =$$

$$\begin{vmatrix} a & 0 \\ 0 & a \end{vmatrix}^2 + \begin{vmatrix} 0 & b \\ b & 0 \end{vmatrix}^2 = \begin{vmatrix} a^2 & 0 \\ 0 & a^2 \end{vmatrix} + \begin{vmatrix} 0 & b^2 \\ b^2 & 0 \end{vmatrix} = A^2 + B^2$$

The elements of the resultant matrix are the result of taking the “dot” product of each component separately in each dimension; again, changing the value of the scalar a in matrix A will not change matrix B , and vice versa.

Note that the different powers (e.g. $\begin{vmatrix} a^n & 0 \\ 0 & a^m \end{vmatrix}$ are independent with respect to (n,m) , since changing n will not change m and vice versa.

Consider the case where a and b are dependent in each dimension; that is, changing the value in the top row changes the value in the bottom row of each column.

$$C' = \begin{vmatrix} a & b \\ a & b \end{vmatrix} = \begin{vmatrix} a & 0 \\ 0 & b \end{vmatrix} + \begin{vmatrix} 0 & b \\ a & 0 \end{vmatrix} = A_{ab\|} + A_{ab\perp} = A' + B'$$

$$C'^2 = (A'+B')^2 = \begin{vmatrix} a & b \\ a & b \end{vmatrix}^2 = A_{ab\|}{}^2 + A_{ab\|}A_{ab\perp} + A_{ab\perp}A_{ab\|} + A_{ab\perp}{}^2$$

$$= \begin{vmatrix} a^2 & 0 \\ 0 & b^2 \end{vmatrix} + \begin{vmatrix} a & 0 \\ 0 & b \end{vmatrix}\begin{vmatrix} 0 & b \\ a & 0 \end{vmatrix} + \begin{vmatrix} 0 & b \\ a & 0 \end{vmatrix}\begin{vmatrix} a & 0 \\ 0 & b \end{vmatrix} + \begin{vmatrix} 0 & b^2 \\ a^2 & 0 \end{vmatrix}$$

$$= \begin{vmatrix} a^2 & 0 \\ 0 & b^2 \end{vmatrix} + \begin{vmatrix} ab & 0 \\ ba & 0 \end{vmatrix} + \begin{vmatrix} 0 & ba \\ 0 & ab \end{vmatrix} + \begin{vmatrix} 0 & b^2 \\ a^2 & 0 \end{vmatrix}$$

$$= \begin{vmatrix} a^2 & 0 \\ 0 & b^2 \end{vmatrix} + \begin{vmatrix} ab & ba \\ ab & ba \end{vmatrix} + \begin{vmatrix} 0 & b^2 \\ a^2 & 0 \end{vmatrix}$$

$$A^2 + B^2 + \begin{vmatrix} ab & ba \\ ab & ba \end{vmatrix} = A^2 + B^2 + rem(A,B,2)$$

In this case, changing the value of a will result in a change in A but not B as before, but will also result in a change in the extra term $rem(A,B,2)$, which is analogous to the remnant term of the Binomial Theorem for the case n=2, where $c^2 = a^2 + b^2 + rem(a,b,2)$; note that the terms in each element of the rem(a,b,2) commute as well. The remainder exists because the vectors in each column are dependent.

Consider the case of

$$C^n = \begin{vmatrix} a & b \\ b & a \end{vmatrix}^n = \left(\begin{vmatrix} a & 0 \\ 0 & a \end{vmatrix} + \begin{vmatrix} 0 & b \\ b & 0 \end{vmatrix} \right)^n = \left(A+B\right)^n \quad \text{where } n > 2$$

By the Binomial Theorem, this will result in the expression

$C^n = A^n + B^n + \mathrm{Rem}(A,B,n)$ consist of interaction terms involving Pascal's coefficients and terms varying in powers of n. This is also true of the individual elements of the matrices.

The process of multiplication of these terms for dependent variables can be characterized by the cross product, where:

$$\begin{vmatrix} a^n & 0 & 0 \\ 0 & b^m & 0 \\ 0 & 0 & 0 \end{vmatrix} \rightarrow \begin{vmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & a^n b^m \end{vmatrix}$$ corresponding to the operation $a^n \vec{i} \otimes b^m \vec{j} = a^n b^m (\vec{i} \otimes \vec{j}) = \left(a^n b^m\right)\vec{k}$,

and

$$\begin{vmatrix} 0 & a^n & 0 \\ b^m & 0 & 0 \\ 0 & 0 & 0 \end{vmatrix} \rightarrow \begin{vmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & b^m a^n \end{vmatrix}$$ corresponding to $b^m \vec{j} \otimes a^n \vec{i} = b^m a^n (j \otimes i) = \left(b^m a^n\right)\left(-\vec{k}\right)$, where the change in sign is due to the vector juxtaposition, not the scalars, which commute.

Each term in the remainder of the expansion of the Binomial Theorem for each value of n consists of the sum of terms consisting of a Pascal Coefficient and terms such as $c_p a^n b^m$ where c is the Pascal coefficient and the other terms are of the form $\left(a^n b^m\right)\left(\pm\vec{k}\right)$, where the coefficients of the vector remain positive. If the "outer product" of dependent variables is simply re-defined as simple multiplication (e.g. $(\sqrt{a^n b^m}) \pm \vec{k} \cdot (\sqrt{b^m a^n}) \pm \vec{k} = a^n b^m = b^m a^n$ and all terms will remain positive, where the terms are integers, even though their square roots are not. Alternatively, taking the absolute value (the modulus. magnitude) of the vectors defined in the rem(a,b,n) defines positive multiplication w.r.t. the interaction terms.

$$\begin{vmatrix} a^n \\ 0 \end{vmatrix} \vec{i} \otimes \begin{vmatrix} 0 \\ b^m \end{vmatrix} \vec{j} = \left(a^n b^m\right)\vec{k}$$

$$\begin{vmatrix} 0 \\ b^m \end{vmatrix} \vec{j} \otimes \begin{vmatrix} a^n \\ 0 \end{vmatrix} \vec{i} = \left(-b^m a^n\right)\left(\vec{k}\right)$$

$$\left|a^n b^m\right| \vec{k} = \left|-b^m a^n\right| \vec{k}$$

$$\left((P_c)_{(m,n)} \sum_{m,n} \left(\left|a^n b^m\right| + \left|-b^m a^n\right| \right) \right) \vec{k} =$$

$$\left((P_c)_{(m,n)} \sum_{m,n} \left(\left|a^n b^m\right| + \left|b^m a^n\right| \right) \right) \vec{k} = rem(a,b,n) > 0,\ m < n$$

Then $rem(a,b,n) = 0$ only if $a = 0, b = 0,$ or $a = b = 0$ (in which case $c^n = b^n,\ c^n = a^n,$ or $c^n = 0,$ respectively.

Higher powers of n are represented by

$$
\begin{aligned}
c^n &= (a+b)^2(a+b)^{n-2} = (a+b)^n \\
&= \left[(a^2+b^2+rem(a,b,2)\right](a+b)^{n-2} \\
&= a^2(a+b)^{n-2} + b^2(a+b)^{n-2} + rem(a,b,2)(a+b)^{n-2} \\
&= a^n + T_1 + T_2 + b^n + rem(a,b,2)(a+b)^{n-2} \\
&= a^n + b^n + T_1 + T_2 + rem(a,b,2)(a+b)^{n-2}
\end{aligned}
$$

Where T1 and T2 are positive sums, powers, and multiples of a and b, so that all terms are non-zero and positive.

Since the remainder term remains for all powers of $n > 2$, Fermat's Theorem is proven:

$$
\begin{aligned}
&C^n \neq A^n + B^n,\ n > 2 \\
&c^n \neq a^n + b^n,\ n > 2
\end{aligned}
$$

(QED)