The Mathematical Physics and the Derivative

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The Relativistic Unit Circle (RUC)

Proof of Fermat's Last Theorem

Any number line of positive unit integers $|c| = |a| + |b|$ can be divided into two distinct sets $\{|c|\} = \{|a|\} + \{|b|\} \triangleq c = a + b$ where $a = \sum_1^a 1,\ \ b = \sum_1^b 1\ \ c = a + b = \sum_1^{a+b} 1 = \sum_1^a 1 + \sum_1^b 1$

Then by the Binomial Theorem, for $n > 1$ where n is a positive integer,

$c = a + b \Leftrightarrow c^n = a^n + b^n + rem(a,b,n)$ and in particular,

$c^2 = a^2 + b^2 + 2ab$.

$$c^n = a^n + b^n \Leftrightarrow rem(a,b,n) = 0$$
$$rem(a,b,n) > 0$$
$$c^n \neq a^n + b^n$$

Q.E.D.

Note that for $a \neq 0$, $c^n = a^n + b^n \Leftrightarrow b = 0$ so that $c^n = a^n$ and $c^n - a^n = 0$ which is a statement of equivalency for two numbers c and a.

This result is true for any two positive single valued functions:

$$g \triangleq g(x_1, x_2, ...x_n),\ h \triangleq h\left(y_1, y_2, ..., y_m\right)$$
$$f\left(g,h\right) \triangleq f = g + h$$
$$f^n = g^n + h^n + rem(g,h,n) \neq g^n + h^n$$

It is also true for multinomials in the same or different powers (Euler's Conjecture) by extrapolation.

(**Pythagorean Triples**)

A Pythagorean Triple results from eliminating the interaction term for the case $n = 2$ by conjugation:

$c = a + ib$

$cc^* = (a+ib)(a-ib) = a^2 + a(ib) - a(ib) - (i^2)b^2 = a^2 + b^2$

However, $cc^* \neq c^2$ because $i^2 = -1 \neq -1$ and $2 = \log_i i^2 = \log_i(-1) \neq 1 = \log_1(-1)$.

e.g., for the Pythagorean Triple $(3,4,5)$:

$c = (3+4) = 7$

$c^2 = (3+4)^2 = 3^2 + 4^2 + 2(3*4) = 9 + 16 + 2(12) = 25 + 24 = 49 = 7^2$

but

$c = (3+4i)$

$cc^* = (3+4i)(3-4i) = 3^2 + 4^2 = 25$

Note that these results are true for the positive real numbers (proved by Newton), as will be applied to physical systems in the following sections.

Physical Systems

Physical Systems are characterized by all existing elements are positive definite, where $x \triangleq \left(\sqrt{x}\right)^2 \in |\mathbb{R}|$

The physical state of a system can be in either a state of no change $\tau' = \tau,\ v = 0$, a state of increasing change $\gamma = \frac{\tau'}{\tau} > 1$, or a state of decreasing change $\gamma = \frac{\tau'}{\tau} < 1$. For two **interacting** elements (or discrete sets), this can be represented by the parameterized relation

$(c\tau') = (c\tau) + (\pm v)\tau' \triangleq (c\tau) + (v)\tau'$,

where

$(c\tau')^2 = \left[(c\tau) + (v\tau')\right]^2 = (c\tau)^2 + (v\tau')^2 + 2(c\tau)(v\tau')$

and the color red indicates that v can take on positive and negative values. The relations between interacting elements can be further characterized by the definitions:

$\gamma \triangleq \frac{\tau'}{\tau},\ \beta \triangleq \frac{v}{c}$

Note that for $\frac{1}{\gamma} = \beta = 1 \ (v = c)$, the above expression is represented in matrix form as

$$2 = 1 + 1$$

$$2^2 = (1+1)^2 = 1^2 + 1^2 + 2(1^2) = 4 = Tr\begin{vmatrix} 1^2 & 0 \\ 0 & 1^2 \end{vmatrix} + Det\begin{vmatrix} 1 & -1 \\ 1 & 1 \end{vmatrix} = 4$$

Note that $Tr\begin{vmatrix} 1^2 & 0 \\ 0 & 1^2 \end{vmatrix} = 1^2 + 1^2 = \left[\left(\vec{i}\cdot\vec{i}\right) + \left(\vec{j}\cdot\vec{j}\right)\right] = 2(1^2)$ (a sum of dot products) which is a scalar and can be reassigned as a coefficient of $2\left(1^2\right)\vec{k}$. $Det\begin{vmatrix} 1 & -1 \\ 1 & 1 \end{vmatrix} = (1,1)\vec{i} \otimes (-1,1)\vec{j} = \left[2(1)(1)\right]\vec{k}$ (as a cross product); both mapping two dimensions into one ($\vec{k}$), so that (e.g.)

$2^2\vec{k} = (1+1)^2\vec{k} = 1^2 + 1^2 + 2(1^2)\vec{k} = 4\vec{k}$ which is equivalent to

$2^2\vec{i} = (1+1)^2\vec{i} = 1^2 + 1^2 + 2(1^2)\vec{i} = 4\vec{i}$,

and where all the numbers are scalars in the single (arbitrary) dimension $\vec{i}$.

Final State (See RUC) ("radiation", reduction from initial state)

The final state of the system referenced to unity $1\left(\frac{(c\tau')}{(c\tau')}\right)^2 \triangleq \frac{(c\tau')^2}{(c\tau')^2}$ in second order (just before $v=0$) is obtained by dividing both sides of the equation $(c\tau')^2 = (c\tau)^2 + (v\tau')^2 + 2(c\tau)(v\tau')$ by the final state $(c\tau')^2$, where $\gamma < 1, \quad \beta < 1$

$$1\left(\frac{(c\tau')}{(c\tau')}\right)^2 = \left(\frac{1}{\gamma}\right)^2 + \beta^2 + 2\frac{\beta}{\gamma}$$

$$\Leftrightarrow 1\left(\frac{(\tau')}{(\tau')}\right)^2 \triangleq \left(\frac{1}{\gamma}\right)^2 + \beta^2 + 2\frac{\beta}{\gamma}$$

where $1\left(\frac{(\tau')}{(\tau')}\right)^2$ characterizes the final state, $\left(\frac{\tau'}{\tau}\right)^2 \triangleq \frac{1}{\gamma^2} + \beta^2 \le 1\left(\frac{(\tau')}{(\tau')}\right)^2$, and

$\left(\frac{1}{\gamma}\right)^2 + \beta^2 + 2\frac{\beta}{\gamma} > 0$ for $\left(\frac{1}{\gamma}\right)^2 > 0,$ and

$$1\left(\frac{(\tau')}{(\tau')}\right)^2 = \left[\left(\frac{1}{\gamma}\right)^2 + \beta^2 + 2\frac{\beta}{\gamma}\right] = \left[\left(\frac{1}{\gamma}\right)^2 + \beta^2 + (h_{\tau'})^2,\ (h_{\tau'})^2 \triangleq 2\frac{\beta}{\gamma}\right] < 2$$

This result is represented in matrix form as:

$$1\left(\frac{(\tau')}{(\tau')}\right)^2 = Tr\begin{vmatrix} \left(\frac{1}{\gamma}\right)^2 & 0 \\ 0 & \beta^2 \end{vmatrix} + Det\begin{vmatrix} \left(\frac{1}{\gamma}\right) & -\beta \\ \left(\frac{1}{\gamma}\right) & \beta \end{vmatrix}, \text{ where } Det\begin{vmatrix} \left(\frac{1}{\gamma}\right) & -\beta \\ \left(\frac{1}{\gamma}\right) & \beta \end{vmatrix} = 2\frac{\beta}{\gamma} = (h_{\tau'})^2$$

Note that for $\frac{1}{\gamma}=\beta=1\ (v=c)$, the above expression is

$$2=1+1$$

$$2^2=(1+1)^2=1^2+1^2+2(1^2)=4=Tr\begin{vmatrix}1^2 & 0\\ 0 & 1^2\end{vmatrix}+Det\begin{vmatrix}1 & -1\\ 1 & 1\end{vmatrix}=4,\quad 1\left(\frac{(\tau')}{(\tau')}\right)^2\triangleq 1^2$$

$$\cos\theta\triangleq\frac{1}{\gamma},\ \sin\theta\triangleq\beta$$

$$1\left(\frac{\tau'}{\tau'}\right)=\cos\theta+\sin\theta$$

$$\frac{1}{\gamma^2}\triangleq\cos^2\theta,\ \beta^2\triangleq\sin^2\theta$$

$$1\left(\frac{\tau'}{\tau'}\right)^2=\cos^2\theta+\sin^2\theta+2\cos\theta\sin\theta$$

$$(h_{\tau'})^2\triangleq 2\cos\theta\sin\theta$$

$$1\left(\frac{\tau'}{\tau'}\right)^2=\cos^2\theta+\sin^2\theta+(h_{\tau'})^2$$

The relation

$$1\left(\frac{\tau'}{\tau'}\right)^2=\cos^2\theta+\sin^2\theta+2\cos\theta\sin\theta$$

is represented in matrix form by:

$$1\left(\frac{\tau'}{\tau'}\right)^2=Tr\begin{vmatrix}\cos^2\theta & 0\\ 0 & \sin^2\theta\end{vmatrix}+Det\begin{vmatrix}\cos\theta & -(\sin\theta)\\ \cos\theta & \sin\theta\end{vmatrix}$$, where $-\sin(\theta)=\sin(-\theta)$

So that $Det\begin{vmatrix}\cos\theta & \sin(-\theta)\\ \cos\theta & \sin\theta\end{vmatrix}=(h_{\tau'})^2=2cos\theta\sin\theta$

Note that $1\left(\frac{\tau'}{\tau'}\right)^2-\left[\cos^2\theta+\sin^2\theta\right]>2\cos\theta\sin\theta$, so that $2\cos\theta\sin\theta>0$ is positive in the context of the full system.

Plot of h^2 **vs** $(\pm)2cos\theta\sin\theta$

graph.pdf

Note that the positive and negative values are for half of the RUC for $\pm(v\tau')<(c\tau)$; i.e in the interval $-\frac{\pi}{2}<\theta<\frac{\pi}{2}$ This means that the "spin" interaction in the diagram is only one-half the total value. The plot thus represents or photo polarization or "fermions" with "Spin" $\frac{1}{2}$ (For matter, there will be a binding energy of "holes' and "electrons'; released as "neutrinos" in the case of the hydrogen atom….)

Initial State (See RUC) ("absorption", increase to initial state)

$$\gamma^2 \triangleq 1\left(\frac{(c\tau)}{(c\tau)}\right)^2 + (\beta\gamma)^2 + 2(\beta\gamma) = 1\left(\frac{\tau}{\tau}\right)^2 + (\beta\gamma)^2 + 2(\beta\gamma),$$

$$\gamma^2 \geq 1\left(\frac{(\tau)}{(\tau)}\right)^2, \ |\beta| \geq 0$$

where $1\left(\frac{\tau}{\tau}\right)^2$ characterizes the initial state, and

$$\gamma^2 = 1\left(\frac{\tau}{\tau}\right)^2 + (\beta\gamma)^2 + (h_\tau)^2, \ (h_\tau)^2 \triangleq 2(\beta\gamma)$$

$$\gamma^2 - \left[1\left(\frac{\tau}{\tau}\right)^2 + (\beta\gamma)^2\right] = (h_\tau)^2$$

$$\cosh\theta \triangleq \gamma, \ \sinh\theta \triangleq (\beta\gamma)$$

$$\cosh\theta = 1\left(\frac{\tau}{\tau}\right) + \sinh\theta$$

$$\gamma^2 \triangleq \cosh^2\theta, \ (\beta\gamma)^2 \triangleq \sinh^2\theta$$

$$\cosh^2\theta = 1\left(\frac{\tau}{\tau}\right)^2 + \sinh^2\theta + 2\left[1\left(\frac{\tau}{\tau}\right)\right]\sinh\theta$$

$$(h_\tau)^2 \triangleq 2\left[1\left(\frac{\tau}{\tau}\right)\right]\sinh\theta$$

$$\cosh^2\theta = 1\left(\frac{\tau}{\tau}\right)^2 + \sinh^2\theta + (h_\tau)^2$$

Note that for $\gamma = \beta = 1 \ (v = c)$, the above expression is

$$2 = 1 + 1$$

$$2^2 = (1+1)^2 = 1^2 + 1^2 + 2(1^2) = 4 = Tr\begin{vmatrix} 1^2 & 0 \\ 0 & 1^2 \end{vmatrix} + Det\begin{vmatrix} 1 & -1 \\ 1 & 1 \end{vmatrix} = 4$$

$$4\gamma^2 = 1^2 + (\gamma\beta)^2 + 2\gamma\beta = 4, \ 1\left(\frac{(\tau)}{(\tau)}\right)^2 \triangleq 1^2$$

Entropy

In both Decreasing ($\gamma, \beta < 1$, "radiation") and Increasing ($\gamma, \beta < 1$, "absorption") Final States from an initial state, the "interaction energies", $(h_{\tau'})^2$ and $(h_\tau)^2$ respectively, characterize the change in entropy between the initial and final states.

If there is no interaction, then the system is represented by the matrices

$$|M| = \begin{vmatrix} (c\tau) & 0 \\ 0 & (v\tau') \end{vmatrix}, \text{ where } |M|^2 = \begin{vmatrix} (c\tau) & 0 \\ 0 & (v\tau') \end{vmatrix}^2 = \begin{vmatrix} (c\tau)^2 & 0 \\ 0 & (v\tau')^2 \end{vmatrix} \text{ where } Tr|M|^2 = (c\tau)^2 + (v\tau')^2$$

The Pauli Matrices

Note that the "interaction" term $2(1^2) = Det\begin{vmatrix} 1 & -1 \\ 1 & 1 \end{vmatrix}$ is the cross product of two vectors:

$\begin{vmatrix} 1 & -1 \\ 1 & 1 \end{vmatrix} = \begin{vmatrix} 1 \\ 1 \end{vmatrix} \otimes \begin{vmatrix} -1 \\ 1 \end{vmatrix}$ which maps the matrix into a single "dimension"

$\begin{vmatrix} \vec{i} & \vec{j} & k \\ 1 & -1 & 0 \\ 1 & 1 & 0 \end{vmatrix} \Rightarrow 2\overleftrightarrow{k}$ where the opposite arrows over the $\overleftrightarrow{k}$ vector indicates that it is the product $2(1^2)$

Since $1 = \log_1(1) \neq 2 = \log_1(1^2)$, it can now be included in the single "dimension"

$$2^2 = (1+1)^2 = [1^2 + 1^2] + [2(1^2)]$$

Note that the matrices $|I| = \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix}$ and $\sigma_3 = -\begin{vmatrix} -1 & 0 \\ 0 & 1 \end{vmatrix} = \begin{vmatrix} 1 & 0 \\ 0 & -1 \end{vmatrix}$ represent the identity matrix (representing two non-interacting particles) and the (negative) Pauli matric $-\sigma_3$

The remaining two matrices can be represented as a sum, where

$$\sigma_1 + \sigma_2 = \begin{vmatrix} 0 & 1-i \\ 1+i & 0 \end{vmatrix}$$

The cross product of the two vectors is represented by

$$\begin{vmatrix} \vec{i} & \vec{j} & k \\ 0 & 1-i & 0 \\ 1+i & 0 & 0 \end{vmatrix} \triangleq \begin{vmatrix} \vec{i} & \vec{j} & k \\ 0 & \psi^* & 0 \\ \psi & 0 & 0 \end{vmatrix} \Rightarrow\Rightarrow -2\left(1^2+i^2\right)\vec{k}$$ where the interaction product $2(1)(i)$ has been eliminated by conjugation

$$\psi = 1+i$$
$$\psi\psi^* = (1+i)(1-i) = 1^2 + i^2$$

But then $1^2 + i^2 = 1 + (-1) = 0$

The three matrices do not include the identity matrix or its square, which means that the Pauli matrices exclude the factor $|I| = \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} \Leftrightarrow |I|^2 = \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix}^2 = \begin{vmatrix} 1^2 & 0 \\ 0 & 1^2 \end{vmatrix}$ where $Tr\begin{vmatrix} 1^2 & 0 \\ 0 & 1^2 \end{vmatrix} = 1^2 + 1^2$ and so exclude the "non-interacting" term (sum of squares) $\left[1^2+1^2\right]$ in the expression

$2^2 = (1+1)^2 = \left[1^2+1^2\right] + \left[2(1^2)\right]$ and eliminate the "interacting" term $2(1)(i)$ by conjugation.

Then the system represented by 2^2 does not exist (i.e., its "mass" is zero), corresponding to Electromagnetism expressed in terms of global $c = 1 \Leftrightarrow c^2 = 1^2$ rather than Maxwell's $c^2 = \frac{1}{\varepsilon_0\mu_0}$ derived linearly from the Coulomb and Ampere force laws.

$$\varphi = \varepsilon_0 E + \mu_0 B$$

$$\varphi^2 = (\varepsilon_0 E)^2 + (\mu_0 B)^2 + 2(\varepsilon_0 E)(\mu_0 B),$$

where the interaction term $2(\varepsilon_0 E)(\mu_0 B) = 2(\varepsilon_0\mu_0)(EB) = \frac{1}{c^2}(EB) = \frac{1}{c^2} Det\begin{vmatrix} E & -B \\ E & B \end{vmatrix}$

The Pauli Matrices require the product $EB = 0$ and the product $-EB$ to be imaginary, so that the two particles only exist in the imagination (there is no interaction between the E and B fields (the complex plane has its basis element logarithm) in the Cartesian product $(i,1)$.

Note that $\frac{1}{c^2} = \frac{1}{(c\tau)^2} \triangleq \frac{1}{(x)^2} \equiv \frac{1}{(r)^2}$, $\tau^2 = 1^2$ which is the foundation of the inverse force lawL

$$\varphi^2 = (\varepsilon_0 E)^2 + (\mu_0 B)^2 + \left(\frac{1}{r^2}\right) 2(E)(B) = (\varepsilon_0 E)^2 + (\mu_0 B)^2 + \left(\frac{1}{r^2}\right)(h_r)^2$$

For two particles (or charges), the interaction term then represents gravity in terms of Electomagnetism, where $(h_r)^2 \triangleq 2S^2$ Note, however, that the expression $S = \frac{h_r}{\sqrt{2}}$ requires the sum of squares for its definition, since its logarithm is 1, not 2.

(Note: a similar analysis applies to the Minkowski Metric (as a quaternion)

$$\begin{vmatrix} 1 & 0 & 0 & 0 \\ 0 & i & 0 & 0 \\ 0 & 0 & i & 0 \\ 0 & 0 & 0 & i \end{vmatrix} \Leftrightarrow \begin{vmatrix} 1 & 0 & 0 & 0 \\ 0 & i & 0 & 0 \\ 0 & 0 & i & 0 \\ 0 & 0 & 0 & i \end{vmatrix}^2 = \begin{vmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{vmatrix}^2$$

and to the Dirac "0 Matrix":

$$\begin{vmatrix} 1 & 0 & 0 & 0 \\ 0 & i & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & i \end{vmatrix} \Leftrightarrow \begin{vmatrix} 1 & 0 & 0 & 0 \\ 0 & i & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & i \end{vmatrix}^2 = \begin{vmatrix} 1^2 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1^2 & 0 \\ 0 & 0 & 0 & -1 \end{vmatrix}$$

Compare with the Identity Matrix:

$$|I| \triangleq \begin{vmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{vmatrix} \Leftrightarrow \begin{vmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{vmatrix}^2 = \begin{vmatrix} 1^2 & 0 & 0 & 0 \\ 0 & 1^2 & 0 & 0 \\ 0 & 0 & 1^2 & 0 \\ 0 & 0 & 0 & 1^2 \end{vmatrix}$$, where $Tr|I| = 4(1)$ and $Tr|I|^2 = 4(1^2)$

(TBD – in progress) ---

Entropy and the Rate of Change

It might be thought that for both initial and final states $\theta \neq 0,\ v \neq 0$ $\tan\theta = \frac{\beta}{(1/\gamma)} = \gamma\beta$ characterizes the rate of change, but this is only possible if entropy is eliminated by conjugation.

(Radiation)

$$1\left(\frac{\tau'}{\tau'}\right)^2 = \cos^2\theta + \sin^2\theta + 2\cos\theta\sin\theta$$

$$\frac{d}{d\theta}\left[1\left(\frac{\tau'}{\tau'}\right)^2\right] = \frac{d}{d\theta}\left[\cos^2\theta + \sin^2\theta + 2\cos\theta\sin\theta\right]$$

(Absorption)

$$\cosh^2\theta = 1\left(\frac{\tau}{\tau}\right)^2 + \sinh^2\theta + 2\cosh\theta\sinh\theta$$

The Derivative

$$\gamma^2 = 1\left(\frac{\tau}{\tau}\right)^2 + (\gamma\beta)^2 + 2\beta\gamma$$

$$f'(\gamma^2) = \lim_{[\gamma^2-(\gamma\beta)^2]\to 0} \frac{f(\gamma^2) - f(2\beta\gamma)}{(2\beta\gamma)} = \lim_{[\gamma^2-(\gamma\beta)^2]\to 0} \frac{f\left((\gamma\beta)^2 - 2\beta\gamma + 1\left(\frac{\tau}{\tau}\right)^2\right) - f(2\beta\gamma)}{(2\beta\gamma)}$$

$$= \lim_{[\gamma^2-(\gamma\beta)^2]\to 0} \frac{f\left(\left[(\gamma\beta)^2 + 1\left(\frac{\tau}{\tau}\right)^2\right] - h^2\right) - f(h^2)}{(h^2)}$$

Note that $f'(\gamma^2) \Rightarrow 1\left(\frac{\tau}{\tau}\right)^2 \to 0$ as $(\gamma\beta)^2 \to \infty$

Hyperbolic Functions

$$\frac{d}{d\theta}\left[\cosh^2\theta\right] = \frac{d}{d\theta}\left[1\left(\frac{\tau}{\tau}\right)^2 + \sinh^2\theta + 2\cosh\theta\sinh\theta\right]$$

$$\sinh\theta = \frac{e^{\theta} - e^{-\theta}}{2} = \frac{1-e^{-2\theta}}{2e^{-\theta}}$$

$$\cosh\theta = \frac{e^{\theta} + e^{-\theta}}{2} = \frac{1-e^{-2\theta}}{2e^{-\theta}}$$

$$\lim_{\theta\to\infty}\left[\sinh^2\theta\right] = \lim_{\theta\to\infty}\left[\frac{e^{\theta}-e^{-\theta}}{2}\right]^2 = \left[\frac{e^{\theta}}{2}\right]^2$$

$$\lim_{\theta\to\infty}\left[\cosh\theta\right]^2 = \lim_{\theta\to\infty}\left[\frac{e^{\theta}+e^{-\theta}}{2}\right]^2 = \left[\frac{e^{\theta}}{2}\right]$$

$$\lim_{\theta\to\infty}\left[\sinh\theta\right] = \lim_{\theta\to\infty}\left[\sinh\theta\right] = \lim_{\theta\to\infty}\left[\frac{e^{\theta}}{2}\right]$$

$$f'(\theta) = \lim_{\theta\to\infty} f'(\theta) = \lim_{\left[\gamma^2-(\gamma\beta)^2\right]\to 0}$$

Hyperbolic cosine

$$\varphi = \left[\cosh\theta \pm \sinh\theta\right]$$

$$\varphi^2 = \left[\cosh\theta - \sinh\theta\right]^2 = \cosh^2\theta + \sinh^2\theta \pm 2\cosh\theta\sinh\theta$$

$$\varphi = \cosh\theta + \sinh\theta = e^{\theta}$$

$$\cosh\theta - \sinh\theta = e^{-\theta}$$

$$\psi \triangleq \left[\cosh\theta + i\sinh\theta\right] = e^{\theta}$$

$$\psi\psi^* = \cosh^2\theta + \sinh^2\theta$$

$$\psi = \cosh\theta = 1\left(\frac{c\tau}{c\tau}\right) + i\sinh\theta$$

$$\psi\psi^* = \cosh^2\theta = 1\left(\frac{c\tau}{c\tau}\right)^2 + \sinh^2\theta$$

$$\text{Area} = \int_a^b \cosh\theta d\theta = \int_a^b \sqrt{1(\frac{c\tau}{c\tau}) + \left(\frac{d}{d\theta}\cosh\theta\right)^2} d\theta = \text{arc length}$$

$$\sinh^2\theta = \frac{1}{2}\left(\cosh 2\theta - 1(\frac{c\tau}{c\tau})\right)$$

$$\cosh^2\theta = \frac{1}{2}\left(\cosh 2\theta + 1(\frac{c\tau}{c\tau})\right)$$

Entropy

$$\sinh\theta = \frac{e^{\theta} - e^{-\theta}}{2}$$
$$\cosh\theta = \frac{e^{\theta} + e^{-\theta}}{2}$$

$$S = k_B \log\Omega = k_B 1_\Omega \quad 1_\Omega = \log_\Omega \Omega = \frac{S}{k_B} = \frac{h^2}{n}$$

$$h^2 = k_B 1_\Omega = k_B \log_\Omega(\Omega)$$
$$k_B = \log(e)^{k_B} = \log\left(e^{k_B}\right)$$
$$h^2 = \log_e\left(e^{k_B}\right)\log_\Omega(\Omega)$$

$$S = k_B \log_\Omega \Omega = k_B 1_\Omega$$

The Thermal Voltage is then given by

$$\Delta \triangleq \frac{N_D}{N_A}\exp\left(\frac{k\mathrm{T}}{q}\right) = 2\frac{\beta}{\gamma}\exp\left(\frac{k\mathrm{T}}{q}\right), N_A > 0, N_A \geq N_D$$

Increasing Entropy

$$\left(h_\tau\right)^2 = 2\cosh\theta\sinh\theta = 2\gamma\beta \triangleq S = k_B 1_\Omega$$
$$\left(h_\tau\right)^2 = 2\cosh\left(\frac{kT}{q}\right)\sinh\left(\frac{kT}{q}\right)$$

$$\theta = \frac{k_B T}{q}$$

$$f'(x) \triangleq \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$h \triangleq \gamma, \ \beta \triangleq f(x+h) - f(x)$$

$$\left(f'(x)\right)^2 = \left(\frac{\beta}{1/\gamma} \right)^2 = (\beta\gamma)^2 = \frac{\sin^2\theta}{\cos^2\theta} = \frac{(v\tau')^2}{(c\tau')^2} = \frac{(\sinh\theta)^2}{1(\frac{c\tau}{c\tau})^2}$$

where

$$\gamma = \frac{\tau'}{\tau}, \ \beta = i\frac{v}{c}$$

$$\left[1\left(\frac{c\tau'}{c\tau'} \right) \right] \triangleq \cos\theta + i\sin\theta$$

$$\left[1\left(\frac{c\tau'}{c\tau'} \right) \right]\left[1\left(\frac{c\tau'}{c\tau'} \right) \right]^* = [\cos\theta + i\sin\theta][\cos\theta - i\sin\theta] = 1^2 = \cos^2\theta + \sin^2\theta = \left(\frac{1}{\gamma} \right)^2 + \beta^2$$

And