Vector Products

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4/13/2024

Vector Products in two dimensions

Vector products in two dimensions are represented by 2x2 matrices, where the identity matrix characterizes the existence of two vectors under the group operation of addition.

$\left|I\right| := \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} = \begin{vmatrix} 1 \\ 0 \end{vmatrix} + \begin{vmatrix} 0 \\ 1 \end{vmatrix}$ where $Tr\left|I\right| = 1+1$ and $Det\left|I\right| = 1^2$ where the Trace means that both elements exist and the Determinant means that the product of the elements exist in second order.

The “dot” product of two vectors consists of a component-by-component product of the elements of the vectors, so that

$$\begin{vmatrix} 1 \\ 0 \end{vmatrix} \cdot \begin{vmatrix} 0 \\ 1 \end{vmatrix} = \begin{vmatrix} 0 \\ 0 \end{vmatrix}, \quad \begin{vmatrix} 1 \\ 0 \end{vmatrix} \cdot \begin{vmatrix} 1 \\ 0 \end{vmatrix} = \begin{vmatrix} 1^2 \\ 0 \end{vmatrix}, \text{ and } \begin{vmatrix} 0 \\ 1 \end{vmatrix} \cdot \begin{vmatrix} 0 \\ 1 \end{vmatrix} = \begin{vmatrix} 0 \\ 1^2 \end{vmatrix}$$

The cross product of two vectors consists of a mapping of a two dimensional vector into one dimension of second order:

$$\begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{vmatrix} \Rightarrow \begin{vmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1^2 \end{vmatrix} \equiv 1 \otimes 1 = 1^2 \text{ where } \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{vmatrix} \Rightarrow \begin{vmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1^2 \end{vmatrix} \equiv 1 \otimes 1 = -1^2$$

Note that the cross product eliminates the original vectors, so that $\overrightarrow{1} \otimes \overleftarrow{1} = \widehat{1^2} = \overrightarrow{1} \otimes \overleftarrow{1} + \overrightarrow{1} \cdot \overleftarrow{1}$ where $\overrightarrow{1} \cdot \overleftarrow{1} = \overrightarrow{1} \cdot \overleftarrow{1} = \overleftrightarrow{0}$

For $i := \sqrt{-1},\ i^2 = -1$, the second cross product with the 2D vectors interchanged maps two real elements on to the imaginary axis:

$$\begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{vmatrix} \Rightarrow \begin{vmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & i^2 1^2 \end{vmatrix} = \begin{vmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1^2 \end{vmatrix} \equiv 1 \otimes 1 = -1^2, \text{ and } -1^2 = (i1)^2$$

Prime Numbers

$\begin{vmatrix}1\\0\end{vmatrix}\cdot\left|\frac{{}^{1}\!/_{1}}{0}\right|=\frac{(1)(1_1)}{0}$ so that the element product $(1)(1_1)=1\left(\frac{1}{1}\right)=1$ defines a prime number in one dimension, and similarly for $n\begin{vmatrix}1\\0\end{vmatrix}\cdot\left|\frac{{}^{1}\!/_{1}}{0}\right|=\left|\frac{n(1)(1_1)}{0}\right|$ where $n=n\left(\frac{n}{n}\right)$ so that

$n(1)=n(1)\left(\frac{n(1)}{n(1)}\right)=n(1)1_{n(1)}$ is also prime and in particular for $n=1$, $1^2=1^2\left(\frac{1^2}{1^2}\right)=1^2\left(1_{1^2}\right)$ is also prime.

Note that if $n=ct$, then the product ct is also prime, but for $x=vt\rightarrow\frac{x}{t}=v\frac{t}{t},\ v\neq t$, v and t are not prime.

However, for the interaction equation where # is interpreted as the count of elements:

$$\#:=ct'=ct+vt'$$
$$\#^2=(ct')^2=(ct+vt')^2=\left[(ct)^2+(vt')^2\right]+\left[2(ct)(vt')\right]$$

The count is preserved under both addition (“existence”) and multiplication (Interaction, Entanglement, Entropy, etc.) Note that $\#^2=(\#)(\#)$ is a group under mutliplication, but that $(ct')^2$ is not, since it includes both operations.