

# The Wave Equation

Charles Keyser

10/01/2023

## The Relativistic Unit Circle

The traveling wave equation is given as  $\varphi = \sin(kx - vt)$  where  $x$  is a length,  $k$  is the number of  $x$  lengths (assumed to be arranged end-to-end) and  $x = vt$  relates the length  $x$  to velocity  $v = \frac{x}{t}$ . (The length  $x$  can also be defined as a "wavelength"  $x := \lambda$ ) In the context of this analysis,  $x := \lambda$  is considered to be invariant, and thus represented by a prime number

$$x = x\left(\frac{x}{x}\right) = x(1_x) = (vt)\left(\frac{vt}{vt}\right) = (vt)(1_{(vt)}) = \lambda(1_\lambda)$$

The wave equation then becomes  $\varphi = \sin(k\lambda - \lambda) = \sin[\lambda(k-1)] = \sin\theta$ ,  $\theta = \lambda(k-1)$  where  $\theta$  is expressed as radians, so that  $\varphi = \sin\{2\pi[\lambda(k-1)]\}$  Note that  $\theta = \lambda(k-1) \approx \lambda k$  for  $k \gg 1$

Note that the angle  $\theta_k = \lambda_k := \lambda + [\lambda(k-1)] = k\lambda$  expresses  $\theta$  as an initial wavelength  $\lambda$  and a n additive "traveling" phase  $[\lambda(k-1)]$  so that  $\varphi_k = \sin\{\lambda(2\pi k)\} = \sin\{\lambda C_k\}$  where  $C_k$  has the form of a circumference.

However, the term  $\{\lambda(2\pi k)\}$  only exists as an interaction term between  $\lambda$  and  $C_k$  without expressing the existence of either parameter separately.

To do this, consider the expression  $\lambda' = \lambda + C_k$

Then  $(\lambda')^2 = (\lambda + C_k)^2 = [(\lambda)^2 + (C_k)^2] + \lambda(2C_k)$  so that as a "radial" representation (think  $\lambda' := r'$ ,  $\lambda := r_0$ ,  $k := r_C$ ):

$\pi(\lambda')^2 = [\pi(\lambda)^2 + \pi(C_k)^2] + \{\lambda(2\pi C_k)\}$  where the term  $[\pi(\lambda)^2 + \pi(C_k)^2]$  expresses the existence of the parameters  $\lambda$  and  $C_k$  via addition, and the term  $\{\lambda(2\pi C_k)\}$  represents their interaction ("entanglement") via multiplication. Note that this expression is that of areas (energies) not coordinates.

Therefore the expression  $\varphi = \sin \left\{ 2\pi \left[ \lambda (k-1) \right] \right\}$  omits the initial condition  $\lambda$

Consider the expressions

$$(c\tau') = (c\tau) + (v\tau')$$

$$(c\tau')^2 = [(c\tau) + (v\tau')]^2 = [(c\tau)^2 + (v\tau')^2] + 2(c\tau)(v\tau')$$

With the identifications  $(c\tau) := \lambda$  is the initial state,  $(c\tau') := \#$  is the final state and  $(v\tau') := k$  is the perturbing state. If the condition that the terms be prime (i.e., invariant), then the initial state  $(1_{cr})$  can be expressed as:

$$\frac{(c\tau')}{(c\tau)} = 1_{(cr)} + \left( \frac{v\tau'}{c\tau} \right) \text{ so that}$$

$$\frac{(c\tau')^2}{(c\tau)^2} = \left[ \frac{(c\tau)^2}{(c\tau)^2} + \frac{(v\tau')^2}{(c\tau)^2} \right] + 2 \frac{(c\tau)(v\tau')}{(c\tau)^2} \text{ and}$$

$$\gamma^2 := \frac{(\tau')^2}{(\tau)^2} = \left[ (1_{cr})^2 + (\beta\gamma)^2 \right] + 2\beta\gamma, \text{ where } \gamma = \frac{\tau'}{\tau}, \beta = \frac{v}{c}, \text{ and } (v\tau') > (c\tau)$$

Note the following:

1. The expression  $\gamma^2 := \frac{(\tau')^2}{(\tau)^2} = \left[ (1_{cr})^2 + (\beta\gamma)^2 \right]$  has the hyperbolic form  $\cosh^2 \theta = 1^2 + \sinh^2 \theta$

where  $\tan \theta = \beta\gamma$ . This expression is derived from the complex relationships where  $i = \sqrt{-1}$  and

$$\psi := 1_{cr} + i(\gamma\beta) = 1 + \gamma\beta$$

$$\psi^* := 1_{cr} - \gamma\beta$$

$$\psi\psi^* := (1_{cr})^2 + (\gamma\beta)^2$$

Note that if  $\psi\psi^*$  is included in the full expression so that

$$\gamma^2 := \frac{(\tau')^2}{(\tau)^2} = \left[ (1_{cr})^2 + (\beta\gamma)^2 \right] + 2\beta\gamma \equiv [\psi\psi^*] + 2\beta\gamma$$

the expression in terms of complex numbers is unnecessary.

2. The concept of curvature can be introduced by multiplying by  $\pi$  in the second order expression, so that:

$\pi(\gamma^2) := \left[ \pi(1_{cr})^2 + \pi(\beta\gamma)^2 \right] + \gamma(2\pi\beta)$  where  $\gamma(2\pi\beta) := r_\gamma(2\pi r_\beta) := r_\gamma(C_\beta)$  and the other terms are areas of circles.

For  $(v\tau') < c\tau$  the final state can be expressed as:

$$\frac{(c\tau')}{(c\tau')} = 1_{(cr)} = \frac{c\tau}{c\tau'} + \left( \frac{v\tau'}{c\tau'} \right) = \frac{1}{\gamma} + \beta \text{ so that}$$

$$(1_{(cr')})^2 = \left[ \left( \frac{1}{\gamma} \right)^2 + \beta^2 \right] + 2\frac{\beta}{\gamma}$$

Then the expression  $(1_{(cr')})^2 = \left[ \left( \frac{1}{\gamma} \right)^2 + \beta^2 \right]$  has the trigonometric form  $1^2 = \cos^2 \theta + \sin^2 \theta$  derived

from the expressions

$$\psi = \frac{1}{\gamma} + \beta$$

$$\psi^* = \frac{1}{\gamma} - \beta$$

$$\psi\psi^* = \left( \frac{1}{\gamma} \right)^2 + (\beta)^2$$

Which can be substituted in the full expression in the same way as above, so that

$$(1_{(cr')})^2 = [\psi\psi^*] + 2\frac{\beta}{\gamma} \text{ so complex numbers are unnecessary in the full expression.}$$

The full expression can be multiplied by  $\pi$  as above so that

$$\pi(1_{(cr')})^2 = \left[ \pi \left( \frac{1}{\gamma} \right)^2 + \pi\beta^2 \right] + \left( \frac{1}{\gamma} \right) \left( 2\pi \frac{\beta}{\gamma} \right) \text{ where } C_{\left( \frac{\beta}{\gamma} \right)} := \left( 2\pi \frac{\beta}{\gamma} \right) \text{ has the form of a circumference}$$

interacting with the radius  $\frac{1}{\gamma}$ .

As will be seen later,  $\left( \frac{2\pi\beta}{\gamma} \right)$  can model a charge to mass ratio. Note that  $\tan \theta = \frac{\beta}{(1/\gamma)} = \gamma\beta$  as above.

If  $\lambda$  and  $C_k$  do not interact, the system is represented by the matrix  $\# = Tr \begin{vmatrix} \lambda & 0 \\ 0 & k \end{vmatrix} = \lambda + k$  where

$$\#^2 = \begin{vmatrix} \lambda & 0 \\ 0 & k \end{vmatrix}^2 = \begin{vmatrix} (\lambda)^2 & 0 \\ 0 & (k)^2 \end{vmatrix} \text{ but component by component multiplication is not defined so that}$$

$$\lambda(C_k) = 0 \leftrightarrow (C_k) = 0 \leftrightarrow k = 0$$

However, if they do interact, the expression is

$$\#^2 = Tr \begin{vmatrix} (\lambda)^2 & 0 \\ 0 & (k)^2 \end{vmatrix} + Det \begin{vmatrix} \lambda & \lambda \\ -k & k \end{vmatrix} = [(\lambda)^2 + (k)^2] + \{\lambda(2k)\} \text{ so that}$$

$$\pi \#^2 = \pi Tr \begin{vmatrix} (\lambda)^2 & 0 \\ 0 & (k)^2 \end{vmatrix} + \pi Det \begin{vmatrix} \lambda & \lambda \\ -k & k \end{vmatrix} = [(\pi\lambda)^2 + (\pi k)^2] + \{\lambda(2\pi k)\} \text{ where } C_k := (2\pi k)$$