Russell's Paradox

And its relation to the Pythagorean Theorem

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Wikipedia Links

Russell's Paradox

Binomial Theorem

Pythagorean Triples

Fermat's Last Theorem

Goldbach's Conjecture

Natural Numbers

Linear System

Non-Linear Systems

My Links

Invalidity of Pythagorean Theorem

Working Document

Parity

The Relativistic Unit Circle

This article addresses issues concerning Russell's Paradox and its relation to the Binomial Expansion for the cases $n=1$ and $n=2$, and thus the Pythagorean Theorem in the context of the Natural Numbers $\{N\}$, where odd numbers and even numbers complete the set $\{o\}\cup\{e\}=\{N\}$

Preliminary

The Natural Numbers are prime invariants

$$n = n\left(\frac{n}{n}\right) = n\left(1_n\right),\ 1_n := \left(\frac{n}{n}\right)$$

Each number is prime invariant relative to its own base

$$n = n\left(\frac{m}{n}\right) \wedge n\left(\frac{n}{m}\right) \leftrightarrow m = n$$

Goldbach's conjecture is proven immediately:

$$n + n = 2n$$

Fermat's Last Theorem $c^n \neq a^n + b^n\ \forall\{n,a,b,c\} \in \{N\}$

(Note: Fermat specified $n > 2$ The proof that follows is for all $n \in \{N\}$

$$\begin{aligned}
&\text{Let } c = a + b \\
&c^n = (a+b)^n = [a^n + b^n] + [f(a,b,n,m)] \ \ \text{(via the Binomial theorem)} \\
&c^n = [a^n + b^n] \leftrightarrow [f(a,b,n,m)] = 0 \\
&[f(a,b,n,m)] \neq 0 \\
&\therefore\ c^n \neq [a^n + b^n] \ \ \text{QED}
\end{aligned}$$

For the case $n = 2$, $c^2 = [a^2 + b^2] + [2ab]$, Binomial theorem for the case $n = 2$

Pythagorean Triples (and complex numbers)

Pythagorean Triples are characterized by the relation $c^2 = [a^2 + b^2]$, which appears as a subset of natural numbers in the Binomial Theorem for $n = 2$. For Pythagorean Triples to be valid, $a = 0 \wedge b = 0$ (exercise for the student).

(One can achieve the relation by incorporating complex numbers (indicated here by the color magenta, together with the conjugation operation, but they are outside the scope of the natural numbers. $c \notin \{N\}$). If there are no negative numbers ($-n \notin \{N\}$, there are no square roots of negative numbers.

$$c := a + ib$$
$$c^* := a - ib$$
$$cc^* := (a + ib)(a + ib) = a^2 + b^2 [+iab - iab] \neq a^2 + b^2$$

Note that the expression $[+iab - iab] = 0$ means only that $[+iab - iab] = 0 \leftrightarrow iab = iab$; otherwise $a = 0 \wedge b = 0$

"Imaginary numbers are complex only for those who think they are somehow real." – C. Keyser

Example $\{c, a, b\} = \{5, 4, 3\}$ "right triangle"

$$7 = 4 + 3$$
$$72 = 49 = (4+3)^2 = [16+9] + [2(4)(3)] = [25] + [24]$$
$$c = 4 + 3i$$
$$cc^* = 25 \neq 49$$

Note that for a Pythagorean triple, the expression $\sqrt{cc^*}$ must also be an integer, which further limits the resultants.

Russell's Paradox

"A barber in a village shaves all those and only those that don't shave themselves. Does the barber shave himself?" – Bertrand Russel

The Binomial theorem is a single valued function:

$$c^2 = Tr\begin{vmatrix} a^2 & 0 \\ 0 & b^2 \end{vmatrix} + Det\begin{vmatrix} a & b \\ -b & b \end{vmatrix} = [a^2 + b^2] + [2ab] \neq Tr\begin{vmatrix} a^2 & 0 \\ 0 & b^2 \end{vmatrix} = [a^2 + b^2]$$

For the Pythagorean Triple, where $a > b$, note that $a \in \{e\}$, $b \in \{o\}$ Factoring out even multiples, one is left with the expression

$$c = a + b = 2 + 1$$
$$c^2 = [2^2 + 1^2] + 2(2)(1) = [4 + 1] + [4] = [5] + [4]$$

Since $(o)(o) \in \{o\}, \quad (e)(e) \in \{e\}$, this result can be characterized by the expression

$$c = e + o = 2 + 1$$
$$c^2 = [e^2 + o^2] + [2(e)(o)] = [e + o] + [e] = [5] + [4]$$

In this expression, the elements in the set $\left\{e^2+o^2\right\}$ characterize villagers ($c^2=9$) that only shave themselves (multiplication being the substitute for "shaves"), where the expression $\left[2(e)(o)\right]$ characterizes villagers that shave other villagers, but do not shave themselves.

That means that a villager can either belong in $c\in\left\{e^2+o^2\right\},c\notin\left\{2ab\right\}$ or $c\in\left\{2ab\right\},c\notin\left\{e^2+o^2\right\}$. If the barber is assumed to be a villager $b\in\left\{c^2\right\}$, then such a barber cannot exist (be in both sets at the same time.) That is, a barber cannot shave himself or shave others and not shave himself at the same time (implying $1^2\neq 1$

For the Pythagorean Triple, $\left\{5,4,3\right\}$ this means that the barber can either be in the set $b^2\in\left\{a^2+b^2\right\}=\left\{25\right\}=\left\{5^2\right\}=\left\{[4^2+3^2]\right\}$ or $b\in\left\{2ab\right\}=\left\{2(4)(3)\right\}=24$ but not both, noting that b^2 in the first case is second order (squared), while b in the second case is first order.

That is, the Pythagorean triple and the relation $c^2=a^2+b^2$ is invalid in the set of natural numbers (The Pythagorean triple excludes the products of sums e.g. ab).

Large Scale Limits

Note that for $n=1$ as an odd initial state, the expression $\#(n)=n\pm 1$ approaches n for very large n $\lim\limits_{n\to\infty}\#\,(n\pm 1)\simeq n$, so the initial state becomes insignificant. This also means that the expression $\lim\limits_{n\to\infty} n+(n\pm 1)\simeq 2n$ approaches $\#=2n$ and the expression $n+(n\pm 1)$ and that

$$\lim_{n\to\infty}\#\,[(n\pm 1)^2+(n)^2]=\lim_{n\to\infty}\#\,[(e)^2+(o)^2]=\lim_{m,n\to\infty}\#\,[(m)^2+(n)^2]=n^2+n^2=m^2+m^2$$

Where $n\in\left\{o\right\}$, $m\in\left\{e\right\}$, so the distinction between odd and even becomes insignificant at large m and n in terms of the initial state $n=1,m\simeq n$ That is, for m approximately equal to n where n is the initial state, for large values, the Pythagorean elements become farther and farther apart, so the elements of the subset $\left\{a^2+b^2\right\}$ approach either

$$c^2=\left[o^2+o^2\right]+\left[2(o)(o)\right]\in\left\{e+e\right\}\in\left\{e\right\}$$
$$c^2=\left[e^2+e^2\right]+\left[2(e)(e)\right]\in\left\{e+e\right\}\in\left\{e\right\}$$

That is, $o^2+o^2=2o^2$ or $e^2+e^2=2e^2$ so that c^2 is always even in second order if the (odd) initial state is insignificant compared to the scales involved, and non-linear systems approach linear systems as an approximation at large scales of even numbers compared to odd numbers (as initial states).

(I think, anyway, at least for now.)