Real Fermions and the Stern-Gerlach Experiment

And Special Theory of Relativity

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05/22/2024 – Added section on Spin and Probability

My pdfs:

SU(2)

Fermions (Complex analysis)

“Relativistic” Fermions

“From MM to Relativity”

Foundations of Mathematical Physics

Other related pdf’s of mine linked at the Physics Discussion Forum

The interacting elements of the experimental apparatus are defined as:

First element

$$(ct) = \int_0^{(ct)} d(ct) = (ct)\frac{(ct)}{(ct)} = (ct)1_{(ct)}$$

$$f_0 := (ct)$$

$$\#_0 := (ct) = \left(\frac{ct}{2} + \frac{ct}{2}\right)$$

$$m_0 := (\#_0)^2 := (ct)^2 = \left(\frac{ct}{2} + \frac{ct}{2}\right)^2 = \left[\left(\frac{ct}{2}\right)^2 + \left(\frac{ct}{2}\right)^2\right] + \left[2\left(\frac{ct}{2}\right)\left(\frac{ct}{2}\right)\right]$$

$$\left[2\left(\frac{ct}{2}\right)\left(\frac{ct}{2}\right)\right] = 4\left\{\frac{1}{2}\left(\frac{ct}{2}\right)\left(\frac{ct}{2}\right)\right\} = 4A_{Triangle}$$

$$\pi(m_0) := \pi(\#_0)^2 := \pi(ct)^2 = \left[\pi\left(\frac{ct}{2}\right)^2 + \pi\left(\frac{ct}{2}\right)^2\right] + \left[\left(\frac{ct}{2}\right)\left\{2\pi\left(\frac{ct}{2}\right)\right\}\right]$$

$$\left[\left(\frac{ct}{2}\right)\left\{2\pi\left(\frac{ct}{2}\right)\right\}\right] = \left[\left(\frac{ct}{2}\right)C_{\left(\frac{ct}{2}\right)}\right] = A_{Cylinder}$$

Transition to 2nd element

$$0 < vt' < ct$$

$$\# := (ct') = (ct) + (vt')$$

$$\#^2 = (ct')^2 = \left[(ct)^2 + (vt')^2\right] + \left[2(ct)(vt')\right],\ \left[2(ct)(vt')\right] = 4\left[\left(\frac{1}{2}\right)(ct)(vt')\right]$$

$$\left(\frac{1}{2}\right)(ct)(vt') = A_{Triangle}$$

$$\#^2 = Tr\begin{vmatrix}(ct)^2 & 0 \\ 0 & (vt')^2\end{vmatrix} + Det\begin{vmatrix}(ct) & (ct) \\ -(vt') & (vt')\end{vmatrix}$$

$$\Delta\# = \left[(ct)^2 + (vt')^2\right] - \left[2(ct)(vt')\right]$$

$$\pi(ct')^2 = \left[\pi(ct)^2 + \pi(vt')^2\right] + \left[(ct)\{2\pi(vt')\}\right], \left[(ct)\{2\pi(vt')\}\right] = \left[(ct)C_{(vt')}\right]$$

$$\left[(ct)C_{(vt')}\right] = A_{cylinder}$$

Final State

$$(ct')^2 + (ct')^2 = 2(ct')^2$$

$$2(\#) = 2(ct) = 2\left(\frac{ct}{2} + \frac{ct}{2}\right) = (ct) + (ct)$$

If the forces interact, then

$$2(m_0) = 4(\#)^2 = 4(ct)^2 = \left((ct) + (ct)\right)^2 = \left[(ct)^2 + (ct)^2\right] + \left[2(ct)(ct)\right]$$

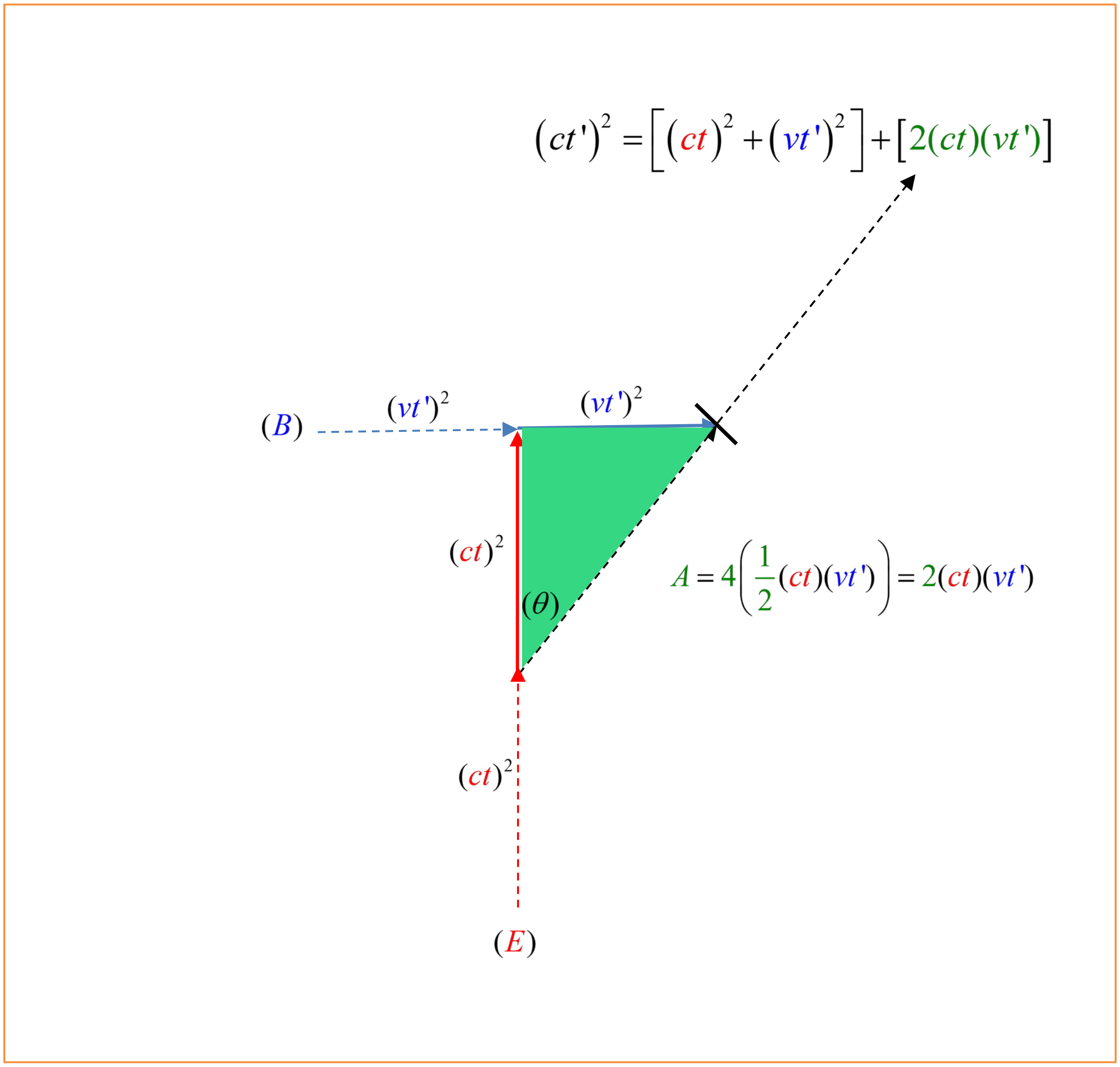
$$2(m_0) = 4(ct)^2 = Tr\begin{vmatrix}(ct)^2 & 0 \\ 0 & (ct)^2\end{vmatrix} + Det\begin{vmatrix}(ct) & (ct) \\ -(ct) & (ct)\end{vmatrix}$$

If they do not interact, then

$$4(ct)^2 = Tr\begin{vmatrix} (ct)^2 & 0 & 0 & 0 \\ 0 & (ct)^2 & 0 & 0 \\ 0 & 0 & (ct)^2 & 0 \\ 0 & 0 & 0 & (ct)^2 \end{vmatrix} = Tr\begin{vmatrix} m_0 & 0 & 0 & 0 \\ 0 & m_0 & 0 & 0 \\ 0 & 0 & m_0 & 0 \\ 0 & 0 & 0 & m_0 \end{vmatrix}$$

Curvature

The Stern-Gerlach Experiment



Initial States:

$$(ct) = (ct)\left(\frac{(ct)}{(ct)}\right) = (ct)\left(1_{(ct)}\right) \leftrightarrow (ct)^2 = (ct)^2\left(\frac{(ct)^2}{(ct)^2}\right) = (ct)^2\left(1_{(ct)^2}\right)$$

$$(vt') = (vt')\left(\frac{(vt')}{(vt')}\right) = (vt')\left(1_{(vt')}\right) \leftrightarrow (vt')^2 = (vt')^2\left(\frac{(vt')^2}{(vt')^2}\right) = (vt')^2\left(1_{(vt')^2}\right)$$

Final State:

$$(ct')^2 = \left[(ct)^2 + (vt')^2\right] + \left[2(ct)(vt')\right]$$

interaction with the second state $(vt')^2 := (B)^2$ where the interaction is $2(ct)(vt') = 4\left\{\frac{1}{2}(ct)(vt')\right\}$,

which continues on with no further interaction if there is no sensor, or impacts the sensor at the diagonal line and continues no further, since it is is completely absorbed by the sensor.

In this diagram, the initial state $(ct) := (E)$ is unperturbed in first order where $m_0 = (ct)^2 = (f)^2 = E^2$ in second order.

The second initial state in first order is $(vt') := (B)$ where $m_0 := (vt')^2 := (E)^2 = f^2$ in second order.

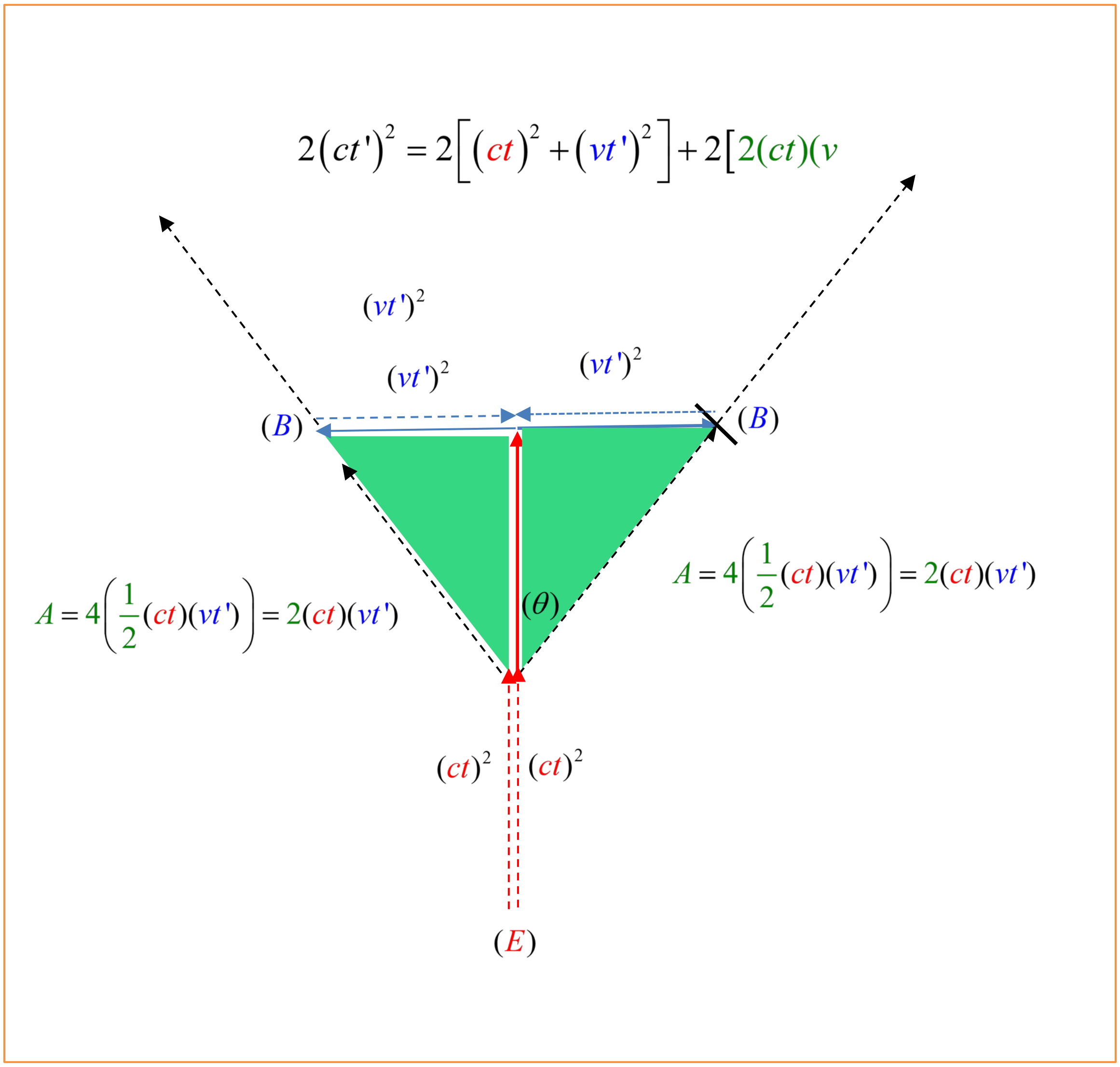
The sum in first order is $\# = (ct) + (vt')$.

The final state in second order is $\#^2 = \left[(ct)^2 + (vt')^2\right] + \left[2(ct)(vt')\right]$

Note that $\tan(\theta) = \frac{(vt')^2}{(ct)^2} = (\beta\gamma)^2$, $\beta := \frac{v}{c}$, $\gamma := \frac{t'}{t}$

Two systems

In the above diagram, the displacement and final path of the interacting particles is to the right. For the complete Stern- Gerlach result, the displacement to the left must be included. (This is effectively including both the "left-hand" and the "right-hand" rule of cross products in vector physics).



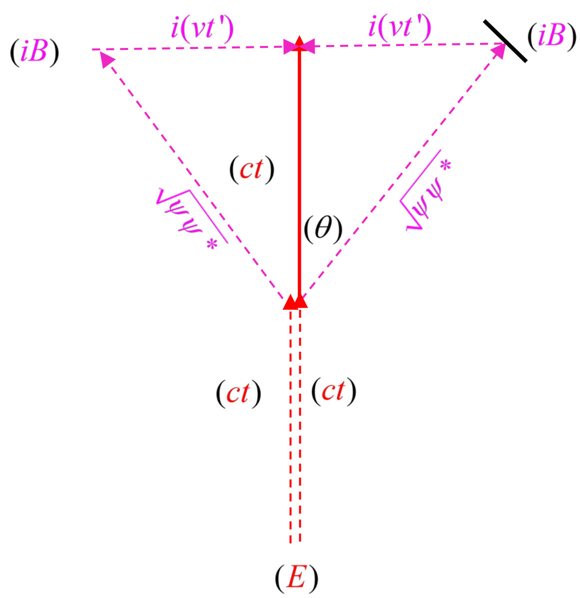
Note that the intersection of the B fields that deflect the particles are at in the flight path of the unperturbed particles, so if there is no B field the two particles will continue on their original path.

The deflected particles are called "fermions", and the undeflected particles are called "bosons".

Compare this with the imaginary states:

$$\psi\psi^* := \left[(ct)^2 + (vt')^2\right] \leftrightarrow \sqrt{\psi\psi^*} = \sqrt{[(ct)+i(vt')][(ct)-i(vt')]}$$

$$(\psi\psi^*) = \left(E^2 + B^2\right) \leftrightarrow \sqrt{\psi\psi^*} = \sqrt{[(E)+i(B)][(E)-i(B)]}$$



Note that the interaction "area" is not defined.

$$\psi := \left[(ct)+i(vt')\right]$$

$$\psi^* := \left[(ct)-i(vt')\right]$$

$$\psi\psi^* := \left[(ct)^2 - \{i(vt')\}^2\right] + \left[(ct)\otimes i(vt')\right] - \left[(ct)\otimes i(vt')\right]$$

$$\psi\psi^* := \left[(ct)^2 + (vt')^2\right] \leftrightarrow \sqrt{\psi\psi^*} = \sqrt{\left[(ct)+i(vt')\right]\left[(ct)-i(vt')\right]}$$

However, note that $(i)^4 = 1 \neq 1^2$ in second order, so that $(1)(\vec{i})\cdot(i)^4(\vec{j}) = (1)(\vec{i})\cdot(1)(\vec{j}) = 0$ and similarly for $(1)(\vec{i})\otimes(i)^4(\vec{j}) = (1)(\vec{i})\otimes(1)(\vec{j}) = 0$

So that $\psi\psi^* \neq \#^2$

Example:

$\# = 7 = 4 + 3$

$\#^2 = (7)^2 = [4^2 + 3^2] + 2(4)(3) = [25] + [24] = 49$

$\psi := 4 + 3i$

$\psi^* := 4 - 3i$

$\psi\psi^* := (4 + 3i)(4 - 3i) = [4^2 + 3^2] + 3i(4) - 4(3i) = [25]$

So that

$\#^2 = (7)^2 = 49 = [\psi\psi^*] + [2(4)(3)] = [25] + [24] = 49$ so the count is preserved in the full expansion, and the application of complex numbers is irrelevant.

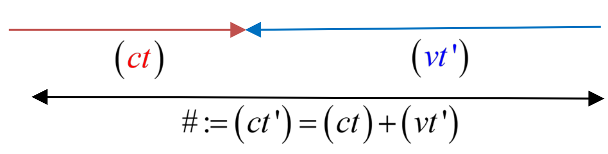
Spin and Probability

The interaction can be characterized as $h^2 := 2(ct)(vt') := 2S^2$ where $S = \dfrac{h}{\sqrt{2}}$,

In first order, $(ct') = (ct) + (vt') \leftrightarrow 1_{(ct')} = \left(\dfrac{t'}{t}\right) + \left(\dfrac{vt'}{ct'}\right)$ then $\beta := \dfrac{vt'}{ct'} = \dfrac{v}{c}$ represents the probability that $vt' = ct$ for $0 \le vt' \le ct$ for the two fields/particles (e.g., "positrons, electrons"). In second order $h^2 := 2(ct)^2 := 2S^2$, for $\theta = \dfrac{\pi}{4}$, $(vt') = (ct) \leftrightarrow (vt')^2 = (ct)^2$ so the probability is conserved for both orders.

The Special Theory of relativity

In first order, the relation between lengths is $(ct') = (ct) + (vt')$:



The Time Dilation equation is generated by solving the second order equation:

$(ct')^2 = (ct)^2 + (vt')^2$ for t', resulting in the expression $t' = t\Gamma,\ \Gamma = \frac{1}{\sqrt{1-\beta^2}},\ \beta = \frac{v}{c}$

However, the second order equation cannot be generated from the first order equation, and requires that the expression $i(vt')$ be imaginary, so that $t' = t\Gamma,\ \Gamma = \frac{1}{\sqrt{1^2-\beta^2}},\ \beta = \frac{v}{c}$ and

$\psi\psi^* = (ct)^2 + (vt')^2$ where $\psi\psi^* \neq \#^2$

That is, the "inertial frame" defined by (ct) does not interact with that defined by (vt'); there is no mutual "acceleration" defined by the "scalar product" $(ct)(vt')$

For more information, see "From MM to Relativity".

Curvature

Returning to the second order interaction equation, note that multiplying by π yields the relation(s):

$\pi\left(\#^2\right)=\pi\left(ct'\right)^2=\left[\pi\left(ct\right)^2+\pi\left(vt'\right)^2\right]+\left[\left(ct\right)\left\{2\pi\left(vt'\right)\right\}\right]$ where $\left(ct\right)\left\{2\pi\left(vt'\right)\right\}$ is characterized by a radius multiplied by a circumference (the area of a cylinder), while the other terms are areas of disks, but the complex relation yields $\pi\left(\psi\psi^*\right)=\pi\left(ct\right)^2+\pi(vt')^2$

In the real interaction, one end of the cylinder is “pinched off” as $\left(vt'\right)\to 0$ but never is a cone if $\left(vt'\right)>0$, so the “light cone” is truncated t $\pi\left(vt'\right)$. This is not infinitesimal in this model, and so does not apply to General Relativity which characterizes curvature in terms of derivatives (imaginary surfaces).