The Quadratic Equation

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Revised 1/18/2024 to include discussion of Wikipedia's derivation via "completing the square"

Reference: The Quadratic Equation (Wikipedia)

The quadratic equation to be solved is represented by the relation:

$ax^2 + bx + c = 0$ with solution $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ (see Wiki derivations).

This means that the solution to the equation is two-valued, since

$$2ax - b = \pm\sqrt{b^2 - 4ac}\,.$$

Assume there are no negative numbers (only differences between positive numbers), since

$$-c = a - b,\ b > a$$
$$b - c = a$$
$$a + 0 = a,\ a - a = 0$$
$$\therefore a \geq 0$$

That existence of a number implies that it is positive, and it is impossible to subtract more than its existing value.

If there are no negative numbers, there are no square roots of negative numbers, since

$i^2 = \left(\sqrt{-1}\right)\left(\sqrt{-1}\right) = \sqrt{(-1)(-1)} = \sqrt{1^2} = 1 \neq -1$, where $i = \sqrt{-1}$

See Imaginary Numbers – (Not)

Then $-b = i^2(b)$ is imaginary and ($b \leftrightarrow b^2$) are imaginary

$$x = \frac{\sqrt{b^2 - 4ac}}{2a} = \frac{\sqrt{b^2 + i^2(4ac)}}{2a}$$

$$\left(ax^2+bx+c\right)=0$$
$$a\left(ax^2+bx+c\right)=a\left(0\right)=0$$
$$\left(a^2\right)x^2+\left(ab\right)x+ac=a\left(0\right)=0$$

Every number is prime to its own base:

$$x=x\left(\frac{x}{x}\right)=x\left(1_x\right)$$
$$x^n=x^n\left(\frac{x^n}{x^n}\right)=x^n\left(1_{x^n}\right)$$
$$(x)(x^n)=x^{n+1}=x^{n+1}\left(\frac{x^n}{x^n}\right)\neq x^n\left(1_{x^n}\right)=x^n$$
$$\therefore 1^{n+1}\neq 1^n$$

$$a^2=a^2\left(\frac{a^2}{a^2}\right)=a^2\left(1_{a^2}\right)$$
$$ab=ab\left(\frac{ab}{ab}\right)=ab\left(1_{ab}\right)$$
$$ac=ac\left(\frac{ac}{ac}\right)=ac\left(1_{ac}\right)$$

Since this relation is true for all $x\geq 0$,

$$\left(a^2\right)=\left(ab\right)=ac=0$$

So the equation is satisfied for all x and x^2.

Consider the tensor:

$$\left|\vec{x}\right|:=\begin{vmatrix}x^2 & 0 & 0\\ 0 & x & 0\\ 0 & 0 & 1_x\end{vmatrix},\quad \left|\vec{\mathrm{A}}\right|:=\begin{vmatrix}a & 0 & 0\\ 0 & b & 0\\ 0 & 0 & c\end{vmatrix}$$

Where the global "unity" $\left(1\right)$ where $x=\left(1\right)x$; $\left(1\right)$ is only syntactical, and operationally meaningless.

Then $Tr\left|\vec{x}\right| = Tr\begin{vmatrix} x^2 & 0 & 0 \\ 0 & x & 0 \\ 0 & 0 & 1_x \end{vmatrix} = x^2 + x + 1_x$

$$Tr\left|\vec{\mathrm{A}}\right| = Tr\begin{vmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{vmatrix} = a + b + c$$

$$\left|\mathrm{X}\right| := \left|\vec{\mathrm{A}}\right| \times \left|\vec{x}\right| = \begin{vmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{vmatrix} \times \begin{vmatrix} x^2 & 0 & 0 \\ 0 & x & 0 \\ 0 & 0 & 1_x \end{vmatrix} = \begin{vmatrix} ax^2 & 0 & 0 \\ 0 & bx & 0 \\ 0 & 0 & c\left(1_x\right) \end{vmatrix}$$

$$Tr\left|\mathrm{X}\right| = ax^2 + bx + c\left(1_x\right) \neq 0,\ x \neq 0$$

$$x = 0 \leftrightarrow Tr\left|\mathrm{X}\right| = c\left(1_0\right) = c\left(\frac{0}{0}\right) = 0\left(\frac{0}{0}\right),\ c = 0$$

Since the sets $\{a,b,c\}$ and $\{x^2, x, 1_x\}$ form the bases of their respective (dual) spaces:

$\begin{vmatrix} a & b & c \end{vmatrix}$, $\begin{vmatrix} x^2 \\ x \\ 1_x \end{vmatrix}$, and in general a general orthogonal basis in an n-dimensional space can be represented by powers of x: $\{1_x, x, x^2, x^3,, ..., x^n\}$ where $\overrightarrow{x^n} \cdot \overrightarrow{x^{n+1}} = 0$ (i.e., the vectors are affine). Note that the power products in the multinomial expansion are non-zero.

Discussion of "Completing the Square" from Wikipedia

The process of completing the square makes use of the algebraic identity

$$\varphi = x + h$$

$$\varphi^2 = (x+h)^2 = \left[x^2 + h^2\right] + \left[2hx\right],$$

which represents a well-defined algorithm "**that can be used to solve any quadratic equation"**.

(This is actually a consequence of Fermat's Last Theorem for the case $n = 2$ via the Binomial Expansion.)

Starting with a quadratic equation in standard form,

$$ax^2 + bx + c = 0$$

(Note that if the above equation is represented by the relation above, then

$$\varphi = f(x+h) = g(x,a,b,c) = ax^2 + bx + c = 0$$
$$\leftrightarrow$$
$$\varphi^2 = \left(f(x+h)\right)^2 = 0$$

1. Dividing each side by a yields:

$$\left(\frac{a}{a}\right)x^2 + \frac{b}{a}x + \frac{c}{a} = 0$$

$$(1_a)x^2 + \frac{b}{a}x + \frac{c}{a} = 0$$

2. Subtracting the term $\frac{c}{a}$ from both sides yields:

$$(1_a)x^2 + \frac{b}{a}x = -\frac{c}{a} = -k$$

This means that either $c < 0$ or $a < 0$

3. Add $\frac{1}{2}\left(\frac{a}{b}\right)^2$ to both sides, "completing the square":

$$(1_a)x^2 + \frac{b}{a}x + \frac{1}{2}\left(\frac{a}{b}\right)^2 = -\frac{c}{a} + \frac{1}{2}\left(\frac{a}{b}\right)^2 = -k\frac{1}{2}\left(\frac{a}{b}\right)^2$$

$$\left(1_a\right)x^2+\frac{b}{a}x+\frac{1}{2}\left(\frac{a}{b}\right)^2=-\frac{c}{a}+\frac{1}{2}\left(\frac{a}{b}\right)^2=-k+\frac{1}{2}\left(\frac{a}{b}\right)^2$$
$$\left(1_a\right)x^2+\frac{b}{a}x+\frac{1}{2}\left(\frac{a}{b}\right)^2=-\frac{c}{a}+\frac{1}{2}\left(\frac{a}{b}\right)^2$$

Note that if $\{a,b,c\}$ are prime numbers, so that $a=a\left(\frac{a}{a}\right)=1_a,\ b=b\left(\frac{b}{b}\right)=1_b,\ c=c\left(\frac{c}{c}\right)=1_c$

Then division is not allowed, so that only the value $b=a$ and $c=a$ are allowed, so that the expression becomes:

$$\left(1_a\right)x^2+\frac{a}{a}x+\frac{1}{2}\left(\frac{a}{a}\right)^2=-\frac{a}{a}+\frac{1}{2}\left(\frac{a}{a}\right)^2$$
$$\left(1_a\right)x^2+\left(1_a\right)x+\frac{1}{2}\left(1_a\right)^2=-\left(1_a\right)+\frac{1}{2}\left(1_a\right)^2$$
$$\left(1_a\right)x^2+\left(1_a\right)x+\left(1_a\right)=0$$

Therefore, either $\left(1_a\right)=0$ so that either

$a\left(1_a\right)=a=0=b=c$ or $x<0$

But if x is a number, then $x=x\left(\frac{x}{x}\right)=x\left(\frac{-x}{-x}\right)$ (i.e., there are no negative numbers) so the original expression is inconsistent with the concept that every number is prime in terms of its own base.