# Pythagorean Triples

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Pythagorean Triple – (Wikipedia)

Remarks on the Foundation of Mathematical Physics (Working Document) (my pdf)

Relativistic Unit Circle (see General Case below) (my pdf)

See also Pythagorean Triples from Binomial Expansion (Kurmet Sultan)

RUC (link to my pdf

Formulas for generating Pythagorean Triples (Wikipedia) (they all are subject to the same consequences)

Parity – an upated version of this analysis, showing the relation between odd and even numbers.

There is much more to this story, but I don't have the spacetime to write it here…

After setting out some basic concepts, the analysis begins with Euclid's "Prime Number Generator"

**Basic Concepts**

**Existence**

$$0 = -0 \leftrightarrow 0 = 0$$

$$a + 0 = a \leftrightarrow a - 0 = a \leftrightarrow a + a = 2a$$
$$-a = -a \leftrightarrow a - a = 0$$

$$a - a = 0 \leftrightarrow a = a,\ a \neq 0$$

**There are no negative numbers**

$$-c = a - b.\ b > a \leftrightarrow b - c = a > 0$$

**Every number is prime relative to its own base**:

$n = n\left(\frac{n}{n}\right) = n(1_n)$ where $1_n$ defines the base of $n$

**Every even number is the sum of two primes** (Goldbach)

$n+n=2n$

**Proof of Fermat's Last Theorem** $c^n \neq a^n + b^n : \forall a,b,n > 1$ positive natural numbers including 0

(For Village Idiots)

$c = a + b$

$c^n = (a+b)^n = \left[a^n + b^n\right] + \left[f(a,b,n)\right]$ (Binomial Expansion)

$c^n = \left[a^n + b^n\right] \leftrightarrow \left[f(a,b,n)\right] = 0$
$\left[f(a,b,n)\right] \neq 0$

$c^n \neq \left[a^n + b^n\right]$ Q.E.D.

(Note: This result can easily be extended to multinomials.)

In particular, for $n = 2$

$c = a + b$
$c^2 = (a+b)^2 = \left[a^2 + b^2\right] + \left[2ab\right]$
$c^2 \neq \left[a^2 + b^2\right]$

So that Pythagorean triples are inconsistent with the natural number system, and must be represented by complex numbers:

$c = a + ib$
$c^* = a - ib$
$cc^* = a^2 + b^2$

Where the terms $a(ib) - a(ib) = 0$ does not mean that the terms vanish, but simply that they are equal $a(ib) = a(ib)$

The analysis begins with showing that while Euclid's Formula may generate Pythagorean Triples, it cannot be used to identify them as existing within the Natural Numbers (positive integers).

**Euclid's Formula for generating Pythagorean Triples**

Euler's method of generating Pythagorean Triples $\{c,a,b\}$ where $c^2 = a^2 + b^2$ is characterized by the

$$a = m^2 - n^2$$
$$b = 2mn = 2h^2,\ h^2 = mn$$
$$c = m^2 + n^2$$

Note that the set of relations above suggests that $\{c,a,b\}$ are first order, while the terms $\{m^2 - n^2, m^2 + n^2, h^2\}$ suggest second order.

$$c = a + b$$
$$m^2 + n^2 = \left[m^2 - n^2\right] + [2mn]$$
$$n^2 = -n^2 + 2mn$$
$$2n^2 = 2mn \leftrightarrow m = n$$

(Euclid's Formula is invalid for $n \neq m$ )

For $n = m$ , $c = m^2 + m^2 = 2m^2 = 4\left(\frac{1}{2}(mm)\right) = 4A_{\vartriangle}$ , where $A_{\vartriangle}$ is the area of an equilateral right triangle.

Note the right triangle in each of the four quadrants of the RUC (link to my pdf).

Note that this is inconsistent with the expressions

$$c = m + n$$
$$c^2 = (m+n)^2 = \left[m^2 + n^2\right] + [2mn]$$
$$m = n$$
$$c^2 = (m+m)^2 = \left[m^2 + m^2\right] + [2mm]$$

The Binomial Expansion for the case $n=2$

$$m=n$$

$$\#:=c_+=m+m$$

$$\#^2=(c_+)^2=(m+m)^2=\left[m^2+m^2\right]+[2mm]=\left[m^2+m^2\right]+[c_+]$$

$$\#^2=Tr\begin{vmatrix} m^2 & 0 \\ 0 & m^2 \end{vmatrix}+Det\begin{vmatrix} m & m \\ -m & m \end{vmatrix},\quad \#^2\neq Tr\begin{vmatrix} m^2 & 0 \\ 0 & m^2 \end{vmatrix},\quad \#^2\neq Det\begin{vmatrix} m & m \\ -m & m \end{vmatrix}$$

$$(c_+)^2\neq[c_+]\quad \text{Russell's Paradox: } 1^2\neq 1\leftrightarrow(c_+)=(c_+)\left(\frac{c_+}{c_+}\right)=(c_+)\left(1_{(c_+)}\right)\neq(c_+)^2=(c_+)\left(\frac{c_+}{c_+}\right)^2=(c_+)^2\left(1_{(c_+)^2}\right)$$

$$m\neq n$$

$$\#^2=(c_+)^2=(m+n)^2=\left[m^2+n^2\right]+[2mn]=\left[m^2+n^2\right]+[b]$$

$$\#^2=Tr\begin{vmatrix} m^2 & 0 \\ 0 & n^2 \end{vmatrix}+Det\begin{vmatrix} m & m \\ -n & n \end{vmatrix},\quad \#^2\neq Tr\begin{vmatrix} m^2 & 0 \\ 0 & n^2 \end{vmatrix},\quad \#^2\neq Det\begin{vmatrix} m & m \\ -n & n \end{vmatrix}$$

$$a=c-b$$

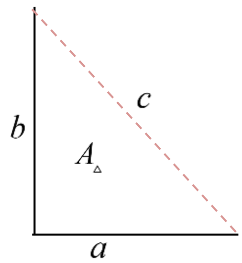
$$m=n\leftrightarrow 0^2=\left[m^2+m^2\right]-2mm\leftrightarrow\left[m^2+m^2\right]=2mm=2m^2$$

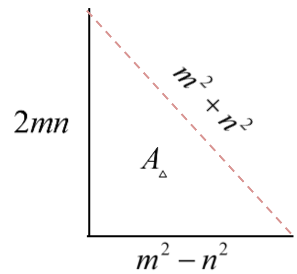
(Goldbach) "Every even number is the sum of two primes"

$$m+m=2m$$

$$m^2+m^2=2m^2$$

$$\#^2=\Delta^2\leftrightarrow 2mn=0\leftrightarrow(m=0\vee n=0)$$




$$A_\triangle = \frac{1}{2}ab \qquad A_\triangle = \frac{1}{2}(2mn)\left(m^2-n^2\right)$$

$$m = n \leftrightarrow A_\triangle = 0$$

$$n = in,\ i := \sqrt{-1}$$

$$c^2 = \left(m+n\right)^2 = \left[m^2+n^2\right]+\left[2mn\right] = \left[\psi\psi^*\right]+\left[2mn\right]$$

$$\psi = m+n$$

$$\psi^* = m-n$$

$$\psi\psi^* = \left[m^2+n^2\right]\left[+2mn-2mn\right]$$

$$c := \psi$$

$$c^* := \psi^*$$

The requirement for a Pythagorean Triple is that $\sqrt{\psi\psi^*}$ is an integer, so the Pythagorean triple is given by $\left\{\sqrt{\psi\psi^*}, m, n\right\}$, $m > n$, since $a^2 := m^2 - n^2 > 0$

The three equations from Euclid's formula then become

$$a = m^2 - n^2$$

$$b = 2mn = \sqrt{\left(2mn\right)^2} = \sqrt{4\left(mn\right)^2}$$

$$c = m^2 + n^2$$

Then

$$c := m^2 + n^2 = m^2 + (in)^2 = m^2 - n^2 = a$$
$$c^* := m^2 - n^2 = m^2 - (in)^2 = m^2 + n^2 = c$$
$$cc^* = (m^2 - n^2)(m^2 + n^2) = \left[m^4 + (in)^4\right] + \left[2(nm)^2 - 2(nm)^2\right] = m^4 + i^4 n^4$$

Note that

$$i^4 = \left(\sqrt{-1}\right)^2 \left(\sqrt{-1}\right)^2 = (-1)(-1) = 1^2 \neq 1^2$$
$$(1+1)^2 = \left[1^2 + 1^2\right] + [2(1)(1)] = \left[1^2 + 1^2\right] + \left[2(1^2)\right] ,$$
$$1^2 = (1)(1) = \frac{1}{2}\left\{(1+1)^2 - \left[1^2 + 1^2\right]\right\}$$

Where the term $(1^2)$ is the result of multiplication (interaction) not addition (existence).

and that $\left[2(nm)^2 - 2(nm)^2\right] = 0 \leftrightarrow 2(nm)^2 = 2(mn)^2,\ m > 0,\ n > 0$

$$cc^* = m^2 + n^2$$

**Pythagorean Triples**

The expression for $n = 2$ is fundamental to physics, especially in the case of complex variables. For $n = 2$, the Binomial Expansion is:

$$\# := c = a + b$$
$$\#^2 = c^2 = \left[a^2 + b^2\right] + [2ab]$$

For the set $\{7,4,3\} := \{\#,4,3\}$

$$\# : 7 = 4 + 3$$
$$\#^2 = 49^2 = \left[4^2 + 3^2\right] + \left[2(4)(3)\right] = [16 + 9] + \left[2(12)\right] = [25] + [24]$$

, where the term $2ab$ defines multiplication (interaction, entropy, entanglement, etc.) between the elements $a$ and $b$ . With this addition, the set $\{c,a,b,+,\times\}$ is no longer a group, but a Ring. The "existence" term $\left[a^2 + b^2\right]$ cannot be generated from the first order expression $c = a + b$ but requires complex numbers. (see below).

A Pythagorean triple $\{c,a,b\}$ is defined by the expression $c^2 = \left[a^2 + b^2\right]$, which is not in the set of positive natural numbers, due to the extension of the proof of Fermat's Last Theorem for $n = 2$ via the

Binomial Expansion as proven above. The most common Pythagorean Triple is the set $\{5,4,3\}$ in which $5^2 = 4^2 + 3^2 = 16 + 9 = 25$

Complex Numbers

A Complex Number of factor is defined by including a factor $i := \sqrt{-1}$ where $i^2 = -1$ but $(i)^4 = 1$

$$\psi = [4 + 3i]$$
$$\psi^* = [4 - 3i]$$
$$\psi\psi^* = [4+3i][4-3i] = [16+9] + [12i] - [12i] = [25] + [12i] - [12i]$$

, where imaginary terms are represented in pink.

Note that the real existence term $25 = [16+9]$ is not equivalent to the term $[16+9]$, since terms $+[12i]-[12i] \leftrightarrow [12i] = [12i]$ is not equivalent to $12 - 12 = 0 \leftrightarrow 12 = 12$ and that $12 \neq 0$ and $12 \neq 0$

If there are no negative numbers, there are no square roots of negative numbers except in the imagination.

In general, expression in complex numbers is given by:

$$\psi = a + i\beta$$
$$\psi^* = a - i\beta$$
$$\psi\psi^* = (a + i\beta)(a - i\beta) = [\alpha^2 + \beta^2] + \alpha i\beta - \alpha i\beta = [\alpha^2 + \beta^2]$$

From a syntactical representation, note that:

$\#^2 = [\psi\psi^*] + [2ab]$ so that $\#^2 \neq [\psi\psi^*]$ (a consequence of Fermat's Last Theorem for $n = 2$

However, the expression $\psi\psi^* = \alpha^2 + \beta^2$ can be expressed as integers (even if real and imaginary), so there must be a way to generate Pythagorean Triples from positive integers in first order. This can be done by applying the Binomial Expansion as outlined below.

(Note that the expression $\#^2 := \psi\psi^* = \alpha^2 + \beta^2$ might be taken as an example of a wff showing its arithmetic system is incomplete (because the expression is "unprovable" as characterized by Peano's

Axioms (since it requires complex evaluation, but if the Multinomial Theorem (products of sums) is included, the system is complete, and Goedel's Proof is irrelevant.

To show that, one must be able to generate Pythagorean Triples from the natural numbers. This can be done via the Binomial Theorem where the real expression $\psi\psi^{*} := \alpha^2 + \beta^2$ only appears in the context of the expansion: $\#^2 = \left[\alpha^2 + \beta^2\right] + \left[2\alpha\beta\right]$

**Corollaries**

1. If $\beta$ is odd, then $\psi\psi^{*}$ is odd ( $\alpha \in \{e\},\ \beta \in \{o\} \leftrightarrow \left(\psi\psi^{*} = \alpha^2 + \beta^2\right) \in \{o\}$

   $\#^2 \in \{N^2\} \leftrightarrow \# \in \{N\}, \{e\} + \{o\} \equiv \{N\}$
2. $\psi\psi^{*} \notin \{N\}, \psi\psi^{*} \in \{\mathbb{C}\}$

   $\psi\psi^{*} \neq \#^2$
3. Since $\{e\} + \{o\} \equiv \{N\}$ is complete where the set $\{o\}$ characterizes the prime numbers in first order $n = n\left\{\frac{n}{n}\right\},\ n \in \{o\}$ and the set $(n+n) = (n+n)\left\{\frac{n+n}{n+n}\right\}, (n+n) \in \{e\} \equiv \{2n\}$ Goldbach), the first order sets $\{N\}$ and $\{N^n\}$ are complete and consistent for both addition and multiplication operators if the multinomial expansion is added to Peano's axioms and subtraction for $n > 1$ represents differences between existing (i.e. $n, b > 0, n > b$ )

   $\{N^2\} \in \{N\}$
4. $\# = (N+N) = 2N \in \{N\}$

   $\#^2 = (N+N)^2 = \left[N^2 + N^2\right] + \left[2N\right] \notin \{N\}$

   (Russell's Paradox)
5. Godel"s "proof" fails because the Fundamental Theorem of Arithmetic (required for his characterization of wff's) fails to distinguish between odd and even numbers (every number is prime relative to its own base ( $e = e\left(\frac{e}{e}\right) = e(1_e),\quad o = o\left(\frac{o}{o}\right) = o(1_o)$ but

   $(e+o) \equiv N,\ |e-o| \neq 0,\ |e-o| > 0$ That is,

   $G(7) = G(7)$

   $G(7) \neq G(4+3), G(6+1), G(5+2), G(4) + G(1) + G(2), etc.$

   $G(7) \neq \text{graffity,my password, modern art, wff} \in \{\text{English Language}\} \neq "7"$

**Special Relativity**

(The "Time Dilation" equation of Special Relativity is generated by solving the equation

$(ct')^2 = (ct)^2 + (vt')^2$ yielding the expression $t' = t\Gamma$ where $\Gamma = \frac{1}{\sqrt{1-\beta^2}}$, $\beta = \frac{v}{c}$

Stephen Hawking discusses "imaginary time" in his book "A Brief History of Time" with Joseph Beckenstein suggesting that entropy is proportional to area.

$2\alpha\beta = 4\left(\frac{1}{2}\alpha\beta\right) = 4(A_{\vartriangle})$ where $A_{\vartriangle}$ is the area of a triangle

However, note that the relation $x = vt$ does not exist in the equation to be solved, and that (classically)

$\beta = \frac{v}{c} = \frac{mv}{mc} = \frac{P_v}{P_c} \leftrightarrow \beta^2 = \frac{E_v}{Ec} = \frac{mv^2}{mc^2} \leftrightarrow \beta^2 = \left(\frac{mv}{mc}\right)^2$ which illustrates that the coordinate system is independent of mass (in STR).

(see **From Lorentz (MM experiment) to Special Relativity**)

This has important consequences on any physics (or mathematical) model that uses imaginary numbers, in particular vectors (e.g., Electromagnetism, Relativity, Quantum Mechanics)

(Imaginary numbers are complex only for those who think they are somehow real) ...

If this analysis is correct, it will have a profound effect on (both) STR and GTR, and Quantum Field Theory, all of which have their foundation in imaginary numbers (the special unitary group SU(2), which is much ado about nothing. There is much more to this story, but I don't have the spacetime to write it here. It will be included in my final pdf to "**Working Document**"

Note: I haven't seen this anywhere else, so if it is original, I claim a beer and pizza from someone…

It would be a simple matter to write a looping program, which I will do in the future if I get time. If someone else wants to do it, I would really appreciate being informed of the result. But first I have to finish and final edit "Working Document" (and I'm 84, so …. )

Note: here is an **example of another generator**
The problem lies in the last step e c^2=n^2 +m ^2 is imaginary (not real positive integers in the general case), as outlined above in the secction on complex numbers.

This also shows that Godel's proof is irrelevant (if he only addresses Peano's axioms);  arithmetic is complete and consistent if it includes **both Peano's axioms and the mulltinomial expansion** (eg. the binomial expansion).  Godel defines wff's only in terms of products of prime numbers (the Fundamental Theorem of Arithmetic), but does not include (as Peano) products of sums e.g. $(a+b)^2$ where **$c^2$ =** $a^2$ **+ $b^2$** is the Pythagorean Theorem.

**Transitions**

An initial state of a system can be characterized as $\# := n+0,\ n>0$ invariant (identical) particles, where $\#$ represents the count of particles in the system.

This state can be changed by the count second set of particles represented by the character $m$, so that the system is now represented by the expression $\# = n \pm m$ where color is used to distinguish between the sets.

**Radiation**

In the above analysis, the radiation change of state is represented by Pythagorean Triples, where

$$\# := n - m,\ m < n$$

$$(\#)^2 = \left[n^2 + m^2\right] + \left[2nm\right]$$

$$n = n + 0$$
$$m = n-1, n-2, \ldots\ldots\ldots\ldots, (1) = (n-n)+1$$

This represents a rotation (ccw in quadrants 1 and 3, cw in quadrants 2 and 4) of the circle described in the article "The Relativistic Unit Circle") and $\left[2nm\right]$ represents a decrease in entropy (radiation) from an initial state $\# = 2n = n + m,\ m = n$ to a final state $\# = n = n - m = 0,\ m = n$

First quadrant: $\theta = \dfrac{\pi}{4}$ ccw decrease to $\theta = 0$. Second quadrant: $\theta = \pi$ cw increase to $\theta = \dfrac{\pi}{4}$

Third quadrant: $\theta = \dfrac{3\pi}{4}$ ccw decrease to $\theta = \pi$. Fourth quadrant: $\theta = \pi$ cw increase to $\theta = \dfrac{\pi}{4}$

**Accretion (Absorption) (from $\# = 0$ to $\# = n$)**

$$\# := n = 0 + m,\ m = 1, 2, 3, \ldots, m = n$$

$$(\#)^2 = \left[n^2 + m^2\right] + \left[2nm\right] = m^2$$

This represents an increase in entropy from $n = 0$ to $n = n$

**Accretion (Absorption) (from $\# = n$ to $\# = 2n$)**

$$\# := n + m,\ m \le n$$

$$(\#)^2 = \left[n^2 + m^2\right] + \left[2nm\right]$$

These expressions an increase in entropy from an initial state $n = n + 0$ to a final state $n = 2n$ where

$$n = n + 0$$
$$m = n + 1, n + 2, \ldots\ldots\ldots\ldots, (n+1) = (n+n) + 2n$$

This represents a rotation (cw in quadrants 1 and 3, ccw in quadrants 2 and 4) of the circle described in the article "The Relativistic Unit Circle") and $[2nm]$ represents an increase in entropy (accretion) from an initial state $\# = n + 0$ to a final state $\# = 2n = n + m,\ m = n$

First quadrant: $\theta = 0$ cw increase to $\theta = \frac{\pi}{4}$. Second quadrant: $\theta = \frac{\pi}{4}$ ccw decrease to $\theta = \pi$

Third quadrant: $\theta = \pi$ cw increase to $\theta = \frac{3\pi}{4}$. Fourth quadrant: $\theta = 2\pi$ cw decrease to $\theta = \frac{3\pi}{4}$

**Accretion (Absorption) (**to #> $2n$**)**

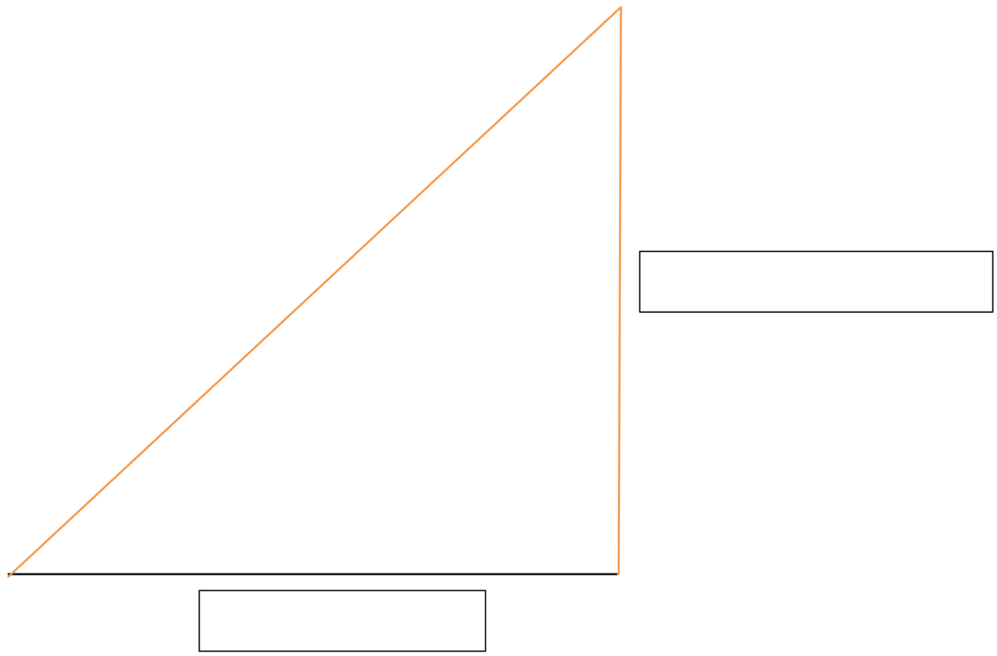
The above cycle is repeated for $\# = kn,\ k = 1, 2, 3\ldots$ to $\# = k(n+1)$

**General Case**

In the more general case, . $n := (ct), m = \left(\frac{vt'}{ct}\right)ct = vt'\left(\frac{ct}{ct}\right) = vt'(1_{ct}),\ vt' < ct$ .The term vt' is interpreted as a coordinate (or momentum ) density, $(vt')$ is is then interpreted as the transition states between invariants (as represented by the expression $n := n(ct),\ n = 0, 1, 2, \ldots.,$ , where

$$\varphi := ct + vt'$$
$$\varphi^2 = (ct + vt')^2 = \left[(ct)^2 + (vt')^2\right] + \left[2(ct)(vt')\right]$$

See the document Relativistic Unit Circle



The Pythagorean Triples are: $\{c_K, a, b_K\} \equiv \left\{\sqrt{\psi\psi^*}, a, b_K\right\} \{c, a, b\}$

The Pythagorean Triples $\left\{\sqrt{\psi\psi^*}, \alpha, \beta\right\}$ are then generated by the relation

$(\varphi)^2 = [\psi\psi^*] + [2\alpha\beta]$ from the relations

$$\psi = \alpha + i\beta$$
$$\psi^* = \alpha - i\beta$$
$$\psi\psi^* = \left[\alpha^2 + \beta^2\right] + \left[\alpha i\beta - \alpha i\beta\right] = \left[\alpha^2 + \beta^2\right]$$

$$\alpha = 10$$
$$b = 13$$