

# Pythagorean Proofs – NOT

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Updated 09/28/2025 Corrected Physics diagram

In this document the argument is presented (by me?) that the Pythagorean Theorem is inconsistent with the Binomial Theorem within the set of the positive Natural Numbers and thus invalid: in particular, that  $c^2 \neq a^2 + b^2$  for  $a, b$  and  $c$  positive natural numbers – in particular the (5, 4, 3) Pythagorean triple as the example. The most general disproof of the Pythagorean Theorem is a special case of Fermat's Last Theorem for the case  $n = 2$  via the Binomial Theorem.

It is then shown that the Pythagorean Theorem can only exist as a subset of the Binomial Expansion, and in the process showing that complex numbers are inconsistent with the Natural Numbers as expected, but can result in the relation  $\psi\psi^* = a^2 + b^2$  where the color magenta is used to indicate complex numbers where  $i := \sqrt{-1}$ ,  $ia := a$ , and the assumption that addition, subtraction and complex conjugation are vector operators in the complex plane.

The fundamental issue is that the most of the proof of the Pythagorean theorem are geometric; i.e., proven by examination of lines, triangles, and squares, connected together in various ways on a flat 2 dimensional surface (piece of paper). These elements are connected visually, but mathematically are represented by vectors, which are translationally and rotationally invariant, meaning that the geometric elements have no common origin – the geometric elements can be separated, moved around, rotated, etc., and it would have no effect on the Pythagorean Theorem.

To understand why the geometric proofs fail, one has to understand Fermat's Last Theorem for the case  $n \geq 2$  and the Binomial Expansion, with the general for proven by Newton some years later after Fermat's death on January 12, 1665. I suspect that Fermat restricted  $n$  to  $n > 2$  because he couldn't imagine that Pythagoras was wrong. The other issue is with negative numbers, which must be rejected in the set of Natural Numbers, where only positive differences between elements can be accepted.

To show geometry inconsistent with the Binomial Theorem, one has to examine the Foundation of Mathematics at a fundamental level; I have provided much of the analysis in the Working Document pdf referenced above, with internal links to relevant articles.

The form of the Pythagorean theorem,  $c^2 = a^2 + b^2$  means that it includes no multiplicative elements  $ab$  which are commutative in the Natural Numbers. In the geometric proofs, this means that the formalism does not include the areas of the four right triangles  $2ab = 4[(1/2)ab]$  This means that

the Pythagorean theorem cannot be derived from the first order relation  $c = a + b$  where the elements represent areas in second order  $c^2 = [a^2 + b^2] + [2ab]$

Note that  $\psi\psi^* = a^2 + b^2$  with  $a$  real and  $b$  imaginary via complex conjugation, but this approach requires a vector relation in the complex plane. This approach is analyzed in the following document and shows why the vector products (dot product, cross product, and conjugation) required to produce the relation fail because of their vector characteristics.

All of the geometric proofs fail for the same reason, in one way or another. I have shown the invalid "proof by area" (from Cambridge University) as a start, but the basic proof that the relation is invalid comes from the proof of Fermat's Last Theorem for  $n \geq 2$ .

[Proofs of the Pythagorean Theorem](#)

[Russell's Paradox](#)

[Working Document](#)

[The Relativistic Unit Circle](#)

[Parity](#)

[Quarks](#) (will be updated shortly)

**Postulate:** There are no negative numbers.

**Proof:**  $-c = a - b, b > a \leftrightarrow b - c = a = a + 0 > 0$

**Corollary:** If there are no negative numbers, there are no square roots of negative numbers.

[Binomial Theorem](#) (Wikipedia)

**Proof that  $c^2 \neq a^2 + b^2$  in the positive natural numbers via Fermat's Last Theorem and the Binomial Theorem**

**Proof of Fermat's Last Theorem for  $n \geq 2$**

$$c = a + b$$

$$c^n = [a^n + b^n] + [f(a, b, n)]$$

$$c^n = [a^n + b^n] \leftrightarrow [f(a, b, n)] = 0$$

$$[f(a, b, n)] \neq 0$$

$$c^n \neq [a^n + b^n]$$

**In particular, for  $n = 2$**

$$c = a + b$$

$$c^2 = [a^2 + b^2] + [2ab]$$

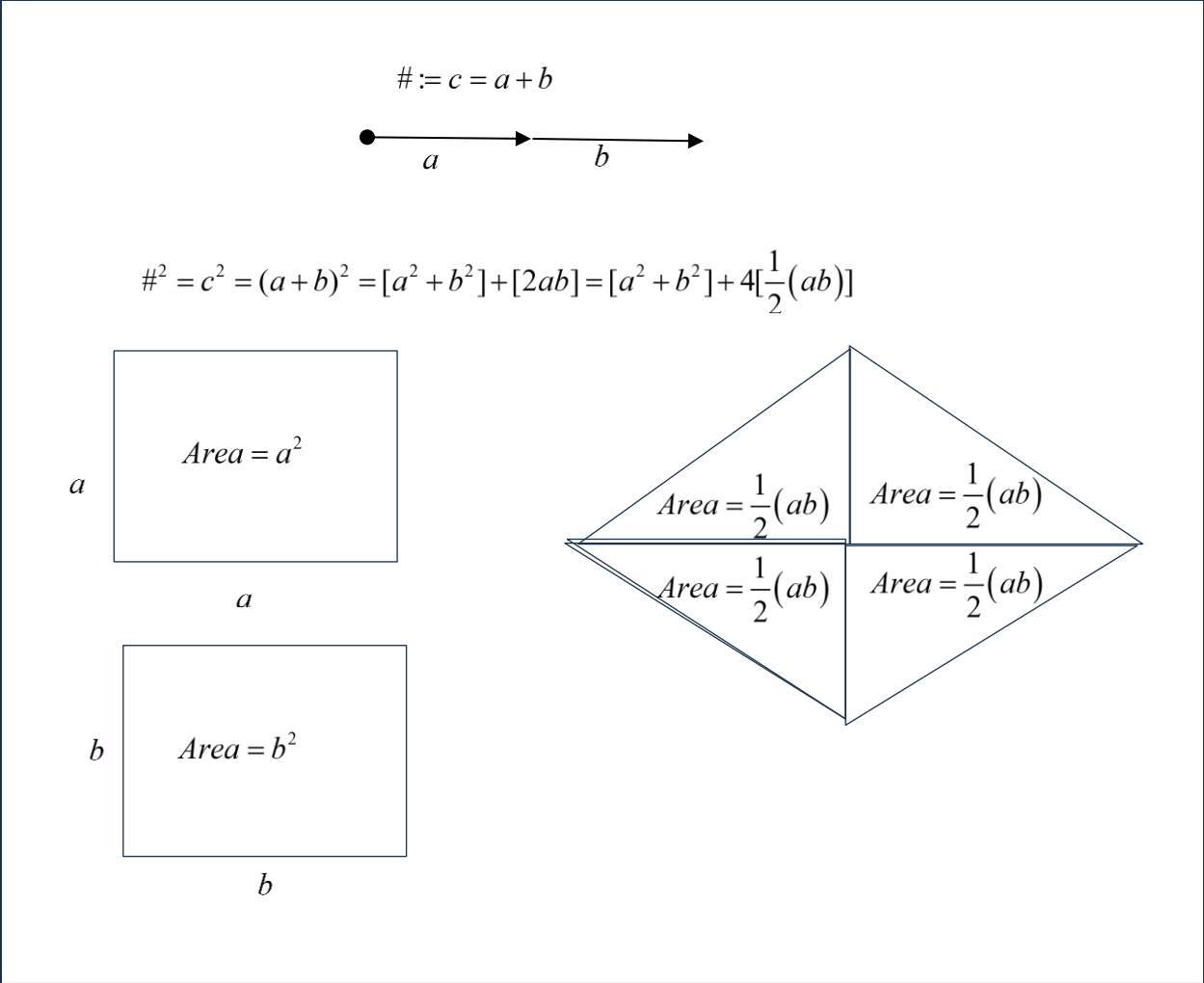
$$c^2 = [a^2 + b^2] \leftrightarrow [2ab] = 0$$

$$[2ab] \neq 0$$

$$c^2 \neq [a^2 + b^2] \quad \text{QED}$$

This means that  $c^2 = a^2 + b^2$  cannot exist outside the context of  $A = [a^2 + b^2] + [2ab]$ , which is the second order formula of the Binomial Expansion (i.e., for  $n = 2$ ).

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$$(\#)\vec{i} := (c)\vec{i} = (a+b)\vec{i}$$

$$(\#^2)\vec{i} = (c^2)\vec{i} = \{[a^2 + b^2] + [2ab]\}\vec{i}$$

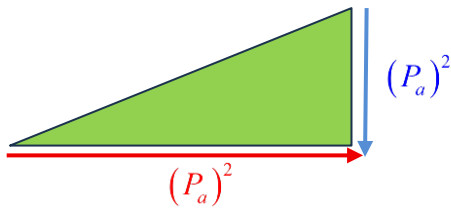
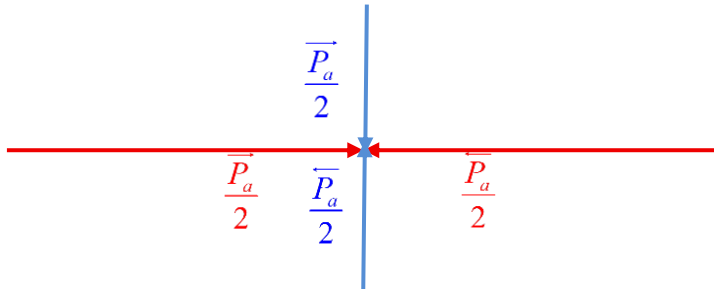
in a physical characterization where each element  $a^2$  and  $b^2$  can be interpreted as equal and opposite momenta at a common interaction site, with  $[2ab]$  representing the interaction (change in entropy, entanglement, etc.)

There is much more to this story, but see the links above for further analysis, e.g..

[The Relativistic Unit Circle](#)

$$P_a = \frac{\vec{P}_a}{2} + \frac{\vec{P}_a}{2} \quad P_a = \frac{\vec{P}_a}{2} + \frac{\vec{P}_a}{2}$$

$$(m_0 v)^2 = (P_a)^2 \quad (m_0 v)^2 = (P_a)^2$$



$$Area = 4 \left[ \frac{1}{2} (P_a)(P_a) \right] \equiv 4 \left[ \frac{1}{2} (P_a)(P_a) \right]$$

$$(\#)\vec{i} := (P_a + P_a)\vec{i}$$

$$(\#^2)\vec{i} = \{(P_a + P_a)^2\}\vec{i} = \{[P_a^2 + P_a^2] + [2P_a P_a]\}\vec{i} = \left\{ [P_a^2 + P_a^2] + 4 \left[ \frac{1}{2} (P_a P_a) \right] \right\} \vec{i}$$

### Matrix Representation

$$|\#| := \begin{vmatrix} a & 0 \\ 0 & b \end{vmatrix}, \quad Tr|\#| = a + b, \quad Det|\#| = ab = ba$$

$$Tr|\#|^2 := (a + b)^2 = [a^2 + b^2] + [2ab]$$

$$= Tr \begin{vmatrix} a^2 & 0 \\ 0 & b^2 \end{vmatrix} + Det \begin{vmatrix} a & a \\ -b & b \end{vmatrix} = [a^2 + b^2] + [2ab]$$

(Note that the element  $-b$  is a convention of the determinant in matrix representation)

## The Complex Plane

In the complex plane, defining addition, subtraction, multiplication, and complex conjugation as vector operations:

$$c := \psi := a + b$$

$$c^* := \psi^* := a - b'$$

$$|\psi| := \begin{vmatrix} a & 0 \\ 0 & b \end{vmatrix}, \text{Tr}|\#| = a + b, \text{Det}|\#| = ab = ba = \vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{a}$$

Note that if the vectors  $\vec{a}$  and  $\vec{b}$  are orthogonal, then  $\vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{a} = 0 = 0$

The vector cross product is a mapping from two dimensions into a one dimensional multiplicative scalar:

$$|a \otimes b| := \begin{vmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & 0 \end{vmatrix} \Rightarrow \begin{vmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & ab \end{vmatrix} \text{ where the result is } a \otimes b = ab \text{ but } a = 0 \text{ and } b = 0$$

If  $a$  is a scalar on the imaginary axis, then  $ab = ba$

Complex conjugation results in

$$\begin{aligned} cc^* &= \psi\psi^* = (a+b)(a-b) \\ &= (a^2 + b^2) + (ab - ba) \\ &= (a^2 + b^2) \end{aligned}$$

$$\begin{aligned} \text{Tr}|\psi\psi^*| &:= (a+b)^2 = [a^2 + b^2] + [2ab] \\ &= \text{Tr} \begin{vmatrix} a^2 & 0 \\ 0 & b^2 \end{vmatrix} + \text{Det} \begin{vmatrix} a & a \\ -b & b \end{vmatrix} = [a^2 + b^2] + [2ab] \end{aligned}$$

For the Pythagorean Triple  $\{5, 4, 3\} \equiv \{c, a, b\}$

$$(c)(c^*) = a^2 + b^2 = 16 + 9 = 25$$

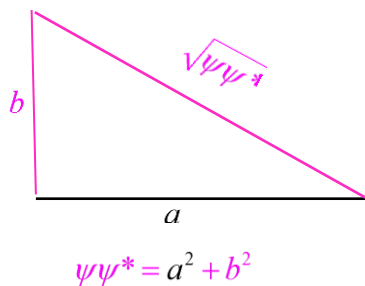
$$(5)(5^*) = 4^2 + 3^2 = 16 + 9 = 25$$

However, in the context of

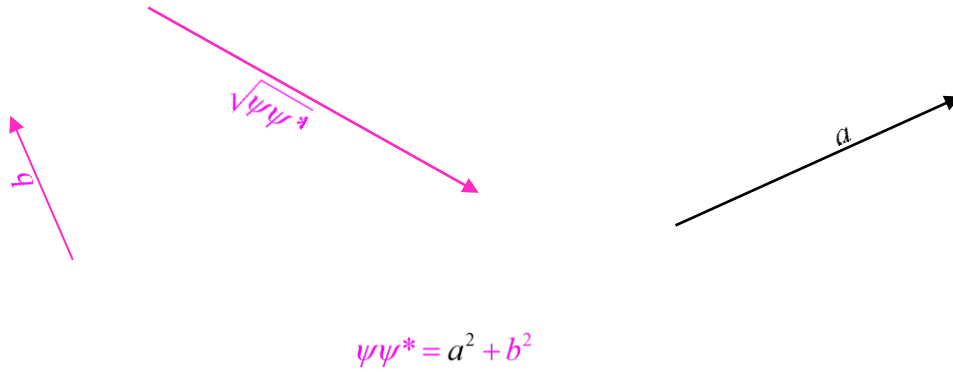
$$\begin{aligned} A = 7^2 = 49 &= [a^2 + b^2] + [2ab] = 25 + 24 \\ &= 7^2 = 49 = [\psi\psi^*] + [2ab] = 25 + 24 \end{aligned}$$

the analysis has simply shifted from the real axis to the imaginary axis, and  $[\psi\psi^*] = [25]$  appears again as a subset of  $7^2 = 49$ .

In the context of the complex plane, the perimeter of the right triangle is represented by the relation:

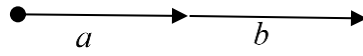


Note that the legs of the triangle are vectors in the complex plane; they can be moved arbitrarily and the mathematical relation still holds (i.e., the vectors have no common origin.)

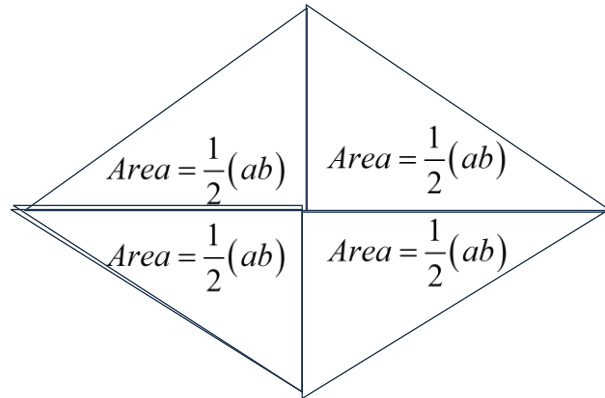
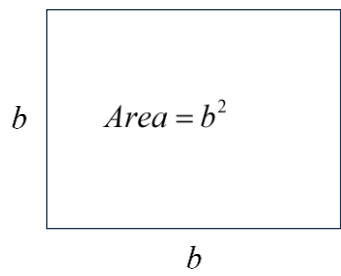
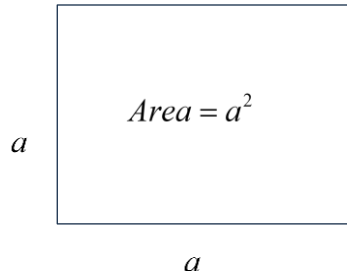


In the real analysis, the hypotenuse doesn't exist; the "triangle" in each quadrant is characterized held together by the values on the real axes ( $a^2$  and  $b^2$ ) and by the areas in each of the four quadrants.

$$\# := c = a + b$$

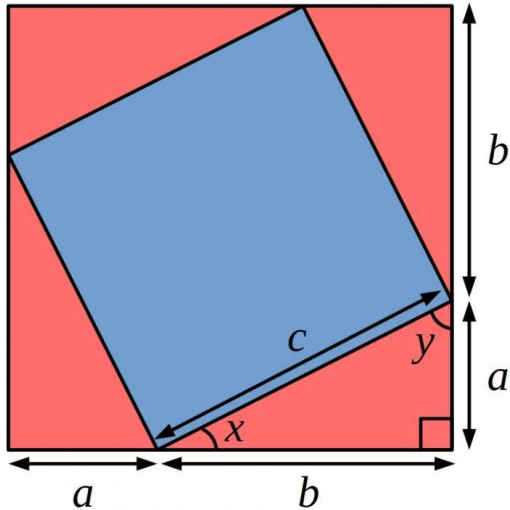


$$\#^2 = c^2 = (a+b)^2 = [a^2 + b^2] + [2ab] = [a^2 + b^2] + 4\left[\frac{1}{2}(ab)\right]$$



## Invalid Pythagorean “Proof”

[Proof 1 - Not](#) (University of Cambridge)



1. Take a square with side lengths  $a + b$ , divided up into four identical right-angled triangles and an enclosed quadrilateral of sides  $c$ .
2. The area of the small square is  $\alpha = c^2$
3. The area of the large square is  $A = (a + b)^2 = [a^2 + b^2] + [2ab] = [a^2 + b^2] + [4\{(1/2)ab\}]$   
where  $a_{\Delta} = [(1/2)ab]$  is the area of each of the four right triangles.
4. The area of the enclosed square is also equal to the area of the large square minus the sum of the areas of the four right triangles.  
$$a = c^2 = A - 4a_{\Delta} = [a^2 + b^2] + [4\{(1/2)ab\}] - [4\{(1/2)ab\}] = [a^2 + b^2]$$
5. Subtracting  $4a_{\Delta}$  from both sides results in  $c^2 = a^2 + b^2$

The reason that this proof fails is that while  $4a_{\Delta} - 4a_{\Delta} = 0$  it does not imply that  $4a_{\Delta} = 4(1/2)ab = 0$ , which would require that either  $a = 0$  or  $b = 0$ . In particular, if  $4a_{\Delta}$  can be subtracted from both sides, it can be added just as easily, restoring  $A$  to the original expression. However, subtraction invalidates the equality, since it introduces an imaginary vector operation of subtraction. Restoring  $4a_{\Delta}$  restores the equality.

The bottom line is that geometry requires complex variables in two dimensions. In the diagram, the corners of the squares and triangles have no common origin, and so their sides must be characterized as vectors, which are translationally and rotationally invariant. The sum of the areas of the geometric units would be the same even if the triangles and squares (and/or their components) are separated on the page.

That is, a vector  $\mathbf{a}$  (as a vector component) in the first "dimension" could be on the earth, and a vector  $\mathbf{b}$  in the second "dimension" (as a vector component) could be in Andromeda, and so multiplication  $\mathbf{ab} = \mathbf{ba} = \mathbf{0}$  both for dot products and cross products

Consider the mathematical difference between an iron wheel rim and a smoke ring - the rim is unchanged by interaction with the atmosphere (i.e., any change), while the smoke ring disintegration with any such change. However, if the ring is placed on the nose of a re-entry vehicle in the atmosphere, it will heat up and disintegrate due to thermal interaction.

Proponents of vectors attempt to get around this by introducing complex variables in  $SU(2)$  in the

$|\sigma_2| := \begin{vmatrix} 0 & i \\ -i & 0 \end{vmatrix}$  matrix as a group element with multiplication as the operation, via the Pauli matrices

$\{\sigma_1, \sigma_2, \sigma_3\} := \left\{ \begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix}, \begin{vmatrix} 0 & i \\ -i & 0 \end{vmatrix}, \begin{vmatrix} 1 & 0 \\ 0 & -1 \end{vmatrix} \right\}$  where the traces are all 0, and  $Tr|\sigma_3| = 1 - 1 \leftrightarrow 1 = 1$ , not that  $1 = 0$ .

The sum of  $\sigma_1$  and  $\sigma_2$  provides an additive "connection"

$|\sigma_{1+2}| := |\sigma_1| + |\sigma_2| = \begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix} + \begin{vmatrix} 0 & i \\ -i & 0 \end{vmatrix} = \begin{vmatrix} 0 & 1+i \\ 1-i & 0 \end{vmatrix}$ , but  $|\sigma_{1+2}|$  is no longer a group element.

Note that the identity matrix  $|I| := \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix}$  is not a member of the group, so  $Tr|I| := 1 + 1 = 2$  and

$Det|I| = 1^2$  are not members of the group either. Also,

$Det \begin{vmatrix} 0 & 1+i \\ 1-i & 0 \end{vmatrix} = -(1+i)(1+i) = 1^2 + i^2 = 1^2 + 1$ , noting that  $1^2 \neq 1$  because they are of different order.

The problem is that adding the two matrices introduced the vector operation  $\mathbf{a} + \mathbf{b}$ , as conjugate elements which invalidates SU(2) as a group, since it now includes addition, so two operations are defined, and former group is now a ring (without division), but one of the elements are real and the other is imaginary.

But if there are no negative elements, there are no square roots of negative elements, so the  $|\sigma_3|$  matrix (whose trace is additively zero -  $Tr|\sigma_3| = 0$  only means that the elements are equal. The identity matrix is not included in SU(2) (or SO(3) - rotations) so neither system characterizes existence.

### Matrix (vector) representation

Note that the expression  $\# := a + b$  is the first order expression where

$$\begin{aligned}\# &:= a + b \\ \#^2 &= A = [a^2 + b^2] + [2ab]\end{aligned}$$

This result can be expressed as matrices:

$$\begin{aligned}|\#| &:= \begin{vmatrix} a & 0 \\ 0 & b \end{vmatrix} \\ Tr|\#| &= Tr \begin{vmatrix} a & 0 \\ 0 & b \end{vmatrix} = a + b, \quad Det|\#| = ab \\ \#^2 &= Tr \begin{vmatrix} a^2 & 0 \\ 0 & b^2 \end{vmatrix} + Det \begin{vmatrix} a & a \\ -b & b \end{vmatrix} = [a^2 + b^2] + [2ab]\end{aligned}$$

Note that in particular,

$$\#^2 \neq Tr \begin{vmatrix} a^2 & 0 \\ 0 & b^2 \end{vmatrix} = [a^2 + b^2]$$

These relations are critical for preserving integer count in the Binomial Expansion; e.g.:

$$\begin{aligned}\# &:= 7 = 4 + 3 \\ \#^2 &= 7^2 = A = [16^2 + 9^2] + [2(4)(3)] = [25] + [24] = 49\end{aligned}$$

Where 25 appears as a subset of the expanded expression  $A$

This means that integer count is not preserved in the **pseudo** expression

$$\#\#^2 = "[a^2 + b^2]" = "(4^2 + 3^2)" = "25"$$

## Complex Analysis

Assume that the operations of addition, subtraction, and complex conjugation are vector operators in the complex plane.

Note that a 2<sup>nd</sup> order pseudo expression  $\psi\psi^* = c^2 = a^2 + b^2$  can only be generated by specifying complex variables (here emphasized by the color magenta) where  $i := \sqrt{-1}$ .

$$\psi := a + b$$

$$\psi^* := a - b$$

$$\psi\psi^* = [a^2 - (-b^2)] + [ab - ba], \quad -b^2 := (ib)^2$$

$$\psi\psi^* = [a^2 + b^2] + [ab - ba]$$

$$\psi\psi^* + ba = [a^2 + b^2] + [ab]$$

$$ba = ab$$

$$\psi\psi^* = [a^2 + b^2]$$

Discussion: starting with:

$$\psi := a + ib$$

$$\psi^* := a - ib'$$

The result is:

$$\psi\psi^* = [\vec{a} \cdot \vec{a} - \vec{b} \cdot \vec{b}] [ +(\vec{a} \cdot \vec{b}) - (\vec{b} \cdot \vec{a}) ] = [a^2 - b^2],$$

$$\text{where } (\vec{a} \cdot \vec{b}) = (\vec{b} \cdot \vec{a}),$$

and

$$i^2 (i^2 b^2) = -(-b^2) = b^2$$

$$\text{so that } \psi\psi^* = [a^2 + b^2] [ + (ab) - (ab) ] = [a^2 + b^2],$$

noting that  $+(iab) - (iba) = 0$  does not imply that the single element  $(iab) = 0$ , and

$$\psi\psi^* = (25) = [4^2 + 3^2] [ + (4)(3) - (4)(3) ] \text{ only obtains for } a \neq 0, b \neq 0, \text{ i.e., the dot product}$$

$\vec{a} \cdot \vec{b} - \vec{a} \cdot \vec{b} = 0$  is aligned along the imaginary axis.

Note that  $+(a \otimes b) - (a \otimes b) = 0$  implies that  $(ab) - (ba) = 0$  which is the result of the cross product

$\vec{a} \otimes \vec{b} + \vec{b} \otimes \vec{a}$  where  $\vec{a} \otimes \vec{b}$  is a mapping into the scalar  $ab$  where  $a = 0$ , and  $b = 0$

$$a \otimes b := \begin{vmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & 0 \end{vmatrix} \Rightarrow \begin{vmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & ab \end{vmatrix}, \text{Tr}(a \otimes b) = ab, a = b = 0$$

$$b \otimes a := \begin{vmatrix} b & 0 & 0 \\ 0 & a & 0 \\ 0 & 0 & 0 \end{vmatrix} \Rightarrow \begin{vmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & ba \end{vmatrix}, \text{Tr}(b \otimes a) = ba = -ab, a = b = 0$$

$$\text{Tr}(a \otimes b) + \text{Tr}(b \otimes a) = ab - ba = 0$$

Side note – this is why the [Hamiltonian defined in Quantum Mechanics](#) is negative (I .e., imaginary, since  $i^2 = -1$ )

Note that integer count is preserved for  $a = 4$ ,  $b = 3$  ( $\neq 0$ ) for

$$\begin{aligned}7^2 &= [4^2 + 3^2] + [2(4)(3)] = [\psi\psi^*] + [24] = \\i^2(7)^2 &= i^2 \{[4^2 + 3^2] + [2(4)(3)]\} = \\-(7)^2 &= -\{[4^2 + 3^2] + [2(4)(3)]\} = \\(7)^2 &= \{[4^2 + 3^2] + [2(4)(3)]\}\end{aligned}$$

which is just the Binomial Expansion defined along the “imaginary” axis.

The formalism  $c^2 = a^2 + b^2$  that eliminates the multiplicative product  $2ab$  can be interpreted as a [first order Presburger arithmetic](#) if powers of its elements are included:  $c^n = a^n + b^n$ , but not multiplication between elements (e.g.  $a^m b^n$ ).

Proof of Fermat’s Last Theorem

$$\begin{aligned}c &= a + b \\c^n &= (a + b)^n = [a^n + b^n] + f(a, b, n) \text{ (Binomial Expansion)} \\c^n &= a^n + b^n \leftrightarrow f(a, b, n) = 0 \\f(a, b, n) &\neq 0 \\c^n &\neq a^n + b^n \quad \text{qed}\end{aligned}$$

This proof can be extended immediately to multinomials.