Parity

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(updated 10/20/25), (12/15/2025)

(updated 02/09/2026 to include relation to Pythagorean Triples)

Parity – Wikipedia

Binomial Theorem – Wikipedia

Pythagorean Triple - Wikipedia

Pythagorean Triples – (my pdf)

"Geometric Proofs" (Not) of the Pythagorean Theorem (my pdf)

Working Document – (my pdf)

The constant e – (my pdf) – note: although I related this to odd/even, the physical significance is actually $\frac{(n+1)}{n}$

3/14/2025 – added comment(s) on general solution

Edited 9/25/2025 – numerous changes

Added 3/12/2026) **Natural Number condition for Pythagorean Triples**

All Natural numbers are positive, since

$$a = a + 0,\ a - a = 0 \leftrightarrow a = a$$
$$-c = a - b,\ b > a \leftrightarrow b - c = a > 0$$

Every number $n \in \{N\}$ is prime (invariant) relative to its own base, where 1_n represents "unity to the base n, so that $n = n\left(\frac{n}{n}\right) = n(1_n) \in \{N\}$

Ratios are not prime numbers

$$n = n\left|\frac{m}{n}\right| \leftrightarrow m = n\,,\ \ n^2 = n^2\left|\frac{m}{n^2}\right| \leftrightarrow m = n^2$$

Parameterization

Consider the expression $x = vt$, and note that dividing both sides by t results in the expression $\frac{x}{t} = v\frac{t}{t} = v(1_t)$; the expression $\frac{x}{t} = v$ is not correct. The expression $v = v\left(\frac{v}{v}\right) = v(1_v)$ characterizes v as a number, where $x = x\left(\frac{x}{x}\right) = (vt)\left(\frac{vt}{vt}\right)$. Note that replacing t by m_0 results in the expression for momentum $P = m' = m_0 v$ where velocity is a scaling factor on m_0. For $v = c$ the relation becomes

$$P_c := m_c = m_0 c = (m_0 c)\left(\frac{m_0 c}{m_0 c}\right)$$

First Order

"Parity" is the relationship between odd and even numbers. These can be expressed as invariants (prime numbers) as:

Odd Number

$$o \in \{o\}$$
$$o = o\left(\frac{o}{o}\right) = o(1_o)'$$

Even Number

$$e \in \{e\}$$
$$e = e\left(\frac{e}{e}\right) = e(1_e)$$

Then the totality of natural (prime) numbers are the sets $\{N\} = \{e\} \cup \{o\}$ where $\{0\} = \{e\} \cap \{o\}$, so the pair $(\{e\},\{o\})$ form a Cartesian space where $e \perp o \leftrightarrow \vec{e} \cdot \vec{o} = 0$

Goldbach's Theorem ("Conjecture") states that every even number is the sum of two primes (invariants), since $n + n = 2n$

Addition

$$\#_o := o + o = 2o = (2o)\left(\frac{2o}{20}\right) \in \{e\}$$
$$\#_e := e + e = 2e = (2e)\left(\frac{2e}{2e}\right) \in \{e\}$$
$$\#_{o,e} := o + e = (o + e)\left(\frac{o + e}{o + e}\right) \in \{o\}$$

Consider Goldbach's theorem where $n + n = 2n \leftrightarrow n = \frac{n}{2} + \frac{n}{2}$. This implies that $n \in \{e\}$, and in particular, for $\# = \frac{m}{2} + \frac{n}{2} \leftrightarrow 2\# = m + n$, the integer relation can be set by redistribution to $2\# = \# + \#$ so that $\# = \frac{\#}{2} + \frac{\#}{2}$.

Second Order

Multiplication

Multiplication between two numbers $n, m \in \{N\}$ is defined by the Binomial Theorem:

$$\# := n + m$$
$$\#^2 = (n+m)^2 = \left[n^2 + m^2\right] + \left[2nm\right]$$

Note that the expression $(\#)' := \left[n^2 + m^2\right]$ exists only as a subset of the Binomial Theorem for the case $n = 2$.

For $n, m \in \{o\}$, $\#^2 \in \{e\}$

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$n \in \{o\}$, $m \in \{e\}$ $\#^2 \in \{o\}$

Fermat's Last Theorem $c^n \neq a^n + b^n \ \forall \ \{c, a, b, n\} \in \{N\}$, $n > 2$

Proof (extended to $n \geq 2$ from $n > 2$)

$$c = a + b$$
$$c^n = \left[a^n + b^n\right] + \left[f(a,b,n)\right] \ \text{(Binomial Theorem)}$$
$$c^n = \left[a^n + b^n\right] \leftrightarrow \left[f(a,b,n)\right] = 0$$
$$\left[f(a,b,n)\right] \neq 0$$
$$c^n \neq \left[a^n + b^n\right] \ \text{qed}$$

Note that the expression for $n = 2$ (equivalent to the Pythagorean equation and the geometric equation of a circle) results in $c^2 \neq \left[a^2 + b^2\right]$ because $\left[f(a,b,2)\right] = 2ab \neq 0$

Scaling

The ratio expressed as $\lim_{n \to \infty} \frac{n + 1_n}{n} \simeq \frac{n}{n} = 1_n = (1_n)\left(\frac{1_n}{1_n}\right)$, so the distinction between odd and even (as a ratio) tends to disappear for large $n >> 0$

Squaring both sides of the expression $\# = \frac{\#}{2} + \frac{\#}{2}$ yields

$$\#^2 = \left[\frac{\#}{2} + \frac{\#}{2}\right]^2 = \left[\frac{\#^2}{4} + \frac{\#^2}{4}\right] + \left[2\left(\frac{\#}{2}\right)\left(\frac{\#}{2}\right)\right] = Tr\begin{vmatrix} \frac{\#^2}{4} & 0 \\ 0 & \frac{\#^2}{4} \end{vmatrix} + Det\begin{vmatrix} \frac{\#^2}{4} & \frac{\#^2}{4} \\ -\frac{\#^2}{4} & \frac{\#^2}{4} \end{vmatrix}$$

, where the negative sign is an artifact of matrix representation. In particular, note that the above expression is not equivalent to the misleading expression:

$$\#^2 \neq \frac{\#^2}{4} + \frac{\#^2}{4} + \frac{\#^2}{4} + \frac{\#^2}{4} = Tr\begin{vmatrix} \frac{\#^2}{4} & 0 & 0 & 0 \\ 0 & \frac{\#^2}{4} & 0 & 0 \\ 0 & 0 & \frac{\#^2}{4} & 0 \\ 0 & 0 & 0 & \frac{\#^2}{4} \end{vmatrix} = Tr\left(\begin{vmatrix} \frac{\#}{2} & 0 & 0 & 0 \\ 0 & \frac{\#}{2} & 0 & 0 \\ 0 & 0 & \frac{\#}{2} & 0 \\ 0 & 0 & 0 & \frac{\#}{2} \end{vmatrix}\right)^2$$

Coordinate length (mass)

Consider the expression for a coordinate ruler expressed as $\lambda := x = ct$ where $\lambda = \lambda\left(\frac{\lambda}{\lambda}\right) = \lambda(1_\lambda)$.

Then any coordinate "distance" can be expressed by the relation $ct' = ct + vt' = \lambda + vt' = \lambda + \delta\lambda$, where $0 \le \delta\lambda \le \lambda$. Then $\delta\lambda$ represents a transition between $\lambda_{\delta\lambda=0}$ to $2\lambda_{\delta\lambda=\lambda}$, where $\lambda_{\delta\lambda=0}$ is represented by one instance of λ and so is odd, and $2\lambda_{\delta\lambda=\lambda}$ is even. Then the expression

$ct' = n\lambda + \delta\lambda$ represents the transition between $n\lambda \to n\lambda + \delta\lambda \to (n+1)\lambda$ noting that the transition tends to zero for large n, so the distinction between odd and even λ becomes insignificant (but not 0) for $n >> 1$

This analysis also holds for momentum where t is replaced by m_0 in the above relations, noting that the result

$P = \frac{p}{2} + \frac{p}{2}$ so that $P = \left[\frac{mv}{2} + \frac{mv}{2}\right] = \left[\frac{m'}{2} + \frac{m'}{2}\right] = \left[\frac{p}{2} + \frac{p}{2}\right]$ and $P^2 = \left[\frac{p}{2} + \frac{p}{2}\right]^2 = (m')^2$ represents the result of equal and opposite momentum at the contact point, where "velocity" is positive if the elements are approaching, but is irrelevant at the contact point (an "origin" for the two "particles)

Pythagorean Triples

Note that Pythagorean Triples $\{a,b,c\}$ are expressed by the relation $c^2 = a^2 + b^2$ where

$c \in \{0\}, c^2 \in \{o\}$
$a \in \{o\}, a^2 \in \{o\}$
$b \in \{e\}, b^2 \in \{e\}$

So that $c^2 = a^2 + b^2 \in \{o\}$ and so is a subset of $\#^2$. In particular, the product $2(ab) \notin \{c^2\}$, so the set is incomplete in $\{N\}$s since it only contains the operation of addition and power, but not multiplication between its elements. That is, one cannot derive the second order relation $c^2 = a^2 + b^2$ from the first order expression $c = a + b$.

Again, note that $o + o \in \{e\}$ and $e + e \in \{e\}$ and $2\# = o + o \leftrightarrow \# = \frac{o+o}{2} = \frac{n}{2}$ which is only half the expression $\# = n = \frac{n}{2} + \frac{n}{2}$

Consider the expression $\#_o = e + o$ where $\#_o \in \{o\}$ so that $\#_o + n_o \in \{e\}$ where $n_o = |e - o|$. Then adding n_o to $\#_o$ results in $\# = e + (o + n_o) = e + e = 2e$ which can be transformed to $\#_e = e = \frac{e}{2} + \frac{e}{2}$

The expression

$$(ct') = (ct) + 0 \leftrightarrow (ct')^2 = (ct)^2 + 0^2 \text{ where } (ct) = \left(\frac{(ct)}{2} + \frac{(ct)}{2}\right), (ct) \in \{e\}$$

Is then represented by the transition

$$(ct') = (ct) + (vt') \leftrightarrow (ct')^2 = \left[(ct)^2 + (vt')^2\right] + \left[2(ct)(vt')\right], 0 < (vt') < (ct)$$

Noting that if $(vt') \in \{o\}$ then $(ct') \in \{o\}$

To the expression $(ct') = 2(ct) + 0 \leftrightarrow (ct')^2 = 4(ct)^2 + 0^2 = \left[(ct)^2 + (ct)^2\right] + \left[2(ct)^2(ct)^2\right]$ where $(ct') \in \{e\}$

Complex discussion

A "Psuedo" Pythagorean Triple proof can be generated using "complex/imaginary" variables (using the color magenta for emphasis):

$\psi := c = a + ib$

$\psi^* := c^* = a - ib$

$\psi\psi^* = (a+ib)(a-ib) = \left[a^2 + b^2\right] + \left[a(ib) - a(ib)\right]$

$\psi\psi^* = \left[a^2 + b^2\right]$

but

$\psi\psi^* \neq \#^2$

Numerically, it might seem that

$\#^2 = (a+b)^2 = [\psi\psi^*] + [2ab]$, but this is misleading, since $a = a(1)$ and $b = b\sqrt{-1}$ are of different order in their bases.

Natural Number condition for Pythagorean Triples

(added 3/12/2026)

$$\#^2 = (a+b)^2 = [\psi\psi^*] + [2ab]$$

Consider the expression

$$o' = o + e$$
$$(o')^2 = (o+e)^2 = [o^2 + e^2] + [2oe]$$

Contrasted with the Pythagorean Triple

$$\psi = e + o$$
$$\psi^* = e - o$$
$$\psi\psi^* = [e+o][e-o] = [e^2 + o^2] + [eo] - [oe]$$

Where $[eo] - [oe] = 0 \leftrightarrow [eo] = [oe] \leftrightarrow oe \neq 0$

Then the expression

$$(o')^2 = (o+e)^2 = [o^2 + e^2] + [2oe]$$

Yields

$$(o+e)^2 - [o^2 + e^2] - [2oe] = 0$$

And $(o+e)^2 - [o^2 + e^2] = [2oe]$ is the condition for the number $(o') = \sqrt{(o')^2}$ to generate the (Natural Number) equivalent of the complex form

$\psi\psi^* = [e+o][e-o] = [e^2 + o^2] + [eo] - [eo]$ where

$\psi\psi^* = [e^2 + o^2]$ where the expression $+[eo] - [eo] \leftrightarrow [eo] - [oe] = 0 \leftrightarrow eo = 0$

Matrix ("2"D) representation

$$\# := n + m$$

$$\#^2 = (n+m)^2 = \left[n^2 + m^2\right] + \left[2nm\right] = Tr\begin{vmatrix} n^2 & 0 \\ 0 & m^2 \end{vmatrix} + Det\begin{vmatrix} n & n \\ -m & m \end{vmatrix}$$

(the negative number $-m$ is an artifact of the representation).

For $m = n$

$$\#^2 = \left[n^2 + n^2\right] + \left[2n^2\right]$$

Since multiplication is defined by the Binomial Theorem, $\left[2n^2\right]$ exists only in the context of

$$\#^2 = (n+n)^2 = \left[n^2 + n^2\right] + \left[2(n)(n)\right]$$

Note that this is not equivalent to the trace of the 4D matrix

$$|n| := \begin{vmatrix} n & 0 & 0 & 0 \\ 0 & n & 0 & 0 \\ 0 & 0 & n & 0 \\ 0 & 0 & 0 & n \end{vmatrix}, \; Tr\left|n^2\right| = Tr\begin{vmatrix} n & 0 & 0 & 0 \\ 0 & n & 0 & 0 \\ 0 & 0 & n & 0 \\ 0 & 0 & 0 & n \end{vmatrix}^2 = Tr\begin{vmatrix} n^2 & 0 & 0 & 0 \\ 0 & n^2 & 0 & 0 \\ 0 & 0 & n^2 & 0 \\ 0 & 0 & 0 & n^2 \end{vmatrix} = n^2 + n^2 + n^2 + n^2 = "4n^2"$$

Pythagorean Theorem

$$c = a + b$$
$$c^2 = \left[a^2 + b^2\right] + \left[2ab\right] = \left[\psi\psi^*\right] + \left[2ab\right],\ \psi := a + ib$$
$$b < a, b = a \pm \delta$$

$$o \in \{o\}$$
$$e \in \{e\}$$

$$o \pm o \in \{e\}$$
$$e \pm e \in \{e\}$$
$$o + e \in \{o\}$$
$$o - e \in \{o\},\ o > e$$
$$e - o \in \{o\}, e > o$$

Both elements odd:

$$o^2 \in \{o\}$$
$$n_e := (o \pm o) \in \{e\}$$
$$n_e^{\ 2} = (o \pm o)^2 = \left[o^2 + o^2\right] \pm \left[2(o)(o)\right]$$
$$n_e^{\ 2} \in \{e\}$$

Both elements even:

$$e^2 \in \{e\}$$
$$n_e := (e \pm e) \in \{e\}$$
$$n_e^{\ 2} = (e \pm e)^2 = \left[e^2 + e^2\right] \pm \left[2(e)(e)\right]$$
$$n_e^{\ 2} \in \{e\}$$

Pythagorean Triples (updated 02/10/2026)

Pythagorean triples can be characterized as an odd transition between pairs of even elements The actual numbers depend on the base of the number system in which they are expressed:

$$n = n\left(\frac{n}{n}\right) = n\left(1_n\right)$$

First Order Analysis

One element odd, one element even:

$$o' = (o+e) \in \{o\}$$

Case $o > e$

$$(e+e) < (o+e) < (o+o)$$
$$(o-e) := \delta o$$
$$(o+e) - \delta o = (o+e) - (o-e) = (e+e) \in \{e\}$$

Case $e > o$

$$(e+e) > (e+o) > (o+o)$$
$$(e-o) = \delta o$$
$$(e+o) - \delta o = (e+o) - (e-o) = (o+o) \in \{e\}$$

This has fundamental consequences for groups (at least two elements related by the “existence” operator ($"+"$)

$$\lambda \in \{e\}$$
$$\lambda = \frac{\lambda}{2} + \frac{\lambda}{2}$$
$$\lambda^2 = \left[\frac{\lambda^2}{4} + \frac{\lambda^2}{4}\right] + \left[2\left(\frac{\lambda}{2}\right)\left(\frac{\lambda}{2}\right)\right]$$

$$\lambda := ct,\ \lambda > 0$$
$$\delta\lambda := vt',\ 0 < \delta\lambda < \lambda$$
$$\lambda < (\lambda + \delta\lambda) < 2\lambda$$

$$L_0 := (n\lambda)$$
$$(n)\lambda < (n\lambda + \delta\lambda) < (n+1)\lambda$$
$$\equiv (ct) < (ct + vt') < 2(ct),$$
$$n(ct) := (n\lambda + vt'),\ vt' = 0$$
$$,\ n(ct') := (n(ct) + vt')$$
$$(n+1)ct = (n(ct) + vt'), vt' = ct$$

Second Order Analysis

The expression

$$\psi := e + io,\quad e > 0, io > 0$$
$$\psi^* := e - io$$
$$\psi\psi^* = e^2 + o^2 + ioe - ieo,\ ioe - ieo = 0 \leftrightarrow ioe = ieo$$
$$\psi\psi^* \neq e^2 + o^2$$

$$(o') = (o + e)$$
$$(o')^2 = (o+e)^2 = \left[o^2 + e^2\right] + [2oe] \in \left\{o^2\right\}$$
$$\left[o^2 + e^2\right] + [2oe] - [2oe] = \left[o^2 + e^2\right] \in \left\{o^2\right\}$$
$$\left[o^2 + e^2\right] \in \left\{o^2\right\}$$
$$\left[o^2 + e^2\right] \notin \left\{e^2\right\} \notin \{e\}$$
$$\left[o^2 + e^2\right] \notin \mathrm{N}$$

Real condition for Pythagorean Triples

$$\#^2 = (a+b)^2 = [\psi\psi^*] + [2ab]$$

Consider the expression

$$o' = o + e$$
$$(o')^2 = (o+e)^2 = \left[o^2 + e^2\right] + [2oe]$$

Contrasted with the Pythagorean Triple

$$\psi = e + o$$

$$\psi^* = e - o$$

$$\psi\psi^* = [e + o][e - o] = [e^2 + o^2] + [eo] - [eo]$$

Where $[eo] - [oe] = 0 \leftrightarrow [eo] = [oe] \leftrightarrow oe \neq 0$

$$m_o := (o - e) \in \{o\},\ o > e$$
$$(m_o)^2 = (o - e)^2 = \left[o^2 + e^2\right] + \left[2(o)(e)\right] \in \{o^2\}$$
$$(o + o)^2 > (o - e)^2$$

$$m_o := (e + o) \in \{o\}$$
$$m_o := (e - o) \in \{o\},\ e > o$$

$$o - e \in \{o\},\ o > e$$
$$\delta := o - e$$
$$e + \delta = o \in \{o\}$$

Example

$$n = 3$$
$$\# = 3 + 3$$
$$\#^2 = (3+3)^2 = \left[3^2 + 3^2\right] + \left[2(3)(3)\right] = \left[9+9\right] + \left[2(9)\right]$$
$$\#^2 = 36 \in \{e\}$$

$$m = 4$$
$$\# = 4 + 4$$
$$\#^2 = (4+4)^2 = \left[4^2 + 4^2\right] + \left[2(4)(4)\right] = \left[16+16\right] + \left[2(16)\right]$$
$$\#^2 = 64 \in \{e\}$$

$$\Sigma := m + n = 4 + 3$$
$$\Sigma^2 = (m+n)^2 = (4+3)^2 = \left[4^2 + 3^2\right] + \left[2(4)(3)\right] = [25] + [24]$$
$$\Sigma^2 = 49 = 7^2 \in \{o\}$$

In second order, the first term $\left[m^2 + m^2\right], \left[n^2 + n^2\right], \left[m^2 + n^2\right]$ characterizes an "existence" term with a group operator of addition, while the second term $\left[2(m)(m)\right], \left[2(n)(n)\right], \left[2(m)(n)\right]$ characterizes an "interaction" term with a group operator pf multiplication (juxtaposition).

$$n^2 < \Sigma^2 < m^2$$
$$(3+3) < (3+4) < (4+4)$$
$$6 < 7 < 8 \leftrightarrow 36 < 49 < 64$$
$$e = 6, o = 7, e + 2 = 8$$
$$6, 8 \in \{e\}, 7 \in \{o\}$$

Note that the (existence term) for the squares of equal elements is even for both elements even or odd, but is odd if one element is odd and the other is even; however, the interaction term is always even because of the multiplier of 2.

$$\left[9+9\right] = 18 \in \{e\}$$
$$\left[16+16\right] = 32 \in \{e\}$$
$$\left[9+16\right] = 25 \in \{o\}$$

In this case, the difference between the even and odd elements is one, but will be different for different values of m and n. However, factors of (powers of 2) can be extracted from each of the three examples, in which case both the elements in the existence term will be even, or one of the elements will be even and the other odd, with the result even or odd, respectively.

For $m > n,\ m = n + \delta$ so that m will be odd if n is odd but δ is even; otherwise m is even, and vice versa.

This analysis shows that the Pythagorean Triple characterizes a "transition" between two even terms:

$$c = a \pm b$$
$$c = a - b \leftrightarrow b < a, c > 0$$

$$a \in \{o\}, b \in \{e\} \vee a \in \{e\}, b \in \{o\}$$
$$c = (o + e) \leftrightarrow c \in \{o\}, c^2 \in \{o\}$$

If the kernel is odd and it square root is an integer, (i.e., all the first order elements are integers) the three define a “Pythagorean Triple”:

$$\left\{m, n, \sqrt{m^2 + n^2}\right\}$$

$$\{4,3,5\} = \left\{4,3,\sqrt{4^2 + 3^2}\right\} = \left\{4,3,\sqrt{25}\right\}$$