Fermions

SU(2) and the Stern-Gerlach Experiment

(and other topics)

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Update: 02/19/2024: added section relating Stern-Gerlach experiment to Pauli Matrices, SU(2), and Special Relativity

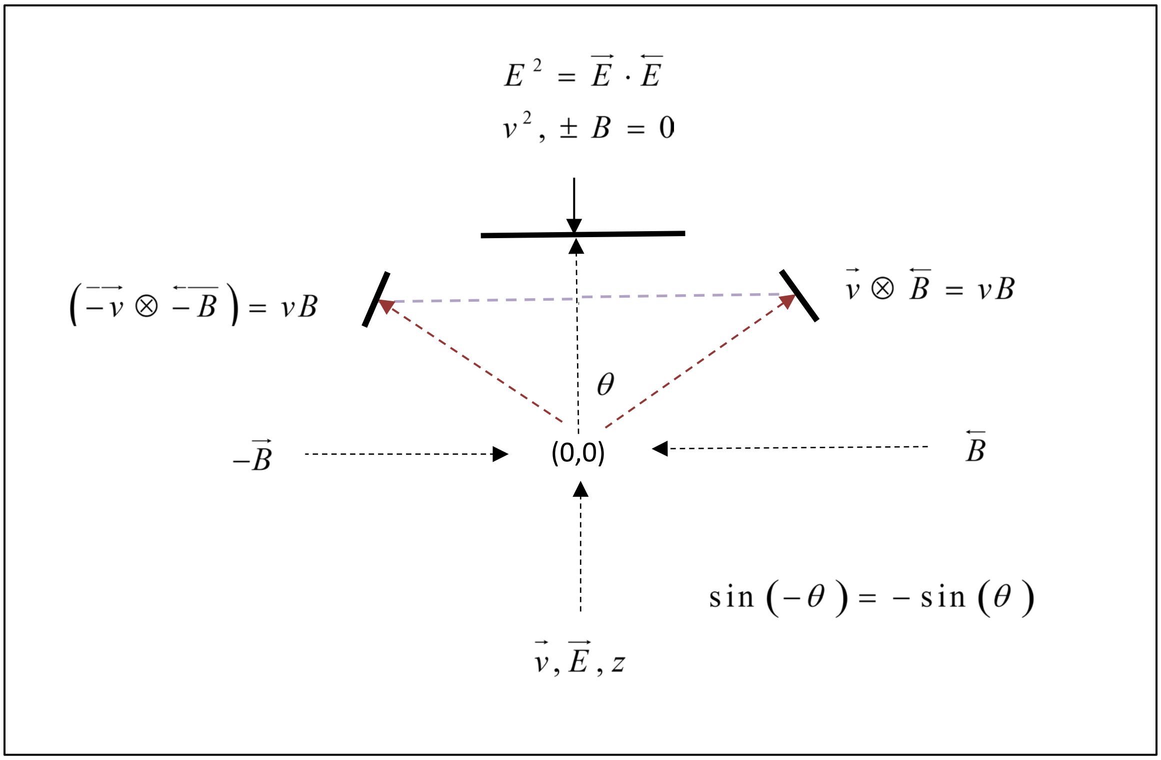
The Lorentz Force

The Lorentz force is defined by the relation: $\overrightarrow{F} = q\left(\overrightarrow{E} + \vec{v} \otimes \overleftarrow{B}\right)$, noting that E is first order (as a coordinate or momentum) but if v and B are first order, then $\vec{v} \otimes \overleftarrow{B}$ is second order, and must be interpreted as area or energy (mass). Note that if

$\# := E + vB$ then $\#^2 = E^2 + vB^2 + E\left(2vB\right)$ in contrast to the term $\vec{v} \otimes \overleftarrow{B}$ in the Lorentz force, which means that the only half the value appears – that is, the Lorentz force only expresses the "right hand" rule, whereas the complete expansion is $2vB = v \otimes B + \left[-\left(v \otimes -B\right)\right] = 2vB$.

The Lorentz force is therefore related to Fermions in the context of the Stern-Gerlach experiment, with the "Spin" S defined as $S = \frac{h}{\sqrt{2}} = \frac{\sqrt{E(vB)}}{\sqrt{2}}$ where $h^2 := 2E\left(vB\right) = 2S^2$

Consider the experimental apparatus where the z-axis represents the initial path of both ionized and un-ionized silver atoms, with a contact along the path at which a B field can be applied.



$\vec{E}$ represents the momentum of a stream of un-ionized atoms (moving bosons, or the atmosphere, which doesn't move, and surrounds the experiment), and does not interact with ionized silver atoms, initially moving toward $(0,0)$ at unit "momentum" v. If there is no B field, both streams impact the target at the top of the page.

When the B field is turned on, the ionized atoms are diverted to each side by the contact existence of the B field at $(0,0)$

ntroduction

A one dimensional vacuum is defined at any point as having an existential value of 0. If a force appears in any region of vacuum, it has a positive definite value of $f = f + 0 = \frac{f}{2} + \frac{f}{2} + 0$. The operation of addition characterizes existence of the force(s) in contrast to the vacuum.

If light is the fundamental force in the universe, this force can be denoted $c \geq 0$ If $c = 0 + 0 = 0$ then the vacuum is undisturbed in its region. If invariant ($c > 0$), it is expressed as a prime number, where $c := c\left(\frac{c}{c}\right) = c\left(1_c\right)$. This can be parameterized to allow for changes as an initial condition, where $f_0 := c_0\tau_0$ an $\tau_0 = 0 \leftrightarrow f_0 = c\tau_0 = 0$ so that τ_0 is analogous to a gain control on a single force in the vacuum.

The mass of light is defined by Newton's Third Law which states that every action (force) must have an equal and opposite reaction. This requires a second equal force where the interaction (collision) is characterized by multiplication (indicated by juxtaposition); if moving toward the first force, the product is taken as positive, and at the point of collision, the mass is then defined as $m_{(f_0)} = \left(f_0\right)\left(f_0\right) = \left(f_0\right)^2 = (c\tau)^2$ at its own origin (0) with no external coordinate system defined.

(There must be at least two interacting forces to define a mass – see Russell's Paradox, the solution of which can be interpreted as $1^2 \neq 1$). $m_{(f_0)} = (c\tau)^2$ is invariant for invariant c and τ since $m_{(f_0)} = (c\tau)^2 = (c\tau)^2\left(\frac{(c\tau)^2}{(c\tau)^2}\right) = (c\tau)^2\left(1_{(c\tau)^2}\right)$ where $\left(1_{(c\tau)^2}\right) = \frac{m_{(f_0)}}{(c\tau)^2}$ and is represented by a prime number.

The P momentum "field"

This field can be considered an initial state.

$$f_E = \frac{f_E}{2} + \frac{f_E}{2}$$

$$m_0 = (f_E)^2 = \left[\left(\frac{f_E}{2}\right)^2 + \left(\frac{f_E}{2}\right)^2\right] + \left[2\left(\frac{f_E}{2}\right)\left(\frac{f_E}{2}\right)\right]$$

$$m_v = m_0\left(\int_0^v dv\right)$$

$$x := vt, \frac{x}{t} := v\left(\frac{t}{t}\right) = v(1_t)$$

$$v = \frac{m_v}{(1_t)}$$

$$\theta = 0$$

$$\cos(\theta) = \cos(-\theta)$$

$$(P)\cos(\theta)(\uparrow) := (m_v v)\cos(\theta)(\uparrow)$$

$$\frac{(P)\cos(\theta)}{(P)\cos(\theta)} := 1_{(P)\cos(\theta)} = 1_{(P)}$$

$$P(\uparrow \vec{i}) = P(1_{(P)})(\uparrow \vec{i})$$

$$2P(1_{(P)})(\uparrow \vec{i}) = Tr\begin{vmatrix} P(1_{(P)})(\uparrow \vec{i}) & 0 \\ 0 & P(1_{(P)})(\uparrow \vec{i}) \end{vmatrix} := Tr\left\{\begin{vmatrix} P & 0 \\ 0 & P \end{vmatrix}\begin{vmatrix} 1_p & 0 \\ 0 & 1_p \end{vmatrix}\right\} = Tr\begin{vmatrix} P & 0 \\ 0 & P \end{vmatrix}$$

$$2P = Tr\begin{vmatrix} P & 0 \\ 0 & P \end{vmatrix}$$

The P "momentum" field

This (momentum) field can be considered a perturbing state and represents the B field in the Stern=Gerlach experiment.

$$f_B = \frac{f_B}{2} + \frac{f_B}{2}$$

$$m_B = (f_B)^2 = \left[\left(\frac{f_B}{2}\right)^2 + \left(\frac{f_B}{2}\right)^2\right] + \left[2\left(\frac{f_B}{2}\right)\left(\frac{f_B}{2}\right)\right]$$

$$m_v = m_B\left(\int_0^v dv\right)$$

$$x := vt,\ \frac{x}{t} = v\left(\frac{t}{t}\right) = v(1_t)$$

$$v = \frac{m_v}{(1_t)}$$

$$[\sin(-\theta)] = [-\sin(\theta)] \leftrightarrow [\sin(-\theta)] + [-\sin(\theta)] = 0$$

$$(P)\sin\left(-\frac{\pi}{4}\right)(\leftarrow \overleftarrow{j}) := (m_v v)\left[\sin\left(-\frac{\pi}{4}\right)\right](\leftarrow \overleftarrow{j})$$

$$\theta = \pm\frac{\pi}{4}$$

$$(P)\sin\left(-\frac{\pi}{4}\right) = -(P)\sin\left(-\frac{\pi}{4}\right)$$

$$(P)\sin\left(\left|\frac{\pi}{4}\right|\right) = (P)\sin\left(\left|\frac{\pi}{4}\right|\right)$$

$$\frac{(P)\sin\left(\left|\frac{\pi}{4}\right|\right)}{(P)\sin\left(\left|\frac{\pi}{4}\right|\right)} := 1_{(P)_r \sin\left(\left|\frac{\pi}{4}\right|\right)} = 1_{(P)_r}$$

$$(P)(\leftarrow) = (P)1_{(P)}(\leftarrow)$$

$$(P)(\leftarrow) = (P)(\rightarrow)$$

$$2P = \begin{vmatrix} (P)(\rightarrow) & 0 \\ 0 & (P)(\leftarrow) \end{vmatrix}$$

This can be represented in a two-dimensional (2×2) matrix as:

$$P = B = 1,$$

$$P\cos(0) = B\sin(\frac{\pi}{4}) = 1$$

$$\# = P + B = 1 + 1 = 2 = Tr\begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix}$$

However,

$\#^2 = (1+1)^2 = [1^2 + 1^2] + [2(1)(1)]$ where $[1^2 + 1^2]$ represents the existence of the two elements via addition, and $[2(1)(1)]$ represents the interaction (multiplication, entanglement, change in entropy) between the two elements.

Note that $\#^2 = (1+1)^2 \neq [1^2 + 1^2]$ via Fermat's Theorem and the Binomial expansion.

The Stern-Gerlach Experiment without interactions.

Here P refers to the "momentum" (which is equivalent to motion without acceleration (force) which is equal and opposite if interacting. It can also represent the momentum of an E "field" expressed as a moving charge. Similarly for P which represents a similar momentum for the B "field".

Note that the relations:

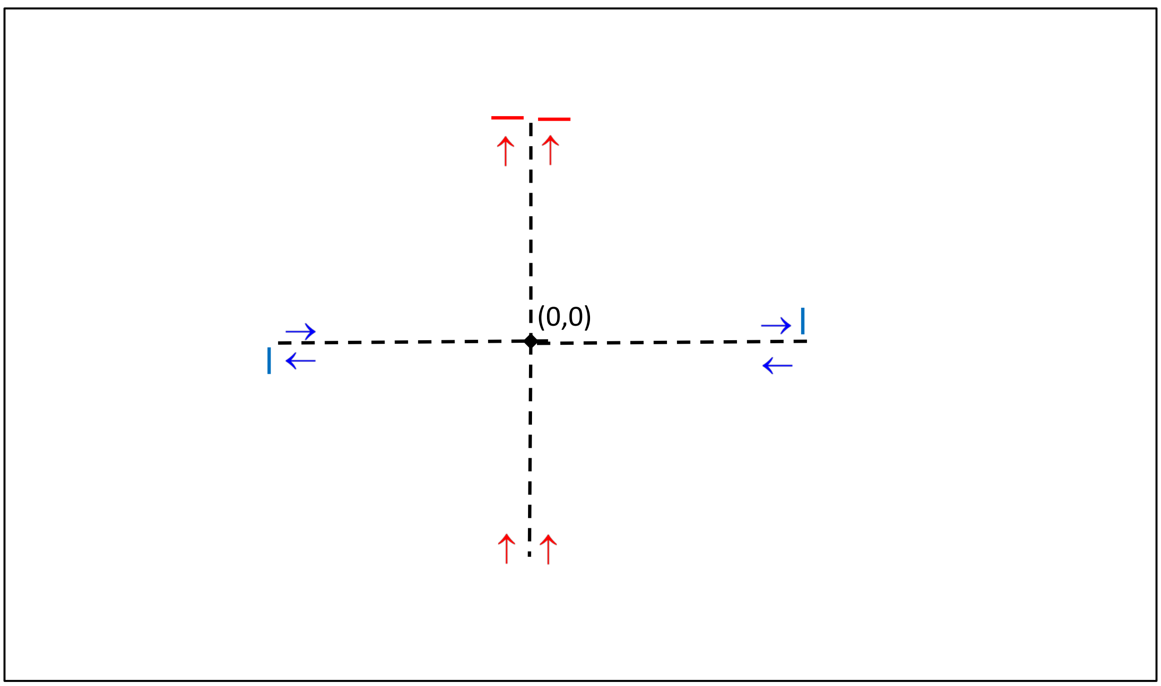
$$(\#) = P + P$$

$$2(\#) = 2P + 2P = [P + P] + [P + P] = [P + P] + [P + P]$$

$$Tr\begin{vmatrix} P & 0 & 0 & 0 \\ 0 & P & 0 & 0 \\ 0 & 0 & P & 0 \\ 0 & 0 & 0 & P \end{vmatrix} = 2P + 2P$$

characterize the existence of four momenta without interaction.

This can be represented by the diagram:



The Stern-Gerlach Experiment with interactions

In the following analysis, the momenta are considered to be equal and opposite, interacting at an origin determined by their constituent forces; therefore, coordinate paths are indicated by dashed lines, meaning that coordinate length does not exist at the site of the interaction (i.e., there is no interaction between momentum and coordinates except at the origin, where length is irrelevant.

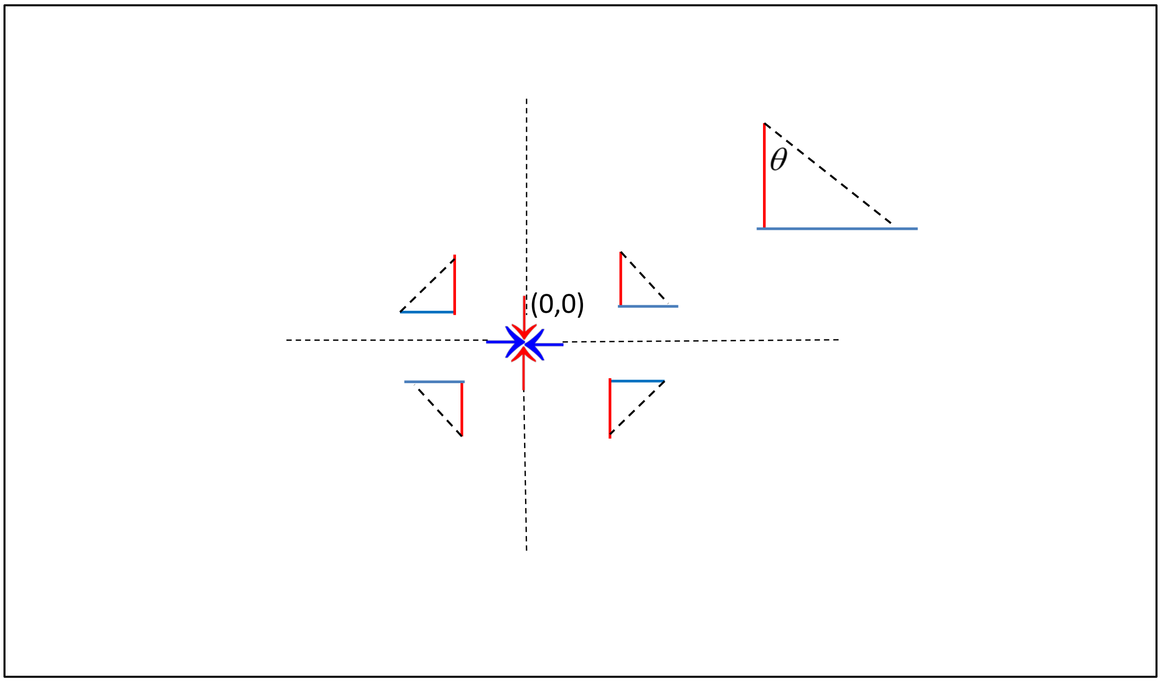
This means that the Kinetic Energy (as momentum) is independent of the Potential energy due to the coordinate system, so the LaGrangian and Hamiltonian are defined by

$$L = T(m,v) - V(x,t)$$
$$H = T(m,v) + V(x,t)$$
$$V(x) = 0 \leftrightarrow L + H = 2T(m,v)$$
$$2H = 2\left(T(P) + T(P)\right)$$

Where $v = v\left(\frac{v}{v}\right) = v\left(1_v\right)$, $\tau = \tau\left(\frac{\tau}{\tau}\right) = \tau\left(1_\tau\right)$, $m_{(v\tau)} = v\tau \in T(m,v)$ where v is the mass creation rate

and τ is the mass creation time if the invariant mass $m_{(v\tau)} = m_{(v\tau)}\left(\frac{m_{(v\tau)}}{m_{(v\tau)}}\right) = m_{(v\tau)} 1_{m_{(v\tau)}}$

but $x = vt \in V(x,t) \leftrightarrow v := \frac{x}{t} = v\left(\frac{t}{t}\right) = v\left(1_t\right)$, $x \in V(x)$ so position (x) is not an invariant with time (t).



(The initial state is represented by P and the "perturbing" state by P

$$\# = P + P$$
$$\#^2 = \left[P^2 + P^2\right] + 2\left[(P)(P)\right] = 4\left\{(\frac{1}{2})\left[(P)(P)\right]\right\}$$
$$(\Updownarrow)^2 + (\Leftrightarrow)^2 + 2(\updownarrow)(\leftrightarrow)$$

Where $\left\{(\frac{1}{2})\left[(P)(P)\right]\right\}$ is the area of each of the triangles in the four quadrants. Note that the product scales each other, so that $\#^2 = \left(P+P\right)^2$ but $\#^2 \neq \left(P+P\right)$ and $\#^2 \neq \left(PP\right)$, e.g.:

$$\# = 4, \# = 3$$
$$\# := 7 = 4 + 3$$
$$\#^2 = 7^2 = 49 = \left[4^2 + 3^2\right] + \left[2(4)(3)\right] = [16+9] + [24] = [25+24]$$

That is, the total count in second order is not preserved in terms of the count of elements in first order.

Note that the introduction of $\pm$ only refers to the "B" as applies to

$$\# = m + v$$
$$\#^2 = (m+v)^2 = \left[m^2 + v^2\right] + 2(mv)$$
$$[m \otimes -v] + [-v \otimes m] = 2(mv) = 2P$$
$$\#^2 = \left[m^2 + v^2\right] + 2[P]$$

Note that $[m \otimes -v] = [-v \otimes m] \leftrightarrow [m \otimes -v] = [-v \otimes m] = 0$

Then $[m \otimes -v]$ and $[-v \otimes m]$ characterize fermions where the sign represents their values at the point of intersection.

In the above characterization, the count of the elements was preserved, since both existence and interaction were included. However, the experiment is unphysical, since the the final result $P\sin(\frac{\pi}{4})$ was at right angles to the initial state $P\cos(\frac{\pi}{4})$ This implies that the interaction preserved the properties of both particles. If P represents mass and P represents light, then the latter changed the direction of the mass at the point of interaction without changing its value, so that even though there was coordinate deflection, the mass of light and the mass of the particle ("ionized silver atoms) did not otherwise interact. Therefore, both momenta are **invariant**, and are represented by **prime numbers**:

$$P_{(i,j)} = P_{(i,j)}\left(\frac{P_{(i,j)}}{P_{(i,j)}}\right) = P_{(i,j)}\left(1_{P_{(i,j)}}\right)$$

This is a representation of Newton's Laws of motion where the momentum $P = P\left(\frac{P}{P}\right) = 1_P = mv = (mv)\left(\frac{mv}{mv}\right) = 1_{(mv)}$ is an invariant. That is, the product of mass and velocity is invariant, but there is no separate existence of mass and velocity defined: $\exists(m+v) := m+v$)

Algebraically, this is expressed as $m+v=0$ so that $m=-v$. However, for a "rest mass", $v=0$ which does **not** imply that invariant "rest mass" m_0 at coordinate position $(0,0)$;

$x = vt \leftrightarrow \frac{x}{t} = v\left(\frac{t}{t}\right) = v(1_t)$, where $x = 0 \leftrightarrow v = 0 \oplus t = t\left(\frac{t}{t}\right) = t(1_t) = 0$.

In this model, $\pm v$ is relegated to coordinates, which only serve to express the change in direction as irrelevant to momenta (even though it is observed at the different sensors; the collisions are then elastic, with no coordinate model (e.g., a sphere) at the interaction point. $\#^2$ then represents a boson, since the "charge" relates only to unobserved coordinates at the sensors originating from the interaction point.

Since coordinates are irrelevant in this model, there is no radiation to the sensors, and the interaction position also has the characteristics of a "singular positive black hole" as well as that of a boson (ie., a positive particle that doesn't interact), in the sense that all momenta are represented as "absorbed" into a single particle at its origin.

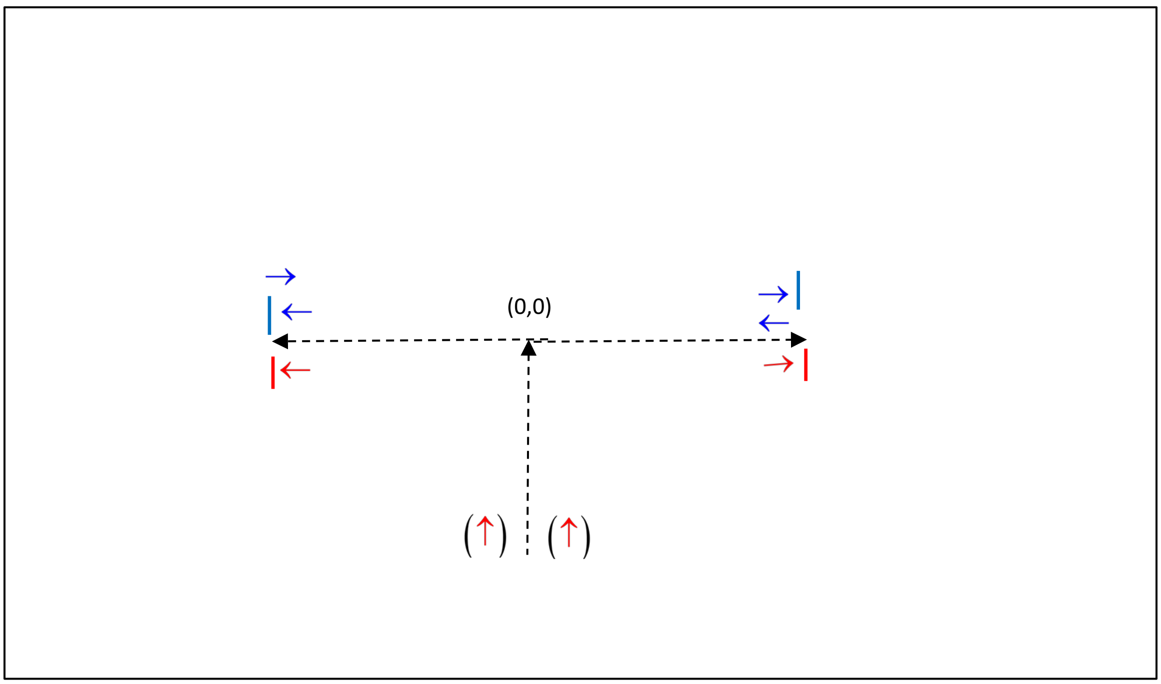
(Note that vectors are inherently affine; that is, they have no common origin since vaectors are both translationally and rotationally invariant and can exist ("anywhere", "anywhen") relative to each other.

Deflection (Existence without Interaction)

In this case, the count $\# = (2)(P+P) = P+P+P+P = 4P = Tr\begin{vmatrix} P & 0 & 0 & 0 \\ 0 & P & 0 & 0 \\ 0 & 0 & P & 0 \\ 0 & 0 & 0 & P \end{vmatrix} = 4P$

, but the expression is first order ($+$ existence) **without** interaction entanglement) h^2 , where

$$h^2 := 2S^2 := 2(P \times P)^2 = 2(PP)^2$$
$$S = \frac{h}{\sqrt{2}} = \sqrt{(PP)^2} = (PP)$$



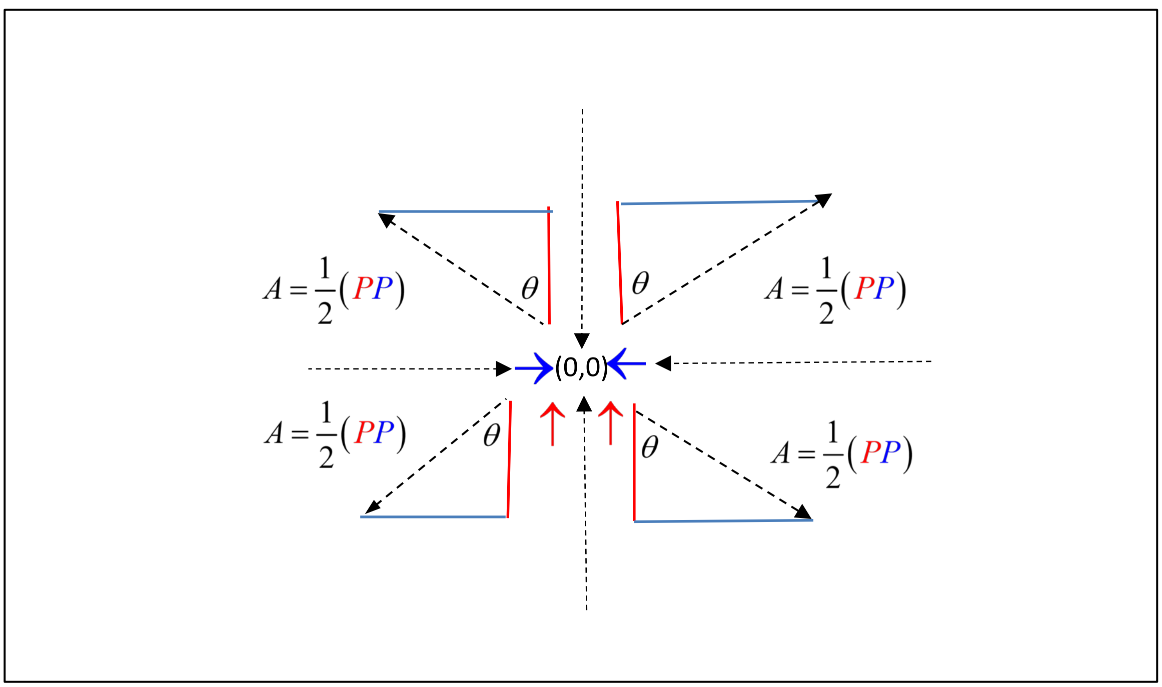
Existence + Interaction $0 \le |\theta| \le \frac{\pi}{4}$

$$P := (c\tau) = (c\tau)\cos(0)$$

$$P := (v\tau') = (v\tau')\sin(\theta),\ 0 \le \theta \le \frac{\pi}{4}$$

$$\sin(-\theta) = -\sin(\theta) = \leftrightarrow \sin(-\theta) - \sin(\theta) = 0$$

(Deflection with interaction) where count is preserved.



Again, note that the coordinate system is irrelevant in the equations themselves, since the hypotenuse is not included in the calculations.

$$\# = (P + P)$$

$$\#^2 = (P + P)^2 = +\left[(P)^2 + (P)^2\right] + 2(P)(P)$$

Where $4\left(\frac{(P)(P)}{2}\right)=2(P)(P)$ and $\left(\frac{(P)(P)}{2}\right)=\frac{1}{2}(P)(P)$ is the (second order) area of the triangles defined by their first order legs.

This is because the use of $\pm$ only refers to the coordinate sines and cosines, and thus directions if vectors, but the elements are not vectors because of the interactions.

$$\#^2=\left[P^2+P^2\right]+2\left[(P)(P)\right]=Tr\begin{vmatrix}P^2 & 0\\ 0 & P^2\end{vmatrix}+Det\begin{vmatrix}P & P\\ -P & P\end{vmatrix}\neg\equiv 4P,$$

which includes interactions in 2D (the dimensions of the system under analysis) and is not equivalent to the 4D model without interactions.

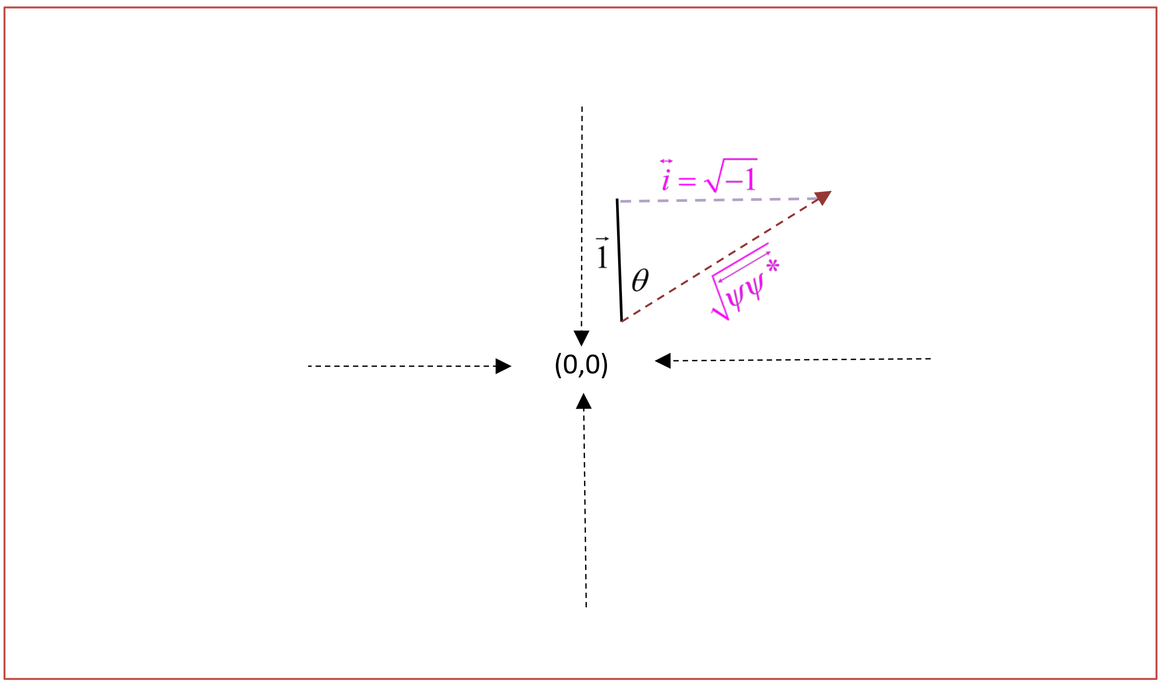
SU(2) and the Pauli Matrices

The Pauli Matrices were an attempt to codify the results of the Stern-Gerlach Experiment, characterized by the matrices:

$$\left|\sigma_1\right| := \begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix},\ \left|\sigma_2\right| := \begin{vmatrix} 0 & i \\ -i & 0 \end{vmatrix} = \begin{vmatrix} 0 & \left(\sqrt{-1}\right) \\ -\left(\sqrt{-1}\right) & 0 \end{vmatrix},\ i := \sqrt{-1}\text{, and } \left|\sigma_3\right| := \begin{vmatrix} 1 & 0 \\ 0 & -1 \end{vmatrix}$$

However, the description is not complete, since the analysis adds an imaginary addition to the single group operation of multiplication, so that the mathematical foundation is no longer a formal group. Doing so makes it consistent (mathematically) with both Quantum Mechanics, and both the Special and General Theories of Relativity.

Consider the upper right corner of the Existence diagram expressed as vectors:



The existence of the two elements is asserted by the identity matrix:

$\left|I\right| := \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix},\ Tr\left|I\right| = 1+1 = 2,\ Det\left|I\right| = -1$ where the negative value of the determinant arises only from the formal definition of vectors, not from any real valued physics, and is therefore irrelevant. This can be expressed by setting $Det\left|I\right| = 0 = Tr\left|\sigma_3\right| = Tr\begin{vmatrix} 1 & 0 \\ 0 & -1 \end{vmatrix} = 1-1 = 0$

The vector $\vec{1}$ is then represented by the matrix $|\sigma_3| := \begin{vmatrix} 1 & 0 \\ 0 & -1 \end{vmatrix}$ which asserts that IF there are two elements, then they must be equal $Tr|\sigma_3| := \begin{vmatrix} 1 & 0 \\ 0 & -1 \end{vmatrix} = 1 - 1 = 0 \leftrightarrow 1 = 1$

Consider the relation where the imaginary procedure of addition (imaginary "existence") $+$ has been introduced to SU(2) (so it is no longer a group.:

$\sigma_+ := \sigma_1 + \sigma_2 = |\sigma_2| := \begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix} + \begin{vmatrix} 0 & i \\ -i & 0 \end{vmatrix} = \begin{vmatrix} 0 & 1+i \\ 1-i & 0 \end{vmatrix}, i := \sqrt{-1}$ where $\sigma_+ := \begin{vmatrix} 0 & 1+i \\ 1-i & 0 \end{vmatrix} = \begin{vmatrix} 0 & \psi \\ \psi^* & 0 \end{vmatrix}$

$Det|\sigma_+| := Det\begin{vmatrix} 0 & 1+i \\ 1-i & 0 \end{vmatrix} = Det\begin{vmatrix} 0 & \psi \\ \psi^* & 0 \end{vmatrix} = -\psi\psi^*$ where the negative sign again arises from the vector formalism, not from real analysis. Note that $1 = \cos(0)$ and $i = \sin\left(\frac{\pi}{4}\right) = \frac{i}{\sqrt{\psi\psi^*}}$.

Then

$\psi := 1 + i$
$\psi^* := 1 - i$
$\psi\psi^* := (1+i)(1+i) = \left[1^2 + i^2\right] + \left[(i1) - (1i)\right] = \left[1^2 + i^2\right]$

Where the anti-commutation has been introduced by the vector cross product:

$\left[\left(\vec{i} \otimes \overleftarrow{1}\right) + \left(\overleftarrow{1} \otimes \vec{i}\right)\right] = 0$, noting that the interaction term have been removed by complex conjugation. The negative sign then refers to imaginary subtraction, not to negative "mass" or "charge"

The above formulation is "geometric" and hence $V(x,t)$ is imaginary, with $h^2 = 2(i1)(i1) = -h^2$ do that $h^2 - h^2 = h \leftrightarrow h^2 = h^2$ and $0 = 0$; that is, the vectors have a common origin in the complex plane. However they do not since the real and imaginary axes are orthogonal: $\vec{1} \perp \vec{i}$ and $\vec{1} \perp \vec{1}, \vec{1} \perp \vec{i^2}, \vec{1} \perp \overrightarrow{-1}$

That is, neither term can exist simultaneously (Russell's Paradox)

Note that $h^2 := 2(1)(1)$ corresponds to the real interaction term in the interaction relationship:

$\# = 1 + 1 = 2$
$\#^2 = (1+1)^2 = \left[1^2 + 1^2\right] + \left[2(1)(1)\right] = \left[1^2 + 1^2\right] + h^2, h^2 := 2[(1)(1)],\ h := 2S,\ S := \frac{h}{\sqrt{2}}$
$\#^2 \equiv \Sigma + \Pi,\ \Sigma := \left[1^2 + 1^2\right],\ \Pi := 2\left[(1)(1)\right] = h^2$

Special Relativity

Pauli's interpretation of the Stern-Gerlach experiment is then solely geometric, and corresponds to the Special Relativistic "Time Dilation" equation where:

$$(ct') := (ct) + (vt')$$
$$\psi : (ct) + (vt')$$
$$\psi^* : (ct) - (vt')$$
$$\psi\psi^* = \left[(ct) + (vt')\right]\left[(ct) - (vt')\right] = \left[(ct)^2 + (vt')^2\right] \leftrightarrow (ct)(vt') = 0$$
$$\psi\psi^* = \left[(ct)^2 + (vt')^2\right] \leftrightarrow (ct)(vt') = 0$$
$$\psi\psi^* = \#^2,\ \#^2 = (ct')^2 = \left[(ct)^2 + (vt')^2\right] + \left[2(ct)(vt')\right] \leftrightarrow \left[2(ct)(vt')\right] = 0$$
$$\left[2(ct)(vt')\right] \neq 0$$
$$\psi\psi^* \neq \#^2 = (ct')^2$$

Solving the equation:

$(ct')^2 := (ct)^2 + (vt')^2$ results in the "Time Dilation" equation: $t' = t\Gamma,\ \Gamma := \dfrac{1}{\sqrt{1-\beta^2}},\beta$

This can be characterized as the relativistic relation between light and matter by substituting

$(ct')^2 := (ct)^2 + (vt')^2$ so that $t = t'(\Gamma),\ \Gamma := \dfrac{1}{\sqrt{1-\beta^2}},\beta = \dfrac{v}{c}$ where $(ct) = \lambda$ represents the mass of light as "wavelength" (as an initial state), and $(vt') := \lambda$ represents an additive change in light mass, noting that $\beta = \dfrac{v}{c} = \dfrac{mv}{mc}$ fore any m so that $\beta^2 = \left(\dfrac{v}{c}\right)^2 = \dfrac{m(v^2)}{m(c^2)}$ showing that momentum is equivalent to kinetic energy in Newton's Laws.

Parametrization

(see the document "The Relative Unit Circle")

Gravity

Consider the relation

$H = T(m,v) + V(x,t) := \#$, which asserts the existence of both an "Inertial Frame" $T(m,v)$ and a coordinate system $V(x,t)$. Their interaction is defined by the relation:

$H^2 = [T(m,v) + V(x,t)]^2 = \left[(T(m,v))^2 + (V(x,t)^2\right] + \left[2(T(m,v))(V(x,t)\right] = \#^2$ where $\left[(T(m,v))^2 + (V(x,t)^2\right]$ expresses the existence of the interacting "Inertial Frame" and coordinate system and $\left[2(T(m,v))(V(x,t)\right]$ is the interaction term between $T(m,v)$ and $V(x,t)$. Note that

$H^2 \neq \left[(T(m,v))^2 + (V(x,t)^2\right]$ but if complex numbers are allowed, then

(Complex numbers)

$$H := (T(m,v)) + i(V(x,t),\ i = \sqrt{-1}$$
$$H^* := (T(m,v)) - i(V(x,t)$$
$$HH^* = \left[(T(m,v)) + i(V(x,t)\right]\left[(T(m,v)) + i(V(x,t)\right]$$
$$= \left[(T(m,v))^2 + ((V(x,t))^2\right] + \left[i(V(x,t)(T(m,v)) - (T(m,v))i(V(x,t)\right]$$
$$= \left[(T(m,v))^2 + ((V(x,t))^2\right]$$

, where the interaction terms have been removed by complex conjugation.

The reader is reminded that

$i^2 := \left(\sqrt{-1}\right)\left(\sqrt{-1}\right) = \sqrt{(-1)(-1)} = \sqrt{(1)(1)} = \sqrt{1^2} = 1 \neq -1$; i.e., if there are no negative numbers, then there are no square roots of negative numbers or products thereof. (The proof that $\sqrt{(-1)(-1)} = 1$ is left as an exercise for the student…. ☺

"Curvature" (radial coordinates)

$$r := \left(T(m,v)\right),\ r := \left(V(x,t)\right)$$
$$\pi H^2 = \left[\pi\left(T(m,v)\right)^2 + \pi\left(V(x,t)\right)^2\right] + \left[\left(T(m,v)\right)2\pi\left(V(x,t)\right)\right] = \#^2$$
$$= \left[\pi r^2 + \pi r^2\right] + r\left(2\pi r\right)$$

where $r := \left(T(m,v)\right)$ can be thought of as an "Inertial Radius" and $2\pi r := 2\pi\left(C_{V(x,t}\right)$ as a "Spacetime" coordinate Circumference where r and $2\pi\left(C_{V(x,t}\right)$ relate curvature at the point of interaction (the "origin" of both systems.

"Quantum Gravity" "Curvature"

Much more to be said, this is only a start.

$$H := \frac{1}{r}\left[m_0 + m_v\right]$$
$$H^2 := \left(m_0 + m_v\right)^2 = \left(\frac{1}{r^2}\right)\left[\left(m_0\right)^2 + \left(m_v\right)^2\right] + \left(\frac{1}{r^2}\right)\left[\left(m_0\right)\left(m_v\right)\right]$$

Where r is the (relative/"observed") distance between $\left(m_0\right)$ and m_v. Then for $\left(m_v\right) = G\left(m_0\right)$, the interaction term becomes $h^2 := \left(\frac{1}{r^2}\right)G\left[\left(m_0\right)\left(m_0\right)\right] = G\frac{\left(m_0\right)^2}{r^2}$ noting that in the "Inertial Frame" relating force and electromagnetism via Maxwell's equation (to be covered in a subsequent section)

$$\left(c\tau\right) = \frac{1}{\sqrt{\varepsilon_0\mu_0}},\ \tau = \left(c\tau\right)1_{(c\tau)} = c = c\left(\frac{c}{c}\right) = c\left(1_c\right),\ \tau = 1\left(\frac{1}{1}\right) = 1\left(1_1\right)$$
$$\varepsilon_0\mu_0 = \frac{1}{\left(c\tau\right)^2} = \frac{1}{r^2} = \frac{1}{c^2},\ \tau = 1$$

Where ε_0 and μ_0 are the permittivity and permeability force constants defined by Coulomb and Ampere laws and interpreted by Maxwell via the geometric Faraday "lines of force"

(Hint: as a precursor to the exposition on Electromagnetism, note that in Maxwell's equations the term $\overrightarrow{\nabla} \otimes \overleftarrow{B} = 0$ means that the $\overleftarrow{B}$ field does not interact with a change in the coordinate field, even when represented by the change from $B(0,0,0)=0$ to $B(x,y,z)$ and that the trace of the 4D Electromagnetic Tensor is equal to 0. (Wikipedia)

Too be continued: Analysis SO(3) , Electromagnetism, Gravity

Stay Tuned