Relativistic (Imaginary) Fermions

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The Relativistic Unit Circle (RUC)

This is a presentation of how complex (imaginary) vectors apply to the concept of fermions as the foundation of the Standard Model via Einstein, Pauli and Dirac. There is much more to say about this, but it is at least a starting point.

The application of complex numbers is shown to be unnecessary in the context of the full interaction equation, since they only apply in the transition regime in the transition from one element to two elements the “trigonometric” region of the generalized transition ($1 < \# < 2,\ 1^2 < \#^2 < 4$). Parametrized according to “relativistic” parameters, the range is

$$(ct) < (ct) + (vt') < 2(ct),\ 0 < (vt') < (ct)$$
$$(ct)^2 < \left[(ct) + (vt')\right]^2 < 4(ct)^2$$

$$1_{ct'} = \frac{1}{\gamma} + \beta,\ \gamma := \frac{t'}{t}\ \beta := \frac{v}{c}$$
$$\left(1_{ct'}\right)^2 = \left(\frac{1}{\gamma} + \beta\right)^2 = \left[\left(\frac{1}{\gamma}\right)^2 + (\beta)^2\right] + \left[2\left(\frac{1}{\gamma}\right)(\beta)\right]$$

Where the transition term $\left[2\left(\frac{1}{\gamma}\right)(\beta)\right] = 2\left(\frac{\beta}{\gamma}\right)$ represents the interaction (entanglement, change in entropy, etc.) by introducing multiplication of the interacting elements. Note that the expression $\left(1_{ct'}\right)^2 = \left[\left(\frac{1}{\gamma}\right)^2 + (\beta)^2\right]$ is analogous to the trigonometric identity $1^2 = (\cos\theta)^2 + (\sin\theta)^2$ but that

$$1 = \cos\theta + \sin\theta \leftrightarrow 1^2 = \left[(\cos\theta)^2 + (\sin\theta)^2\right] + \left[2(\cos\theta)(\sin\theta)\right]$$

In particular,

$$1 = \cos(0) \leftrightarrow \sin(0) = 0$$
$$1^2 = (\cos(0))^2 \leftrightarrow (\sin(0))^2 = 0^2$$

However,

$$\begin{aligned}2&=1+1=\cos(0)+\sin\left(\frac{\pi}{4}\right)\\2^2=4&=\left[\left(\cos(0)\right)^2+\left(\sin\left(\frac{\pi}{4}\right)\right)^2\right]+\left[2\cos(0)\sin\left(\frac{\pi}{4}\right)\right]\end{aligned}$$

Where the term $\left[2\cos(0)\sin\left(\frac{\pi}{4}\right)\right]=4\left\{\frac{1}{2}\cos(0)\sin\left(\frac{\pi}{4}\right)\right\}$ represents the total area of the four right triangles in the RUC.

Particle transitions for greater particle counts ($2<\#$) are expressed in the "hyperbolic" regime

Where $\gamma^2=\left[\left(1_{(ct)}\right)^2+\left(\gamma\beta\right)^2\right]+\left[2\gamma\beta\right]$, noting that the expression $\gamma^2=\left[\left(1_{(ct)}\right)^2+\left(\gamma\beta\right)^2\right]$ is analogous to the hyperbolic identity $\left(\cosh\theta\right)^2=1^2+\left(\sinh\theta\right)^2$ and the transition term $\left[2\gamma\beta\right]=4\left\{\frac{1}{2}(\gamma\theta)\right\}$ represents the transition term between integers in that regime

$$\begin{aligned}&\#<\gamma<\#+1\\&\#^2<\#\left(\gamma\right)^2<\left(\#+1\right)^2\end{aligned}$$

The Pythagorean Triple $\{5,4,3\}$ will be used as an example to illustrate the relevance of complex numbers to the analysis.

Fermions

SU(2)

The addition (existence) operator ($+$)

This operator expresses the existence element of the addition group, where

$$\begin{aligned}&a+0=a\\&a-a=0\leftrightarrow a=a\\&a+a=2a\end{aligned}$$

Note that all existing numbers are positive ; $-c=a-b,\ b>a\ \leftrightarrow b-c=a$.

The Multiplication Operator ($\times$, juxtaposition)

The Multiplication operator can only be applied if both elements exist

$$\# = a + b$$
$$\#^2 = \left[a^2 + b^2\right] + \left[2ab\right]$$

Special Relativity

The “Time Dilation” equation of Special Relativity can be derived from the second order equation:

$\left(ct'\right)^2 = (ct)^2 + \left(vt'\right)^2$ by solving for t', resulting in the expression:

$t' = t\left(\Gamma\right)$, where $\Gamma = \dfrac{1}{\sqrt{1^2 - \beta^2}}$, $\beta = \dfrac{v}{c}$.

However, this equation cannot be derived from the expression for length $\# := \left(ct'\right) = \left(ct\right) + \left(vt'\right)$ in one dimension, since $\#^2 = \left(ct'\right)^2 = \left[\left(ct\right)^2 + \left(vt'\right)^2\right] + \left[2\left(ct\right)\left(vt'\right)\right]$ where $\#$ represents the “count” of the terms, and is preserved from first order to second order.

in second order, where the first term expresses the existence of the interacting terms, and the second expresses their product (interaction, multiplication, entanglement). Note that both terms must exist for the multiplication $\left(ct\right)\left(vt'\right)$ to be defined.

In order to express the Time Dilation equation from, complex numbers must be applied where $i := \sqrt{-1}$, so that

$$\psi := \left(ct\right) + i\left(vt'\right) = \left(ct\right) + \left(vt'\right)$$
$$\psi^* := \left(ct\right) - \left(vt'\right)$$
$$\psi\psi^* := \left[\left(ct\right)^2 + \left(vt'\right)^2\right] + \left[\left(ct\right)\left(vt'\right) - \left(ct\right)\left(vt'\right)\right]$$

, so that $\psi\psi^* := \left[\left(ct\right)^2 + \left(vt'\right)^2\right]$ where the multiplication terms have been eliminated by the conjugation.

As an example, consider the “Pythagorean Triple” $\left\{5, 4, 3\right\}$, noting that $\left(5\right)\left(5\right)^* = 25 = 4^2 + 3^2$. However,

$$\# = 7 = 4 + 3$$
$$\#^2 = 7^2 = 49 = \left(4 + 3\right)^2 = \left[4^2 + 3^2\right] + \left[2\left(4\right)\left(3\right)\right] = \left[25\right] + \left[24\right]$$

So that $\#^2 = 7^2 = \left[\psi\psi^*\right] + \left[2(4)(3)\right] = \left[25\right] + \left[24\right]$ where the interaction terms have been restored, so there is no need for complex representation. However, it is important to emphasize that $\psi\psi^* \neq \#^2$.

The "relativistic" derivation of Fermions has its basis in the Pauli matrices which define a set of group relations under multiplication $SU(2)$:

$|\sigma_1| := \begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix}, |\sigma_2| := \begin{vmatrix} 0 & i \\ -i & 0 \end{vmatrix}, |\sigma_3| := \begin{vmatrix} 1 & 0 \\ 0 & -1 \end{vmatrix}$ Note that the identity matrix $|I| := \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix}$ is not a member of the group, since Tr $Tr|I| := Tr\begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} = Tr\left\{\begin{vmatrix} 1 & 0 \\ 0 & 0 \end{vmatrix} + \begin{vmatrix} 0 & 0 \\ 0 & 1 \end{vmatrix}\right\} = 1 + 1 = 2$ where the operation of addition means the group definition of $SU(2)$ no longer applies.

Note that $Tr|\sigma_3| := Tr\begin{vmatrix} 1 & 0 \\ 0 & -1 \end{vmatrix} = 0 \leftrightarrow 1 - 1 = 0 \leftrightarrow 1 = 1$ which establishes the equality of the group elements, but not their existence (via the Identity matrix). Also, note that $|\sigma_3| := \begin{vmatrix} 1 & 0 \\ 0 & -1 \end{vmatrix} = \begin{vmatrix} i^4 & 0 \\ 0 & i^2 \end{vmatrix}$ so that $Tr|\sigma_3| := 0 \leftrightarrow i^4 = i^2 \leftrightarrow 1 = -1 \leftrightarrow 1 + 1 = 2$ so that

$Tr|\sigma_3| := Tr\begin{vmatrix} 1 & 0 \\ 0 & -1 \end{vmatrix} = Tr\begin{vmatrix} i^4 & 0 \\ 0 & i^2 \end{vmatrix} = 1 - 1 = 0 \leftrightarrow 1 = 1$, again implying equality but not existence.

The complex characterization implies that $\psi := (ct') = (ct) + (vt') \leftrightarrow \psi\psi^* = (ct)^2 + (vt')^2$

Complex Conjugation using the Pauli Matrices is achieved by adding $|\sigma_1 + \sigma_2| = \begin{vmatrix} 0 & 1+i \\ 1-i & 0 \end{vmatrix}$ where the addition operator $\pm$ removes the sum as a group element.. Then

$Det|\sigma_1 + \sigma_2| = Det\begin{vmatrix} 0 & 1+i \\ 1-i & 0 \end{vmatrix} = -(1^2 + 1)$ Note that the terms are of different order, which can be corrected by using $\sqrt{1}$ in σ_1 so that

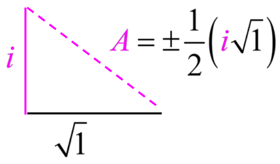
$Det|\sqrt{\sigma_1} + \sigma_2| = Det\begin{vmatrix} 0 & \sqrt{1}+i \\ \sqrt{1}-i & 0 \end{vmatrix} = Det\begin{vmatrix} 0 & \sqrt{\psi} \\ \sqrt{\psi^*} & 0 \end{vmatrix} = -\sqrt{\psi\psi^*} = -(1+1)$ in first order, where the products $\pm i\sqrt{1}$ have been eliminated in the conjugation.

On the other hand, revising $|\sigma_2|$ so that

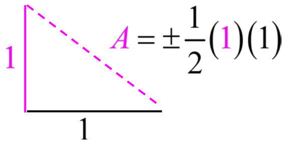
$$\left|\sigma_1\right|+\left|\sigma_2\right|=\begin{vmatrix}0 & 1\\ 1 & 0\end{vmatrix}+\begin{vmatrix}0 & i^4\\ i^2 & 0\end{vmatrix}=\begin{vmatrix}0 & 1+1\\ 1-1 & 0\end{vmatrix}:=\begin{vmatrix}0 & \psi\\ \psi^* & 0\end{vmatrix}$$

$$\leftrightarrow Det\left\{\left|\sigma_1\right|+\left|\sigma_2\right|\right\}=-\psi\psi^*=-\left\{1^2+1^2\right\}$$

in second order, where the products $\pm 1(1)$ have been eliminated by conjugation. Note that $\pi\left(\psi\psi^*\right)=\pi\left\{1^2+1^2\right\}=\pi\left(1^2\right)+\pi\left(1^2\right)$ represents area relationships between real and imaginary circles. In first order, the expression $A=\pm\left(\frac{1}{2}\right)\left(i\sqrt{1}\right)$ is the area of a right triangle with sides i and $\sqrt{1}$, and in second order $A=\pm\left(\frac{1}{2}\right)(1)(1)$:



First Order

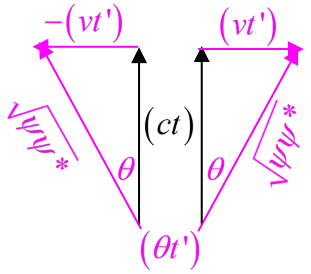


Second Order

In first order, the (eliminated) “Spin” is defined as $S:=\left(i\sqrt{1}\right)$ and in Second order $S^2:=(1)(1)$ These areas are omitted by complex conjugation in both orders.

The Stern-Gerlach Experiment (Relativistic Interpretation)

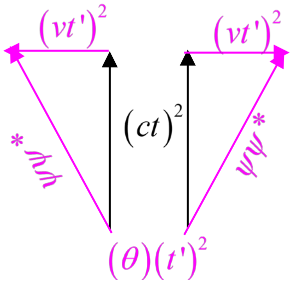
In the Stern-Gerlach experiment, the relativistic representation is expressed as:



$$\sin\left(-(\theta t')\right)=\frac{vt'}{\sqrt{\psi\psi^*}}$$

$$=-\left(\sin(\theta t')\right)=\frac{-vt'}{\sqrt{\psi\psi^*}}$$

First Order



$$\sin\left(-\left(\theta(t')^2\right)\right)=\frac{(vt')^2}{\psi\psi^*}$$

$$=-\left(\sin\left(\theta(t')^2\right)\right)=\frac{(vt')^2}{\psi\psi^*}$$

Second Order

This is interpreted as a source (ct) which has been diverted by an existing field in first order $\pm(vt')$ to a sensor at which point the interaction ceases. The diverted path length is then $\sqrt{\psi\psi^*}$ compared with the un-diverted path length (ct). (Note that this is contrasted with the matrix representation where the determinant(s) are negative). This is because the source elements are receding from the source, so that both path lengths are characterized by a negative relative distance (if they were approaching, the relative distance would be positive.)

Because the "Spin" A is omitted in the above representation, it is assumed to be an intrinsic (imaginary) property of the elements themselves.

Analysis

The pictures above suggest a vector relation between the elements:

First Order

$$\overrightarrow{\sqrt{\psi\psi^*}} = (ct)\vec{i} \pm (vt')\vec{j}$$

$$1_{\sqrt{\psi\psi^*}} := \frac{\overrightarrow{\sqrt{\psi\psi^*}}}{\overrightarrow{\sqrt{\psi\psi^*}}} = \frac{(ct)}{\sqrt{\psi\psi^*}}\vec{i} \pm \frac{(vt')}{\sqrt{\psi\psi^*}}\vec{j}$$

$$\left(1_{\sqrt{\psi\psi}}\right) := (\cos\theta)\vec{i} + (\sin\theta)\vec{j}$$

$$\left(1_{\sqrt{\psi\psi}}\right)^* = (\cos\theta)\vec{i} - (\sin\theta)\vec{j}$$

Second Order

$$1_{\sqrt{\psi\psi^*}} := \frac{\sqrt{\psi\psi^*}}{\sqrt{\psi\psi^*}}$$

$$\left(1_{\sqrt{\psi\psi^*}}\right) := (\cos\theta)\vec{i} + (\sin\theta)\vec{j}$$

$$\left(1_{\sqrt{\psi\psi^*}}\right)^* = (\cos\theta)\vec{i} - (\sin\theta)\vec{j}$$

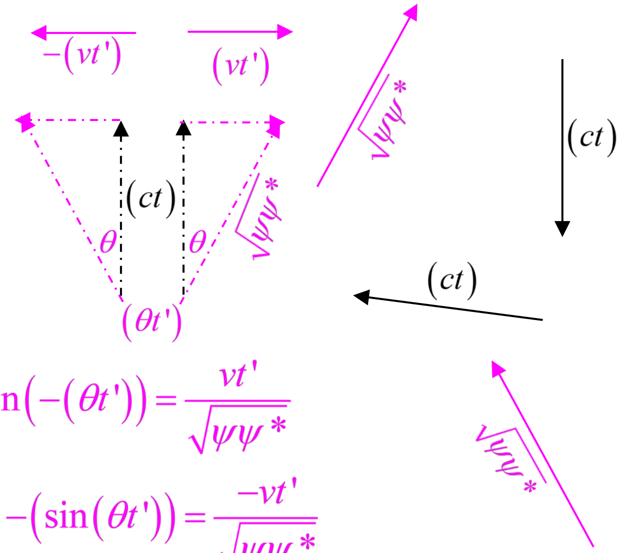
$$\left(1_{\sqrt{\psi\psi^*}}\right)\left(1_{\sqrt{\psi\psi^*}}\right)^* = (\cos\theta)^2\left(\vec{i}\cdot\overleftarrow{i}\right) + (\sin\theta)^2\left(\vec{j}\cdot\overleftarrow{j}\right) = (\cos\theta)^2 + (\sin\theta)^2$$

However, note that $x\overleftrightarrow{i}\cdot x\overleftrightarrow{j} = x\left(\overleftrightarrow{i}\cdot\overleftrightarrow{j}\right) = 0\cdot 0$ because the origins of the imaginary and real axes do not coincide. There are two reasons for this:

1. There are no negative numbers:
   $-c = a - b,\ b > a \leftrightarrow b - c = a$
   $a > 0$
   $a - a = 0 \leftrightarrow a = a$
   $a + a = 2a$
   If there are no negative numbers, there are no square roots of negative numbers. (See the discussion above in the full (binomial) interaction expansion above.
2. 1 and i are of different order $i^2 = -1$ This is why $1(-1) \neq 0 \neq 0$

The interpretation of this result is that the vectors do not have a common origin $0(\neg\equiv)0$ which is why the area A is irrelevant (eliminated by conjugation) in the relativistic ("imaginary") characterization .

Affine vectors have no common origin, so the "connected" first order picture above has an equivalent representation with the translation diagram below (i.e., the scalars for the individual vectors are invariant). (The position of the vectors $\pm(vt')$ corresponds to Einstein's concept of "parallel translation”, and the “Spin” $\pm A$ corresponds to an “affine connection”, which is an oxymoron, since vectors cannot be both connected and not connected. However, the translation need not be parallel, since vectors are invariant under any translation and/or rotation.



$$\sin\left(-(\theta t')\right) = \frac{vt'}{\sqrt{\psi\psi^*}}$$

$$= -\left(\sin(\theta t')\right) = \frac{-vt'}{\sqrt{\psi\psi^*}}$$

First Order

Complex Relationships

The complex (imaginary) relations holds for any set of elements related by the STR non-interaction equation.

$(\psi\psi^*, E, iB)$, $\psi\psi^* = E^2 + B^2$ The Poynting Vector (E doesn’t interact with B)

$(\psi\psi^*, m, iL)$, $\psi\psi^* = m^2 + L^2$ (Relation between Mass and Light. (Mass doesn’t interact with light)

$(\psi\psi^*, m_0, im')$, $\pi\left(\psi\psi^*\right) = \pi\left({m_0}^2\right) + \pi\left(m'\right)^2$ “Relativistic” Masses do not interact (Pauli Exclusion Principle)

$(\psi\psi^*, P, ix)$, $\left(\psi\psi^*\right) = \left(P^2\right) + \left(x\right)^2$ Complementary variables)

Real Force and Mass

The “rest” mass at the common contact point (origin) of two equal and opposite forces is characterized in a single dimension by the relation:

$$f_{(ct)} = \frac{f_{(ct)}}{2} + \frac{f_{(ct)}}{2},\ f_{(ct)} := (ct)$$

$$m_0 := (ct)^2 = {f_{(ct)}}^2 = \left(\frac{f_{(ct)}}{2} + \frac{f_{(ct)}}{2}\right)^2 = \left[\left(\frac{f_{(ct)}}{2}\right)^2 + \left(\frac{f_{(ct)}}{2}\right)^2\right] + \left[2\left(\frac{f_{(ct)}}{2}\right)\left(\frac{f_{(ct)}}{2}\right)\right]$$

Substituting the parameter m for t yields the momentum $P := f = \left(m_0 c\right)$ for a single force defined at an arbitrary velocity c Note that:

$$m_0 = \left[\left(\frac{f_{(ct)}}{2}\right)^2 + \left(\frac{f_{(ct)}}{2}\right)^2\right]$$

$$\leftrightarrow \left[2\left(\frac{f_{(ct)}}{2}\right)\left(\frac{f_{(ct)}}{2}\right)\right] = 0$$

$$\leftrightarrow \left(\frac{f_{(ct)}}{2}\right) = 0$$

Relativistic (Imaginary) Energy

$$(ct')^2 = (ct)^2 + (vt')^2$$
$$(m_{ct'})^2 = (m_{ct})^2 + (m_{vt'})^2$$
$$t' = t\Gamma$$
$$((ct)\Gamma)^2 = (ct)^2 + ((vt)\Gamma)^2$$
$$((c)\Gamma)^2 = (c)^2 + (v\Gamma)^2$$
$$(\Gamma)^2 = (1_c)^2 + (\beta\Gamma)^2$$

Note that the relation $(\Gamma)^2 = (1_c)^2 + (\beta\Gamma)^2$ has the form of the hyperbolic identity

$$(\cosh\theta)^2 = (1_c)^2 + (\sinh\theta)^2$$

$$E_0 = m_0c^2$$
$$Pc := (m_0\Gamma)(\beta c) = (m_0\Gamma)v,\ \Gamma := \frac{1}{\sqrt{1^2 - \beta^2}},\ \beta := \frac{v}{c}$$
$$\psi := E' = (m\Gamma)c^2 = m'c^2$$

$$\psi := E_0 + Pc$$
$$\psi^* := E_0 - Pc$$
$$\psi\psi^* = \left[(E_0)^2 + (Pc)^2\right] + \left[E_0(Pc)\right] - \left[E_0(Pc)\right]$$
$$\psi\psi^* = (E')^2 = \left[(E_0)^2 + (Pc)^2\right]$$
$$(E')^2 = \left[(E_0)^2 + (Pc)^2\right] = \left[(E_0)^2 + \left[(m_0\Gamma)v\right]^2\right]$$
$$(E')^2 = \left[(m\Gamma)c^2\right]^2 = \left[m'c^2\right]^2 = (m')^2(c^4)$$

$$(E')^2 = \left[(Pc)^2 + (E_0)^2\right]$$

(etc. :)

Note that an expression of the interaction equation is only valid for a specific (reified) instance of (vt'), e.g. $(vt')=3$ in the Pythagorean example above, where $(ct)=4$, $\#^2=49$ and $\psi\psi^*=25=\left[4^2+3^2\right]$ so that (taking the lesser of the two elements as imaginary):

$$\#:=4+3=7$$
$$\#^2:=7^2=(4+3)^2=[16+9]+2\left[(4)(3)\right]=[25]+[24]=49$$

Is equivalent to

$\#^2:=7^2=(4+3)^2=[16+9]+2\left[(4)(3)\right]=[25]+[24]=49$ in the full context of the interaction equation.

The second order interaction equation is represented by the matrix relations:

$$0<vt'<ct$$
$$\#:=(ct)+(vt')$$
$$\#^2=Tr\begin{vmatrix}(ct)^2 & 0\\ 0 & (vt')^2\end{vmatrix}+Det\begin{vmatrix}ct & ct\\ -vt' & vt'\end{vmatrix}=\left[(ct)^2+(vt')^2\right]+\left[2(ct)(vt')\right]$$

Quarks

Consider the expression:

$\Delta\left(\#^2\right):=\left[(ct)^2+(vt')^2\right]-\left[2(ct)(vt')\right]$ where $0\le\Delta\left(\#^2\right)\le(ct)^2$

Case $\Delta\left(\#^2\right)\le(ct)^2$: Then $\#^2+\Delta\left(\#^2\right):=\left[(ct)^2+(vt')^2\right]=Tr\begin{vmatrix}(ct)^2 & 0\\ 0 & (vt')^2\end{vmatrix}$, representing the transition from two interacting elements to two non-interacting elements.

Case $(vt')=0$: $\Delta\left(\#^2\right):=(ct)^2$ represents the transition from zero to one positive element

$$\Delta(\#):=(ct)=\int_0^{(ct)}d(ct)$$

Quarks and the generalized equation

In the case if $0<(vt')<(ct'):\gamma:=\frac{t'}{t},\ \beta:=\frac{v}{c}$

$$(1_{(ct')})^2=\left[\left(\frac{1}{\gamma}\right)^2+(\beta^2)\right]+\left[2\frac{\beta}{\gamma}\right]$$

$$\Delta(\#^2):=\left[\left(\frac{1}{\gamma}\right)^2+(\beta^2)\right]-\left[2\frac{\beta}{\gamma}\right]$$

In the case of $(vt')>2(ct')$,

$$\gamma^2=\left[\left((1_{ct})^2+\beta\gamma\right)^2\right]+\left[2(\beta\gamma)\right],\ (\beta\gamma)^2>(1_{ct})^2$$

$$\Delta(\#^2):=\left[\left((1_{ct})^2+\beta\gamma\right)^2\right]-\left[2(\beta\gamma)\right]$$

Note that for $(\beta\gamma)^2>>(1_{ct})^2$ $\Delta(\#^2)\approx(\beta\gamma)^2$ so the initial state fades from existence for $\gamma\beta>>1_{(ct)}$ (the initial state).

Example:

$\#^2+1^2=[25]+[25]=50=Tr\begin{vmatrix}25 & 0\\ 0 & 25\end{vmatrix}=Tr\begin{vmatrix}5^2 & 0\\ 0 & 5^2\end{vmatrix}$ which now represents the existence of two non-interacting elements, so that $(50)-1^2=49$ The transition can then be represented by

$$\#^2+1^2=Tr\begin{vmatrix}4^2 & 0\\ 0 & 3^2\end{vmatrix}+Det\begin{vmatrix}4 & 4\\ -3 & 3\end{vmatrix}+Tr\left|1^2\right|=[25]+\left[\left(\sqrt{24}\right)^2\right]+1^2=50$$

$$Tr\begin{vmatrix}25 & 0 & 0\\ 0 & 24 & 0\\ 0 & 0 & 1^2\end{vmatrix}=49+1^2=50=2(25)=Tr\begin{vmatrix}25 & 0\\ 0 & 25\end{vmatrix}$$

This suggests that quarks $\{R,B,G\}$ arise during the (interacting) transitions between non-interacting particles