Remarks on the Foundations of Mathematical Physics

(Working Document)

(Special Relativity, Quantum Mechanics, General Relativity)

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This paper is a discussion of the mathematical foundation of physics; the first section ("**Mathematical Foundation**") provides the mathematical relationships with only an occasional mention of physics as a basic orientation, while the second section ("**Physical Interpretation**") (to be done) will (hopefully) relate that foundation to the basic concepts of physics as interpreted by that foundation related to the Standard Model.

The perspective is that of rejecting vector physics as its foundation, showing that the Pythagorean Theorem (and thus complex numbers) are inadequate as a foundation, and replacing it with an analysis of a number of assumptions taken for granted, but are highly misleading because of the emphasis on linear systems as the fundamental basis of experimental physics.

Of course, this approach must explain why the Standard Model explains experimental physics so well, which will also be addressed shortly (as of 4/17/2024). Reverse engineering the Standard Model requires analysis of many powerful theories (e.g., Special and General Relativity, Quantum Mechannics, and Quantum Field Theory) as well as mathematical theorems hitherto "unsolved" according to Urban Legend (Fermat's Last Theorem, Goldbach's conjecture, Russell's paradoxes, and the various "paradoxes" associated with Special Theory and Entanglement.

I am working on this by myself, and have no proofreader and at present am 83+ years old, but I soldier on…. In the hope that my journey can help others to understand the foundations of the Standard Model and its consequences.

Note: I am publishing to the "Physics Discussion Forum" forum under the user name BuleriaChk (Web Page(s) need updating)

Updates:

02/03/24 Added section on real geometry model preserving count.

02/04/24 Added section on vector relations, π

02/06/24 Added section on coordinate distance.

02/08/24 Added section on coordinate Matrix.

02/09/24 Added Link to Quark document

03/02/24 Added Link to Groups and Vectors

04/20/2024 Added Link to Relativistic Proof of Fermat's Theorem

04/23/2025 Added Link to The "constant" e

05/20/2024 Additional Links added

References: (all are my pdfs unless otherwise indicated)

Mathematical Foundation

- Fermat's Last Theorem and Multinomials (my pdf)
- The Quadratic Equation (my pdf)
- Pythagorean Triples (my pdf)
- Parity (my pdf)
- The "Constant" e (my pdf)
- Psuedo ("Geometric") Proofs of the Pythagorean Theorem (my pdf)

Physical Interpretation

- From Lorentz (M-M) to Special Relativity (my pdf)
- The Relativistic Unit Circle (my PDF)
- Of Clocks and Rulers (my Pdf) added link 2/6/24
- Quarks (My pdf)
- Fermions (My PDF) added 2/15/2024
- The Pauli Matrices and SU(2) (my PDF) added 2/18/2024
- Boson and Progressions (My PDF) added 3/16/2024 (Higgs Boson is n = 1)
- Quantum Mechanics – (My PDF) added 3/22/2024
- Groups and Vectors – (My PDF) added 04/02/2024
- Existence and Interaction Equations -(My PDF) added 04/06/2024
- Electromagnetism - (My PDF) added 04/24/2024
- The Lorentz Force – (My PDF) added 04/17/2024

"Relativistic" Proof of Fermat's Last Theorem (My PDF added 04/24/2024

Fermat's Little Theorem (my PDF added 04/29/2024

Relativistic Fermions (my PDF added 5/4/2024

Real Fermions (My PDF) added 5/19/2024

Impulse Response (my Pdf added 5/20/2024

Transcendental Numbers (my pdf added 8/20/2025) (Link inactive temporarily, but stay tuned)

Mathematical Foundation

Numbers are real, positive whole numbers (positive integers)

Fundamentals

1. Existence is represented by addition $+$ where $a+0=a$
2. There are no negative numbers (if a exists, then $a>0$:
   Equality
   $-c=a-b, b>a>0$
   $b-c=a>0$
   $a-a=0$
   qed

   This concept is expressed in matrix form as:
   $|\sigma_3|:=\begin{vmatrix}1 & 0\\0 & -1\end{vmatrix}$ Where $Tr|\sigma_3|=1-1=0$. The Pauli Matrices consist of the set $\{\sigma_1,\sigma_2,\sigma_3\}$, of which σ_2 is imaginary (see below), and is the foundation of the Special Unitary Group SU(2), which was developed to spin from the results of the Stern-Gerlach experiment, and so will be discussed in the **Physical Interpretation** subsequently. Note that $Det|\sigma_3|=(1)(-1)=(-1)^2<0$ and therefore does not exist.

3. Interaction (multiplication, entropy, entanglement, etc.) requires at least two elements
   $c=a+b$, $c^n=(a+b)^n=a^n+b^n+f(a,b,n)$, where $f(a,b,n)>0$
   Note that the positive real numbers so not form a group, since there are two operations, existence $(+)$ and multiplication $\times$ (juxtaposition, $a\times b\equiv ab=ba$)
   (Not that addition is not included in Goedel's system of numbering wff with primes)
4. Every number is prime in terms of its own base:
   $n=n(\frac{n}{n})=n(1_n)$
   $m=n\leftrightarrow n=m$
   Where (1_n) means "Unity to the base n
   Note that the "global unity 1" ("1" without a base) applies to all $n=1n$ and is a syntax name with no operational meaning.)
   Note that $0=0\left(\frac{0}{0}\right)=0(1_0)$
   Goldbach's Conjecture: "Every even number is the sum of two primes." Since every number is prime, $n+n=2n, qed$, Goldbach's Conjecture is satisfied.

5. Proof of Fermat's Last Theorem

Fermat is correct for $n \geq 2$ and therefore for $n > 2$

Hypothesis

$c^n \neq a^n + b^n \; \forall \{a,b,c,n\} > 0,\; n \geq 2$

Thesis (via the binomial expansion for $n \geq 2$)

$c = a + b$

$c^n = \left[a^n + b^n\right] + \left[f(a,b,n)\right]$

$c^n = \left[a^n + b^n\right] \leftrightarrow \left[f(a,b,n)\right] = 0$

$\left[f(a,b,n)\right] \neq 0$

$c^n \neq \left[a^n + b^n\right]$

qed

This also true for multinomials:

$$\sum_{i=1}^{n}\left[\left(a_i\right)^m\right] \neq \left(\sum_{i=1}^{n}\left[\left(a_i\right)\right]\right)^m$$

6. There are no imaginary numbers. where $i = \sqrt{-1}$

$i = \left(\sqrt{-1}\right) \leftrightarrow i^2 = -1$

$i^2 = \left(\sqrt{-1}\right)\left(\sqrt{-1}\right) = \sqrt{(-1)(-1)} = \sqrt{(1)^2} = 1 \neq -1$

7. $1^2 \neq 1$ (Russell's Paradox – a number cannot both multiply and not multiply itself.

8. $x^n x^m = 0 \;\; \forall n \neq m$

This comes from the fact that powers of x form an orthogonal basis, where the expansion

$\{x,n\} := \left\{x, x^2, x^3, \ldots\ldots, x^n\right\}$ forms the basis of a vector space, where $\overrightarrow{x^n} \cdot \overrightarrow{x^m} = \overrightarrow{x^n} \otimes \overleftarrow{x^m} = (0,0) \; \forall x$ Note that $\{1,x,n\} := \left\{1, x, x^2, x^3, \ldots\ldots, x^n\right\} \leftrightarrow x = 1 = 0$. In particular, the expression $1 + x + x^2 + \ldots\ldots + x^n = 0 \leftrightarrow (-1) = \left(\sqrt{-1}\right)^2 = i^2 = x + x^2 + \ldots\ldots + x^n$ where $i = x \leftrightarrow \vec{x} = \vec{0} \leftrightarrow \vec{i} = \vec{0}$.However, $\left\{1, 1^2, 1^3, \ldots, 1^n\right\}$ does form an orthonormal basis provided there is no "origin" at $x = 0 = 1$

9. Ratios are not prime numbers

$\left(\frac{x}{x}\right) = \left(1_x\right) \leftrightarrow x\left(1_x\right) = x,\; x \neq 1_x$

$$(1_1) := \frac{1}{1}$$

$$\left(\frac{1}{1}\right) = \left(\frac{1}{1}\right)\left[\frac{\left(\frac{1}{1}\right)}{\left(\frac{1}{1}\right)}\right] = \left(\frac{1}{1}\right)\left[\frac{1^2}{1^2}\right] = 1_{\left(\frac{1}{1}\right)} \neq \left(\frac{1}{1}\right)(1_1)$$

10. Parameterization

    Note that the expression $x = vt = x\left(\frac{x}{x}\right) = vt\left(\frac{vt}{vt}\right)$ is a prime number (representing a coordinate length), but that $x = vt \rightarrow \frac{x}{t} = v\left(\frac{t}{t}\right) = v(1_t)$ where (1_t) represents a single clock "tick" in "coordinate time" is not a prime number unless $v \equiv t \equiv x$ so that $x = t = t\left(\frac{t}{t}\right) = t(1_t)$

    (This will be further discussed below).

11. Covariance and Contravariance (for Vectors - GTR) (Wikipedia Link)

    A basis vector is a prime number in its own dimension (vector component) and cannot be changed (i.e., it is an invariant):

    $1_1 = (1)\left(\frac{1}{1}\right)$, a prime number.

    Similarly, for

    $(a)\vec{i} = a\left(\frac{a}{a}\right)\vec{i} = a(1_a)\vec{i},\ x = a$

    $(b)\vec{j} = (b)\left(\frac{b}{b}\right)\vec{j} = (b)(1_b)\vec{j},\ y = b$

    $(c)\vec{k} = (c)\left(\frac{c}{c}\right)\vec{k} = (c)(1_c)\vec{k},\ z = c$

    $(d)\vec{l} = d\left(\frac{d}{d}\right)\vec{l} = d(1_d)\vec{l},\ z = t$

    "Tensor Analysis" is an oxymoron. (See 11 Above) since its language is in derivatives (ratios).

12. Counting vs. Interaction

    An invariant is countable: $\#_a = \sum_{i=1}^{a} 1_i = a(1_a) = a$ ($a = a + 0$ represents the existence of a)

    In the following discussion, set $a = 4, b = 3$

    Addition (existence)

Invariants can be added:

$$\#_c := c = \#_a + \#_b = \sum_{i=1}^{a} 1_i + \sum_{i=1}^{b} 1_i = a(1_a) + b(1_b) = a + b$$

(c can only exist if both a and b exist.

Multiplication (Interaction)

$$\#_{ab} = (\#_a)(\#_b) = \left(\sum_{i=1}^{a} 1_i\right)\left(\sum_{i=1}^{b} 1_i\right) = a(1_a)b(1_b) = (a)(b)$$

However, the invariants a and b must exist as a precondition for multiplication, so that $(\#_c)^2 = c^2 = (c)(c) = (a+b)^2 = \left[a^2 + b^2\right] + [2ab]$ where the interaction term $[2ab] = 2(\#_{ab})$ defines multiplication in terms of $\#^2$.

$$7^2 = 49 = \left[4^2 + 3^2\right] + 2(4)(3) = [25] + [24]$$

13. Pythagorean Triple e.g. $\{5,4,3\}$ where $25 = 4^2 + 3^2$

    This expression can only be calculated only if imaginary numbers are introduced where:

    $$\psi := 4 + 3i$$

    $$\psi^* := 4 - 3i$$

    $$\psi\psi^* := (4+3i)(4+3i) = 4^2 + \{(3i)(4)\} - \{(4)(3i)\} - i^2(3)(3)$$

    $$= (4)^2 + (3)^2 \leftrightarrow i = \sqrt{-1}$$

    Note that the relation $+\{(3i)(4)\} - \{(4)(3i)\} = 0$ is valid whether the terms are vector dot products or cross products. That is, the complex conjugation eliminates the multiplication product from the countable expression.

14. Relation of imaginary numbers to Real Numbers

    Note that the full expression:

    $7^2 = 49 = \left[4^2 + 3^2\right] + 2(4)(3) = [25] + [24]$ is equivalent to

    $$7^2 = 49 = \left[4^2 + 3^2\right] + 2(4)(3) = [25] + [24]$$

    $$7^2 = [\psi\psi^*] + 8(3) = [25] + 8(3)$$

    Which is equally valid for real number 3 so the expression $i = \sqrt{-1}$ is irrelevant, since $i^2 = \left(\sqrt{-1}\right)^2 = \left(\sqrt{(-1)(-1)}\right)^2 = \left(\sqrt{1}\right)^2 = 1 > 0 \neq -1$ as above.

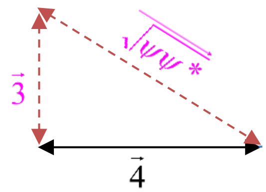
15. The Right Triangle

    The equation of a right triangle is then expressed as the relation:

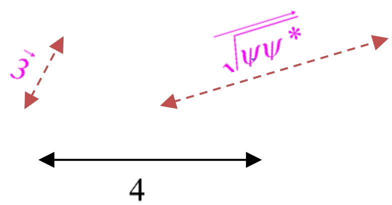
    $$\sqrt{\psi\psi^*} = r^2 = \sqrt{a^2 + b^2}$$

    $$r^2 = 25 = 16 + 9$$

Where r is the imaginary hypotenuse of the "vectors" $\vec{x} = \vec{4} = \overrightarrow{x_4} \equiv 4\vec{i},\ \vec{y} = \vec{3} = \overrightarrow{y_3} \equiv 3\vec{j}$:



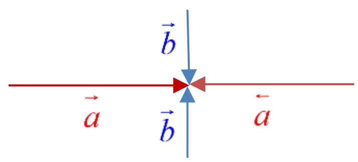
Note that the diagram implies that $\vec{4}$, $\vec{3}$, and $\overrightarrow{\sqrt{\psi\psi^*}}$ 3 are "connected", but the relation is equally valid if there is no common origin:

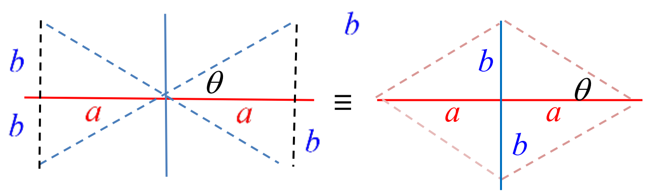


That is, the "vectors" have magnitude and direction but do not intersect unless the magnitudes ae changed and/or rotated in the imagination. (Vectors are translation and rotation invariant). Thus all vectors are "affine" (without a common origin).

16. Real Number "Geometric" Relation – (See the RUC)

Note that the count is preserved in the following relations where all elements have a common center (i. e., are not "affine"):





$$\vec{a}\cdot\overleftarrow{a} = a^2,\ \vec{b}\cdot\overleftarrow{b} = b^2$$

$$A = 4\left[\left(\frac{1}{2}\right)(ab)\right] = 4A_{\triangle} = 2ab$$

$$\# := a + b$$
$$\#^2 := (a+b)^2 = \left[a^2 + b^2\right] + \left[2(ab)\right]$$
$$h^2 := 2ab := 2S^2 \leftrightarrow S^2 := ab \leftrightarrow= S = \sqrt{ab}$$
$$S = \frac{h}{\sqrt{2}}$$

Trigonometrically, $a = a\cos(0) = a\cos\left(\frac{\pi}{2}\right)$ and $b = b\sin(\frac{\pi}{4}) = b\sin\left(-\frac{3\pi}{4}\right)$

Note that:

$$\# = 1\cos(0) + 1\sin\left(\frac{\pi}{4}\right) = 2$$

$$\#^2 = 2^2 = \left[1\cos^2(0) + 1\sin^2\left(\frac{\pi}{4}\right)\right] + \left[2\left(1\cos(0)1\sin\left(\frac{\pi}{4}\right)\right)\right] = 4$$

One can think of a as an initial state, to which is added a "perturbing" term b that interacts with via multiplication (the product of the sum (existence) of the two terms).

The term $S := \frac{h}{\sqrt{2}}$ conforms to the traditional definition of "Spin", where h is Planck's constant.

The "Interaction term" represents the change in entropy ("entanglement)" that "binds" the terms a and the "perturbation" term b together.

$$\# = \left(r_a + r_b\right)$$
$$\pi\left(\#^2\right) = \pi\left(r_a + r_b\right)^2 = \left[\pi r_a^{\;2} + \pi r_b^{\;2}\right] + \left[r_a\left(2\pi r_b\right)\right]$$

Note that π is only relevant in second order. ($2\pi(\#) = \left(2\pi r_a + 2\pi r_b\right)$ only relates two affine "circumferences with no common center.

The term $\left[\pi r_a^{\;2} + \pi r_b^{\;2}\right]$ can be interpreted as the sum of two circular areas, while the term

$r_a\left(2\pi r_b\right) = r_a C_b = 2\pi\left(r_a r_b\right) = 2\pi\left(\sqrt{\left(r_a r_b\right)}\right)^2 := 2\left(\pi S^2\right) := h^2$ can be interpreted as:

1. a radius multiplied by a circumference (e.g. Earth and atmosphere)
2. Two circular areas; i.e. "Spins"

The expression $\#^2 := (a+b)^2 = \left[a^2 + b^2\right] + \left[2(ab)\right]$ can be expressed in matrix form as:

$$\#^2 = Tr\begin{vmatrix} a^2 & 0 \\ 0 & b^2 \end{vmatrix} + Det\begin{vmatrix} a & a \\ -b & b \end{vmatrix} = \left[a^2 + b^2\right] + \left[2(ab)\right]$$

Where $\left[a^2 + b^2\right] = \left[\vec{a}\cdot\overleftarrow{a} + \vec{b}\cdot\overleftarrow{b}\right]$ and $Det\begin{vmatrix} a & a \\ -b & b \end{vmatrix} = 2(ab) = \left(\vec{a}\otimes\overleftarrow{b}\right) - \left(\vec{b}\otimes\overleftarrow{a}\right)$ where

$$2ab = \vec{a}\otimes\overleftarrow{b} = \left[(ab)\sin\left(\frac{\pi}{4}\right) - (ba)\sin\left(-\frac{3\pi}{4}\right)\right] = \left[(ab)\sin\left(\frac{\pi}{4}\right) + (ba)\sin\left(+\frac{3\pi}{4}\right)\right]$$

Physical Interpretation

1. Coordinate Distance in wavelengths

The coordinate distance for any wavelength $\lambda\left(\frac{\lambda}{\lambda}\right)=\lambda\left(1_{\gamma}\right)$ to an atom in the mirror image of any galaxy along a line of sight "geodesic" can be expressed as:

$x=n(ct)+(vt')=n(\lambda)+(\delta\lambda)$ where $\lambda=ct$ is an invariant wavelength, where $0\le\delta\lambda\le\lambda$ so that $x=n(ct),\ (\delta\lambda)=0$, $x=(n+1)(ct),\ (\delta\lambda)=0$, $0<\lambda\le x$, where $(\delta\lambda)$ is the relative distance the galaxy moves the atom in our galaxy between the limits $x_{n\lambda}=n\lambda$ to $x_{(n\pm1)\lambda}=(n\pm1)\lambda$

There are two possible constraints for (arm length) $\le x$

(a) This relationship implies that the "wavelength" terms are arranged end to end. However, if they are invariant, the are arranged as a matrix where:

Distance = $(x_{\lambda})\vec{i}:=Tr\begin{vmatrix}\lambda_1 & 0 & 0 & 0\\ 0 & \dots & 0 & 0\\ 0 & 0 & \lambda_n & 0\\ 0 & 0 & 0 & \delta\lambda\end{vmatrix}\vec{i}=\sum_{i=1}^{n}(\lambda_i)\vec{i}+(\delta\lambda)\vec{i}$

But then each element of the matrix is a vector, and in two (or greater) dimensions (in particular (n) or ($n+1$) as above), the vector components are vectors themselves are affine (have no common origin) and are both translation and rotation invariant.

(b) There may be gaps in the distance where there are no wavelengths (i.e., a vacuum); that is, the distance is not continuous.. Visible wavelengths are between 380nm and 750 nm. (Wikipedia) ($1\,\text{nm}=10^{-9}=.00000001\text{ meter}$).
Now go figure the distance to the mirrored galaxy ….

(c) A photon of wavelength λ from the mirror galaxy may interact with a photon coming from behind the atom in our galaxy, resulting in a loss of energy (a "red shift".

(TBD) (In progress – stay tuned)

2. Force and Mass

A Newtonian force can be represented by the expression $f=\frac{f}{2}+\frac{f}{2}$. A force can be parameterized by the expression $f=c\tau$ where c represents a "force creation rate" and τ

represents a “force creation time”. (Note, a “coordinate ruler” can be characterized as $x := \lambda = vt$ where x represents a length (i.e., a ruler), v represents velocity, and t represents “ruler creation time”. Note that if reified (given values) the left and right sides are invariant prime numbers (e.g., $n = (ct) = (ct)\left(\frac{ct}{ct}\right) = (ct)(1_{ct})$).

A single force is positive, since $f + 0 = f \leftrightarrow f - f = 0$ and is represented by a one dimensional “matrix”: $f = Tr|f|$ A single force has no “origin” since “0” is a syntax”name” in the expression $f + 0 = f$ and has no operational meaning (i.e., it has no “position” since it is always “moving”.

Relative motion in a coordinate system expressed as a velocity is positive if two forces are approaching each other, and negative velocity if the two forces are receding. If the two elements do not interact positive velocity changes to negative velocity at the point of coincidence.

In Progress (TBD) -----------------------------------

$$\begin{vmatrix} a & a \\ -b & b \end{vmatrix} = \begin{vmatrix} a & 0 \\ 0 & b \end{vmatrix} + \begin{bmatrix} 0 & a \\ -b & 0 \end{bmatrix}$$

$$Det\begin{vmatrix} a & a \\ -b & b \end{vmatrix} = Tr\begin{vmatrix} a & 0 \\ 0 & b \end{vmatrix} + Det\begin{bmatrix} 0 & a \\ -b & 0 \end{bmatrix} = 2ab$$

For each quadrant:

$$\frac{1}{4}\left\{ Det\begin{vmatrix} a & a \\ -b & b \end{vmatrix} \right\} = \left(\frac{1}{4}\right)(2ab) = \left(\frac{1}{2}\right)(ab)$$

$$a = b = 1$$

$$\left\{\frac{1}{4}\right\}\left\{ Det\begin{vmatrix} 1 & 1 \\ -1 & 1 \end{vmatrix} \right\} = \left\{\frac{1}{4}\right\}\left\{1^2 + 1^2\right\} = \left\{\frac{1}{4}\right\}\left\{2\left(1^2\right)\right\} = \left\{\frac{1}{2}\right\}\left(1^2\right)$$

Note that this is only true if $\left\{1^2 + 1^2\right\} = 2\left(1^2\right)$

$$(1+1)^2 = \left[1^2 + 1^2\right] + 2(1)(1) = \left[1^2 + 1^2\right] + \left[2\left(1^2\right)\right]$$

$$= Tr\begin{pmatrix} 1^2 & 0 \\ 0 & 1^2 \end{pmatrix} + Det\begin{vmatrix} 1 & 1 \\ -1 & 1 \end{vmatrix}$$

$$\neg\left[1^2 + 1^2 + 1^2 + 1^2\right]$$

$$= Tr\begin{vmatrix} 1^2 & 0 & 0 & 0 \\ 0 & 1^2 & 0 & 0 \\ 0 & 0 & 1^2 & 0 \\ 0 & 0 & 0 & 1^2 \end{vmatrix}$$

Pauli Matrices

$$|\sigma_1| = \begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix},\ |\sigma_2| = \begin{vmatrix} 0 & i \\ -i & 0 \end{vmatrix},\ |\sigma_3| = \begin{vmatrix} 1 & 0 \\ 0 & -1 \end{vmatrix}$$

$$\left|\sigma_3\right| = \begin{vmatrix} 1 & 0 \\ 0 & -1 \end{vmatrix} = \begin{vmatrix} -i^2 & 0 \\ 0 & i^2 \end{vmatrix} = \begin{vmatrix} -\left(\sqrt{-1}\right)^2 & 0 \\ 0 & \left(\sqrt{-1}\right)^2 \end{vmatrix}$$

$$Tr\left|\sigma_3\right| = 0 \leftrightarrow -\left(\sqrt{-1}\right)^2 = \left(\sqrt{-1}\right)^2 \leftrightarrow 1 = -1$$

Einsteins's "Train" Thought Experiment

Einstein's "thought experiment" is characterized by the model of two trains on straight parallel tracks (i.e., without curvature moving at different velocities, and the effect that the different velocities have on our concepts of space and time. Note that one of the trains can be thought of as "stationary" at the station, with the other train moving at velocity v; however, if the stationary velocity is that of the speed of light, then the model suggests that inertia means that the moving train must be moving slower, as suggested by the "Time Dilation" equation:

$$t' = t\Gamma,\ \Gamma := \frac{1}{\sqrt{1-\beta^2}},\ \beta := \frac{v}{c}$$

Note that mass is not explicitly referenced in this equation, since the parameters refer only to "velocity" v related to that of the velocity of light by the term $\beta := \frac{v}{c}$.

Coordinate Definition of Velocity

Newtonian Momentum and Energy

Note that this relation can be expressed as a global momentum relationship (classica/ Newtonl, for any mass m) $\beta := \frac{(m)v}{(m)c} = \frac{P(v)}{P(c)}$ in first order or as $\beta^2 := \frac{(m)v^2}{(m)c^2} = \frac{E(v)}{E(c)}$ as a global (classical/ Newton) Energy in second order.

.

The length relation then means that

$\# := (ct) + (vt')$ (the train lengths can be summed)' note that the product $L' := (ct)(vt') =$ is not defined.

It is worth noting that the relation above is only capable of derivation from the relation

$(ct')^2 = (ct)^2 + (vt')^2$ by solving for t', but the latter cannot be derived from the length relation:

$\left(ct'\right) = \left(ct\right) + \left(vt'\right)$ which relates the "relativistic" sum of the length of the two trains, and the "velocities" are defined by the relations: $x = ct \Leftrightarrow c = \frac{x}{t}$ and $x' = vt' \leftrightarrow \frac{x'}{t'} := v$.

That is, the relation $ct' = ct + vt'$ is not satisfied; the product $\frac{\left(x''\right)}{(vt)} = \left(ct'\right) \leftrightarrow x'' = (vt)(ct')$ means that the "coordinate" x'' scaled by the factor

$$
\begin{aligned}
&c = 4, t = 3 \\
&v = 5, t' = 6 \\
&ct = 12, vt' = 30 \\
&\# = \left(ct\right) + \left(vt'\right) = 12 + 30 = 42 \\
&ct' = (4)(6) = 30 \\
&\therefore \# \neq ct'
\end{aligned}
$$

This is then generalized to General Relativity which models the relation of mass